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The Locating Chromatic Number of Zigzag Graph Z_n

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Abstract

The locating chromatic number is a concept developed from vertex coloring and the partition dimension of a graph, first studied by Chartrand et al. (2002). A connected graph G is said to have a locating coloring when each vertex is assigned a color such that the resulting color code defined by its distances to every color class is unique. The minimum number of colors that satisfies this condition is known as the locating chromatic number, denoted by $\chi_L(G)$. This study investigates the value of χ_L for the zigzag graph Z_n with $n \geq 3$. Although colorings have been studied for various families of graphs, no explicit characterization of zigzag graphs has been established. Our analysis shows that Z_3 has a locating chromatic number of 3, while for all $n \geq 4$, the value increases to 4. These results provide the first complete characterization of locating colorings on zigzag graphs and contribute to the broader study of location-based parameters in graphs with structured topology.

Keywords: Locating chromatic number; Zigzag graph; Color code.

Abstrak

Bilangan kromatik lokasi merupakan konsep pengembangan dari pewarnaan titik dan dimensi partisi suatu graf yang pertama kali dikaji oleh Chartrand dkk (2002). Sebuah graf terhubung G dikatakan memiliki pewarnaan lokasi apabila setiap titik diberi warna sedemikian rupa sehingga kode warna yang dibentuk berdasarkan jaraknya terhadap setiap kelas warna bersifat unik. Banyaknya warna minimum yang memenuhi kondisi tersebut disebut bilangan kromatik lokasi, dilambangkan dengan $\chi_L(G)$. Penelitian ini mengkaji nilai χ_L pada graf zig-zag Z_n untuk $n \geq 3$. Walaupun sejumlah keluarga graf telah diteliti sebelumnya dalam konteks pewarnaan lokasi, graf zig-zag belum pernah memperoleh karakterisasi yang jelas. Hasil analisis menunjukkan bahwa Z_3 memiliki bilangan kromatik lokasi adalah 3, sedangkan untuk semua $n \geq 4$, nilai tersebut menjadi 4. Temuan ini memberikan karakterisasi lengkap pertama untuk pewarnaan lokasi pada graf zig-zag dan memperkaya kajian mengenai parameter lokasi pada graf dengan struktur khusus.

Kata Kunci: *Bilangan kromatik lokasi; Graf zig-zag; Kode warna.*

2020MSC: 05C12, 05C15.

1. INTRODUCTION

Graph theory is a branch of mathematics that studies the relationships between objects called vertices and edges. Rahayuningsih [1] explains graph theory and its applications. Circulant graphs have been widely studied and have numerous significant applications, particularly in multicomputer networks and distributed computation [2][3].

A graph G is defined as an ordered pair of sets (V, E), denoted by $G = (V, E)$, where $V(G)$ is a non-empty set of vertices with $V = \{v_1, v_2, \ldots, v_n\}$ and $E(G)$ is the set of edges that connect two vertices in G, called $E = \{e_1, e_2, \ldots, e_m\}$. The locating chromatic number was first introduced by

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Chartrand et al, in 2002 [4]. The locating chromatic number of a graph G is the minimum number of colors required to color the vertices of the graph such that every vertex has a distinct color code, denoted by $\chi_L(G)$. Chartrand, et al. [4] determined the locating chromatic number of the cycle graph C_n, where $\chi_L(C_n) = 3$ for odd n and $\chi_L(C_n) = 4$ for even n. Chartrand et al. [4] also found that the locating chromatic number of the path graph P_n for $n \geq 3$ is $\chi_L(P_n) = 3$. Chartrand et al, in 2003 [5] also characterized all graphs of order n with the locating chomatic number $n-1$.

The locating chromatic number has been widely studied, including in connected and disconnected graphs. In [6], [7], [8], [9] the locating chromatic number for disconnected graphs is considered. Welyyanti et al. [10] determined the locating chromatic number for graphs with dominant vertices. Asmiati et al. [11] determined the locating chromatic number of firecracker graphs. Asmiati et al. [12] characterizing all graphs containing cycles with locating chromatic number 3. Apriliza et al. [13] determined the locating chromatic number for lobster graphs. Suryaningsih and Baskoro [14] determined the locating chromatic number for the fibonacene graphs. Zulkarnain et al. [15] determined on the locating chromatic number of the disjoint union of buckminsterfullerene graphs. Hartiansyah and Darmaji [16] determined the locating chromatic number for the amalgamation of edges from stars and complete graphs. Yulianti et al. [17] determined the locating chromatic number for amalgamation of some complete graphs.

Several studies on the location of chromatic numbers have been conducted by previous researchers. The locating chromatic number of the book graph [18]. The locating chromatic number of some jellyfish graphs [19]. The locating chromatic number of powers of paths and cycles [20]. On locating chromatic number of complete n-ary tree [21]. The locating chromatic number of origami graphs [22]. On the locating chromatic number of certain barbell graphs [23]. The locating chromatic number for pizza graphs [24]. On some Petersen graphs having chromatic number four or five [25].

The zigzag graph, denoted Z_n, is a connected graph consisting of a cycle graph C_n in which a path graph P_n is embedded, forming a zigzag pattern. Subsequently, a new graph is constructed, namely the zigzag graph Z_n. In this article, we will determine the locating chromatic number of zigzag graph Z_n, for $n \geq 3$.

2. DEFINITION

The zigzag graph (Z_n) is a connected graph that has a vertex set and an edge set defined as follows:

$V(Z_n) = \{v_i \mid 1 \leq i \leq n\}$,
$E(Z_n) = \{v_i v_{i+1}, v_n v_1 \mid 1 \leq i \leq n-1\} \cup \{v_i v_{n-i}, v_{j+1} v_{n-j} \mid 1 \leq i \leq \lfloor (n-2)/2 \rfloor$ and $1 \leq j \leq \lceil (n-4)/2 \rceil\}$.

Figure 1 shows the zigzag graph Z_n. To obtain the locating chromatic number of zigzag graph, first, we are determining the locating coloring on the zigzag graph. Furthermore, determining the upper and lower bound of the locating chromatic number of the zigzag graph, and obtaining the exact value of the locating chromatic number of the zigzag graph Z_n.

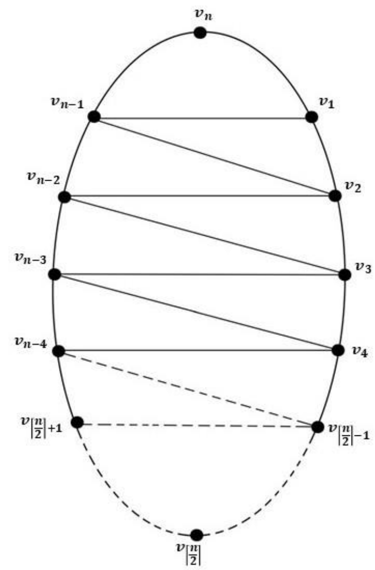

Figure 1. Zigzag graph Z_n

3. RESULTS

In this section, we determine the locating chromatic number of zigzag graph Z_n. Then, we provide the lower and upper bounds of the locating chromatic number of Z_n.

Theorem 1. Let Z_n be a graph zigzag with $n \geq 3$, then

$$\chi_L(Z_n) = \begin{cases} 3, & if\ n = 3, \\ 4, & if\ n \geq 4. \end{cases}$$

Proof.

Proof of the locating chromatic number of the zigzag graph will be divided into two cases as follows.

Case 1. $\chi_L(Z_3) = 3$.

We will find the upper bound of the locating chromