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The Locating Chromatic Number of Pentagonal Circular Ladder Graph PCLn

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Abstract

Locating coloring is a type of vertex coloring applied to connected graphs, where each vertex is assigned a color such that adjacent vertices receive different colors. In this setting, each color corresponds to a color class, which consists of all vertices assigned that color. A central notion in locating coloring is the color code of a vertex, determined by its distances to each color class. A coloring is classified as a locating coloring when every vertex in the graph has a unique color code. The locating chromatic number of a graph is the minimum number of colors needed to achieve such a coloring. The Pentagonal Circular Ladder graph is a structure formed by combining a circular graph with pentagonal components. This article examines the locating chromatic number of the Pentagonal Circular Ladder graph and provides an analysis of the behavior of locating colorings within this graph family.
Keywords: Locating chromatic number; Partition; Locating coloring; Color code; Pentagonal Circular Ladder Graph.

Abstrak

Pewarnaan lokasi merupakan jenis pewarnaan titik yang diterapkan pada graf terhubung, di mana setiap titik diberi warna sehingga titik-titik yang bertetangga tidak memiliki warna yang sama. Dalam konteks ini, setiap warna membentuk sebuah kelas warna yang terdiri atas seluruh titik yang diberi warna tersebut. Salah satu konsep utama dalam pewarnaan lokasi adalah kode warna suatu titik, yang ditentukan berdasarkan jaraknya terhadap setiap kelas warna. Suatu pewarnaan disebut pewarnaan lokasi apabila setiap titik dalam graf memiliki kode warna yang berbeda. Bilangan kromatik lokasi dari suatu graf didefinisikan sebagai jumlah minimum warna yang diperlukan untuk menghasilkan pewarnaan semacam ini. Graf Pentagonal Circular Ladder merupakan struktur graf yang dibentuk melalui penggabungan graf lingkaran dengan komponen-komponen pentagonal. Artikel ini mengkaji bilangan kromatik lokasi dari graf Pentagonal Circular Ladder serta memberikan analisis mengenai perilaku pewarnaan lokasi pada keluarga graf tersebut.
Kata Kunci: *Bilangan kromatik lokasi; Partisi; Pewarnaan lokasi; Kode warna; Graf Pentagonal Circular Ladder.*

2020MSC: 05C12, 05C15.

1. INTRODUCTION

In 2002, Chartrand et al. [1] introduced the locating chromatic number of a graph G, denoted by $\chi_L(G)$, for the first time. The locating chromatic number is the minimum number of colors required for a vertex coloring of a graph. The vertex coloring of a graph is an assignment of colors to all vertices such that any two adjacent vertices receive different colors.

Several studies have been conducted on the locating chromatic number, including both connected and disconnected graphs. For connected graphs, several works have been reported, such

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as [2], [3], and [4]. In 2021, Abdy et al. [5] determined the locating chromatic number of the dual of a wheel graph. Furthermore, Darmawahyuni and Narwen [6] obtained the locating chromatic number of a caterpillar graph. In Rahimah et al. [7], the locating chromatic number of the diamond graph was established for n = 3 and n = 4. Surbakti et al. [8] discussed the locating chromatic number of the pizza graph. In Welyyanti et al. [9], the locating chromatic number was found for a complete n-ary tree. In 2020, Kabang et al. [10] investigated the locating chromatic number of the shadow graph and the middle graph of a star. Moreover, [11] determined the locating chromatic number of a banana tree graph. Several other studies related to the locating chromatic number of connected graphs can be found in [12], [13], [14], [15], [16], and [17].

In disconnected graphs, several studies have been conducted. For instance, Welyyanti et al. [18] introduced a theory regarding the locating chromatic number of disconnected graphs. Furthermore, in 2020, Azhari et al. [19] investigated the locating chromatic number of disconnected graphs whose components are path graphs and double star graphs. Nurinsani et al. [20] determined the locating chromatic number of a disjoint union of palm graphs. Next, Welyyanti et al. [21] determined locating chromatic number of disconnected graph with path and cycle graph as its components. Several studies related to the chromatic number of connected graphs can be found in [22], [23], and [24].

In 2025, Ponraj et al. [25] determined the labeling of several graphs containing cycles. One of the graphs studied was the pentagonal circular ladder graph, denoted by PCL_n. The graph PCL_n is constructed from a cycle graph and a pentagonal graph. In addition, [1] established the locating chromatic number of the cycle graph C_n, namely $\chi_L(C_n) = 3$ when n is odd, and $\chi_L(C_n) = 4$ when n is even.

In this research, the locating chromatic number of the pentagonal circular ladder graph (PCL_n) with $n \geq 3$ will be established. A graph G is an ordered pair $(v(G), E(G))$ where $V(G)$ is a non-empty set whose elements are the vertices of the graph G. The set $E(G)$ consists of ordered pairs of distinct vertices of G, called edges.

Let $G = (V, E)$ be a connected graph and let c be a locating coloring on G where $c : V(G) \rightarrow \{1,2, \dots, k\}$ is a vertex coloring such that any two adjacent vertices receive different colors. Let S_i denote the set of vertices of G that are assigned color i, called the color class. Then, $\Pi = \{S_1, S_2, \dots, S_k\}$ is the collection of color classes of $V(G)$ [1].

The color code of a vertex v in $G\left(c_\pi(v)\right)$, is defined as the k-tuple $c_\Pi(v) = (d(v, S_1), d(v, S_2), \dots, d(v, S_k))$, where $d(v, S_i) = min\{d(v, x) | x \in S_i\}$ is the distance between the vertex v and the i-th color class. If every vertex in G has a distinct color code for a given Π, then c is called a k-locating coloring of G. The locating chromatic number of G is the minimum number of colors used in a k-locating coloring of G, denoted by $\chi_L(G)$ [1].

2. DEFINITIONS

The pentagonal circular ladder graph, denoted by PCL_n, is a graph constructed from a cycle graph and a pentagonal graph. The following is the definition of the pentagonal circular ladder graph (PCL_n).

$$V(PCL_n) = \{u_i | 1 \leq i \leq n\} \cup \{v_i | 1 \leq i \leq n\} \cup \{w_i | 1 \leq i \leq n\}$$

$$E(PCL_n) = \left\{\{(u_i u_{i+1}) | 1 \leq i \leq n-1\} \cup \{(u_n u_1)\}\right\} \cup \{u_i v_i | 1 \leq i \leq n\} \cup \{w_i v_i | 1 \leq i \leq n\} \cup \left\{\{(w_i v_{i+1}) | 1 \leq i \leq n-1\} \cup \{(w_n v_1)\}\right\}.$$

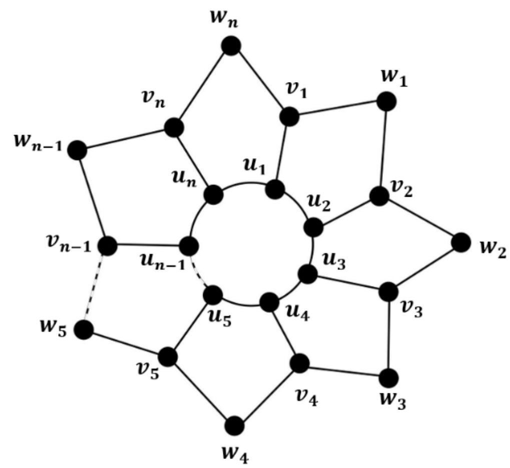

Figure 1. Pentagonal Circular Ladder Graph PCL_n

Figure 1 shows the Pentagonal circular ladder graph PCL_n. To prove the locating chromatic number of the pentagonal circular ladder graph, first, we determine the locating coloring of the pentagonal circular ladder graph. Next, determine the upper and lower bounds of the locating chromatic number of the pentagonal circular ladder graph, and obtain the exact value of the locating chromatic number of the pentagonal circular ladder graph.

3. RESULTS

Theorem 1. Let PCL_n be a pentagonal circular ladder graph for $n \geq 5$, then:

$$\chi_L(PCL_n) = \begin{cases} 4 & for\ 3 \leq n \leq 4, \\ 5 & for\ n \geq 5. \end{cases}$$

Proof.

The proof of the theorem is divided into three cases as follows.

Case 1. $\chi_L(PCL_n) = 4$ for $n = 3$

The upper bound of the locating chromatic number of the graph $PCL_3, \chi_L(PCL_3) \leq 4$ will be determined. Define a coloring $c: V(PCL_3) \rightarrow \{1,2,3,4\}$ such that PCL_3 has a 4-locating coloring as follows.

$$\begin{aligned} &c(u_1) = c(w_1) = 1, \\ &c(u_2) = c(v_1) = c(v_3) = 2, \\ &c(u_3) = c(w_2) = c(w_3) = 3, \\ &c(v_2) = 4. \end{aligned}$$

The partition class is $\Pi = \{S_1, S_2, S_3, S_4\}$ with

$$\begin{aligned} &S_1 = \{u_1, w_1\}, \\ &S_2 = \{u_2, v_1, v_3\}, \end{aligned}$$

$$S_3 = \{u_3, w_2, w_3\},$$
$$S_4 = \{v_2\}.$$

The color codes obtained at each vertex are as follows.

$$c_\Pi(u_1) = (0,1,1,2),$$
$$c_\Pi(u_2) = (1,0,1,1),$$
$$c_\Pi(u_3) = (1,1,0,2),$$
$$c_\Pi(v_1) = (1,0,1,2),$$
$$c_\Pi(v_2) = (1,1,1,0),$$
$$c_\Pi(v_3) = (2,0,1,2),$$
$$c_\Pi(w_1) = (0,1,2,1),$$
$$c_\Pi(w_2) = (2,1,0,1),$$
$$c_\Pi(w_3) = (2,1,0,3).$$

It can be seen that there are no identical color codes. Hence, the upper bound of the locating chromatic number of the graph PCL_3, $\chi_L(PCL_3) \leq 4$.
Next, the lower bound of the locating chromatic number of the graph PCL_3, $\chi_L(PCL_3) \geq 4$. Suppose that PCL_3 admits a 3-vertex coloring. Without loss of generality, there exist two dominant vertices with the same color, namely vertices v_1 dan v_2. This means that the two vertices have the same color code. This contradicts the definition of the locating chromatic number, which states that every vertex in PCL_3 must have a distinct color code. Therefore, the lower bound of the locating chromatic number of the graph PCL_3, $\chi_L(PCL_3) \geq 4$. Jadi PCL_3, $\chi_L(PCL_3) = 4$.

Case 2. $\chi_L(PCL_n) = 4$ for $n = 4$

The upper bound of the locating chromatic number of the graph PCL_4, $\chi_L(PCL_4) \leq 4$ will be determined. Define a coloring $c: V(PCL_4) \rightarrow \{1,2,3,4\}$ such that PCL_4 has a 4-locating coloring as follows.

$$c(u_1) = c(v_4) = c(w_1) = 1,$$
$$c(u_2) = c(u_4) = c(v_1) = c(v_3) = 2,$$
$$c(u_3) = c(w_2) = c(w_3) = 3,$$
$$c(v_2) = c(w_4).$$

The partition class is $\Pi = \{S_1, S_2, S_3, S_4\}$ with

$$S_1 = \{u_1, v_{2,} v_4, w_1\},$$
$$S_2 = \{u_2, u_4, v_1, v_3\},$$
$$S_3 = \{u_3, w_2, w_3\},$$
$$S_4 = \{v_2, w_4\}.$$

The color codes obtained at each vertex are as follows.

$$c_\Pi(u_1) = (0,1,2,2),$$
$$c_\Pi(u_2) = (1,0,1,1),$$
$$c_\Pi(u_3) = (2,1,0,2),$$
$$c_\Pi(u_4) = (1,0,1,2),$$

$$
\begin{aligned}
c_\Pi(v_1) &= (1,0,3,1),\\
c_\Pi(v_2) &= (1,1,1,0),\\
c_\Pi(v_3) &= (2,0,1,2),\\
c_\Pi(v_4) &= (0,1,1,1),\\
c_\Pi(w_1) &= (0,1,2,1),\\
c_\Pi(w_2) &= (2,1,0,1),\\
c_\Pi(w_3) &= (1,1,0,2),\\
c_\Pi(w_4) &= (1,1,2,0).
\end{aligned}
$$

By preserving the same coloring pattern of the pentagonal circular ladder graph PCL_n for $n = 3$, and adding three new vertices with the coloring $c(u_4) = 2$, $c(v_4) = 1$ and $c(w_4) = 4$, it can be seen that no two vertices share the same color code. Therefore, the upper bound of the locating chromatic number of the graph PCL_4, $\chi_L(PCL_4) \leq 4$.

Next, the lower bound of the locating chromatic number of the graph PCL_4, $\chi_L(PCL_4) \geq 4$. Suppose that PCL_4 admits a 3-coloring of its vertices. Without loss of generality, there are two dominant vertices with the same color, namely v_4 dan w_1. This means that these two vertices have the same color code. This contradicts the definition of the locating chromatic number, which requires that every vertex in PCL_4 must have a distinct color code. Consequently, it must be that $\chi_L(PCL_4) \geq 4$. Hence, $\chi_L(PCL_4) = 4$.

Case 3. $\chi_L(PCL_n) = 5$ for $n \geq 5$.

The upper bound of the locating chromatic number of the graph PCL_n for $n \geq 5$, $\chi_L(PCL_n) \leq 5$ will be determined. Define a coloring $c: V(PCL_n) \rightarrow \{1,2,3,4,5\}$ such that PCL_n for $n \geq 5$ has a 5-locating coloring as follows.

$$
\begin{aligned}
&c(u_1) = 1,\\
&c(u_i) = \begin{cases} 2, & \text{for even } i,\\ 3, & \text{for odd } i, i \geq 3. \end{cases}\\
&c(v_2) = 4,\\
&c(v_4) = 1,\\
&c(v_i) = \begin{cases} 2, & \text{for odd } i,\\ 3, & \text{for even } i, i \geq 6. \end{cases}\\
&c(w_1) = 1,\\
&c(w_2) = c(w_3) = 3,\\
&c(w_4) = 4,\\
&c(w_i) = 5, \qquad \text{for } i \geq 5.
\end{aligned}
$$

The partition class is $\Pi = \{S_1, S_2, S_3, S_4, S_5\}$ with

$$
\begin{aligned}
&S_1 = \{u_1, u_4, w_1\},\\
&S_2 = \{u_i | \text{for even } i\} \cup \{v_i | \text{for odd } i\},\\
&S_3 = \{u_i | i \geq 3, \text{for odd } i\} \cup \{v_i | i \geq 6, \text{for even } i\} \cup \{w_2, w_3\},\\
&S_4 = \{v_2\} \cup \{w_4\},\\
&S_5 = \{w_i | i \geq 5\}.
\end{aligned}
$$

From the vertex colorings described above, the distance of each vertex in the pentagonal circular ladder graph PCL_n to the color classes warna S_1, S_2, S_3, S_4 and S_5 is obtained as follows.

Subcases 3.1. For n odd, $1 \leq i \leq n$.

$$d(u_i|S_1) = \begin{cases} i-1, & 1 \leq i \leq 3, \\ i-3, & 4 \leq i \leq \left\lceil \frac{n}{2} \right\rceil + 1, \\ n-(i-1), & \left\lceil \frac{n}{2} \right\rceil + 2 \leq i \leq n. \end{cases}$$

$$d(u_i|S_2) = \begin{cases} 0, & \text{for even } i \\ 1, & \text{for odd } i \end{cases}$$

$$d(u_i|S_3) = \begin{cases} 0, & \text{for odd } i \text{ without } 1, \\ 1, & i = 1 \text{ or for even } i. \end{cases}$$

$$d(u_i|S_4) = \begin{cases} 1, & i = 2 \\ 2, & i = 1,3,4,5, \\ i-3, & 6 \leq i \leq \left\lceil \frac{n}{2} \right\rceil + 2, \\ n-(i-3), & \left\lceil \frac{n}{2} \right\rceil + 3 \leq i \leq n. \end{cases}$$

$$d(u_i|S_5) = \begin{cases} 2, & i = 3 \text{ or } 5 \leq i \leq n, \\ 3, & i = 2,4, \\ 4, & i = 3. \end{cases}$$

$$d(v_i|S_1) = \begin{cases} 0, & i = 4, \\ 1, & 1 \leq i \leq 2, \\ 2, & i = 3,5, \\ i-2, & 6 \leq i \leq \left\lceil \frac{n}{2} \right\rceil + 1, \\ n-(i-2), & \left\lceil \frac{n}{2} \right\rceil + 2 \leq i \leq n. \end{cases}$$

$$d(v_i|S_2) = \begin{cases} 0, & \text{for odd } i \\ 1, & \text{for even } i \end{cases}$$

$$d(v_i|S_3) = \begin{cases} 0, & \text{for even } i, i \geq 6, \\ 1, & 2 \leq i \leq 5, \\ 2, & i = 1. \end{cases}$$

$$d(v_i|S_4) = \begin{cases} 0, & i = 2, \\ 1, & 4 \leq i \leq 2, \\ 2, & i = 1,3, \\ 3, & i = 6, \\ i-2, & 7 \leq i \leq \left\lceil \frac{n}{2} \right\rceil + 2, \\ n-(i-4), & \left\lceil \frac{n}{2} \right\rceil + 3 \leq i \leq n. \end{cases}$$

$$d(v_i|S_5) = \begin{cases} 1, & i = 1 \; or \; 5 \leq i \leq n, \\ 3, & i = 2,4, \\ 5, & i = 3. \end{cases}$$

$$d(w_i|S_1) = \begin{cases} 0, & i = 1, \\ 1, & 3 \le i \le 4, \\ 2, & i = 2, \\ 3, & i = 5, \\ i-1, & 6 \le i \le \left\lceil \frac{n}{2} \right\rceil + 1, \\ n-(i-2), & \left\lceil \frac{n}{2} \right\rceil + 2 \le i \le n. \end{cases}$$

$$d(w_i|S_1) = 1,\ 1 \le i \le n.$$

$$d(w_i|S_3) = \begin{cases} 0, & 2 \le i \le 3, \\ 1, & 5 \le i \le n-1, \\ 2, & i = 1,4\ or\ i = n. \end{cases}$$

$$d(w_i|S_4) = \begin{cases} 0, & i = 4, \\ 1, & 1 \le i \le 2, \\ 2, & i = 3, 5, \\ 4, & i = 6, \\ i-1, & 7 \le i \le \left\lceil \frac{n}{2} \right\rceil + 2, \\ n-(i-4), & \left\lceil \frac{n}{2} \right\rceil + 3 \le i \le n-1, \\ 3, & i = n. \end{cases}$$

$$d(w_i|S_5) = \begin{cases} 0, & 5 \le i \le n, \\ 2, & i = 1,4, \\ 4, & i = 2,3. \end{cases}$$

Subcases 3.2. For n even, $1 \le i \le n$.

$$d(u_i|S_1) = \begin{cases} i-1, & 1 \le i \le 3, \\ i-3, & 4 \le i \le \left\lceil \frac{n}{2} \right\rceil + 2, \\ n-(i-1), & \left\lceil \frac{n}{2} \right\rceil + 3 \le i \le n. \end{cases}$$

$$d(u_i|S_2) = \begin{cases} 0, & \text{for even } i, \\ 1, & \text{for odd } i. \end{cases}$$

$$d(u_i|S_3) = \begin{cases} 0, & \text{for odd } i\ without\ 1, \\ 1, & \text{for even } i, \\ 2, & i = 1. \end{cases}$$

$$d(u_i|S_4) = \begin{cases} 1, & i = 2 \\ 2, & i = 1,3,4,5, \\ i-3, & 6 \le i \le \left\lceil \frac{n}{2} \right\rceil + 2, \\ n-(i-3), & \left\lceil \frac{n}{2} \right\rceil + 3 \le i \le n. \end{cases}$$

$$d(u_i|S_5) = \begin{cases} 2, & i = 3 \text{ or } 5 \le i \le n, \\ 3, & i = 2,4, \\ 4, & i = 3. \end{cases}$$

$$d(v_i|S_1) = \begin{cases} 0, & i = 4, \\ 1, & 1 \le i \le 2 \\ 2, & i = 3,5, \\ i-2, & 6 \le i \le \left\lceil \frac{n}{2} \right\rceil + 2, \\ n-(i-2), & \left\lceil \frac{n}{2} \right\rceil + 3 \le i \le n. \end{cases}$$

$$d(v_i|S_2) = \begin{cases} 0, & \text{for odd } i \\ 1, & \text{for even } i \end{cases}$$

$$d(v_i|S_3) = \begin{cases} 0, & \text{for even } i, i \ge 6, \\ 1, & 2 \le i \le 5, \\ 2, & i = 1. \end{cases}$$

$$d(v_i|S_4) = \begin{cases} 0, & i = 2, \\ 1, & 4 \le i \le 2, \\ 2, & i = 1,3, \\ 3, & i = 6, \\ i-2, & 7 \le i \le \left\lceil \frac{n}{2} \right\rceil + 3, \\ n-(i-4), & \left\lceil \frac{n}{2} \right\rceil + 4 \le i \le n. \end{cases}$$

$$d(v_i|S_5) = \begin{cases} 1, & i = 1 \; or \; 5 \le i \le n, \\ 3, & i = 2,4, \\ 5, & i = 3. \end{cases}$$

$$d(w_i|S_1) = \begin{cases} 0, & i = 1, \\ 1, & 3 \le i \le 4, \\ 2, & i = 2, \\ 3, & i = 5, \\ i-1, & 6 \le i \le \left\lceil \frac{n}{2} \right\rceil + 1 \\ n-(i-2), & \left\lceil \frac{n}{2} \right\rceil + 2 \le i \le n. \end{cases}$$

$$d(w_i|S_1) = 1, \;\; 1 \le i \le n.$$

$$d(w_i|S_3) = \begin{cases} 0, & 2 \le i \le 3, \\ 1, & 5 \le i \le n, \\ 2, & i = 1,4. \end{cases}$$

$$d(w_i|S_4) = \begin{cases} 0, & i = 4, \\ 1, & 1 \le i \le 2, \\ 2, & i = 3,5, \\ 4, & i = 6, \\ i-1, & 7 \le i \le \left\lceil \frac{n}{2} \right\rceil + 2, \\ n-(i-4), & \left\lceil \frac{n}{2} \right\rceil + 3 \le i \le n-1, \\ 3, & i = n. \end{cases}$$

$$d(w_i|S_5) = \begin{cases} 0, & 5 \le i \le n, \\ 2, & i = 1,4, \\ 4, & i = 2,3. \end{cases}$$

Each color code of the vertices u_i, v_i, and w_i with $1 \le i \le n$ in the graph PCL_n is distinguished by the distance of the vertices in PCL_n to the color classes S_1, S_2, S_3, S_4, and S_5, so that the obtained color codes are distinct. Therefore, the upper bound of the locating chromatic number of the graph PCL_n is $\chi_L(PCL_n) \le 5$ for $n \ge 5$.

Next, the lower bound of the locating chromatic number of the graph PCL_n for $n \ge 5$, will be determined, namely $\chi_L(PCL_n) \ge 5$. Suppose that PCL_n for $n \ge 5$ admits a 4-coloring of its vertices. Without loss of generality, there exist two dominant vertices with the same color when $n \ge 5$, namely v_4 and v_6. This means that these two vertices have the same color code. This contradicts the definition of the locating chromatic number, which states that every vertex in PCL_n for $n \ge 5$ must have a distinct color code. Therefore, the lower bound of the locating chromatic number of the graph PCL_n for $n \ge 5$, is $\chi_L(PCL_n) \ge 5$. Therefore, we conclude that the locating chromatic number of the graph PCL_n for $n \ge 5$, is $\chi_L(PCL_n) = 5$. ■

4. DISCUSSIONS

In analyzing the locating chromatic number of the Pentagonal Circular Ladder graph PCL_n, several structural characteristics of the graph become apparent. The interaction between the pentagonal components and the circular framework results in a graph with repetitive yet nontrivial connectivity patterns, which directly influence the uniqueness of color codes in locating colorings. Similar structural influences have been observed in other families of graphs containing cycles or ladder-like constructions, such as chain graphs [14] and diamond graphs [7], where additional complexity in adjacency relations often increases the number of colors required for a valid locating coloring.

Furthermore, the results obtained in this study align with previous findings regarding locating colorings on composite or multi-layer graph structures. Studies on pizza graphs [8] and complete n-ary trees [9] indicate that when a graph exhibits both regularity and branching substructures, the locating chromatic number tends to increase as a means of maintaining unique color codes across vertices with similar distances to color classes. This phenomenon is also observed in the Pentagonal Circular Ladder graph, where the recurrence of pentagonal units requires careful partitioning of color classes to prevent duplication of color codes.

Additionally, the behavior of dominant vertices within PCL_n plays a significant role in determining both the upper and lower bounds of its locating chromatic number. The presence of repeated patterns of dominant vertices reflects characteristics found in studies of amalgamated graphs [13] and one-heart graphs [12], where dominant vertices frequently restrict the possibility of employing fewer colors. Consequently, the identification of dominant vertex pairs in PCL_n provides essential insight into establishing sharp bounds for its locating chromatic number, as verified in this work for cases $n = 1, 2$ and $n \ge 3$.

5. CONCLUSIONS

This work establishes a complete characterization of the locating chromatic number of the Pentagonal Circular Ladder graph PCL_n, demonstrating that $\chi_L(PCL_n) = 4$ for $n = 1$ and $n = 2$, and increases to $\chi_L(PCL_n) = 5$ for all $n \geq 3$. The increase in required colors is attributable to the heightened structural symmetry and repetitive pentagonal layering inherent in larger graphs, which constrain the distinctiveness of vertex color codes. These findings enhance theoretical understanding of locating colorings in structured composite graphs and provide a foundational basis for future extensions involving generalized ladder constructions and related location-based graph parameters.

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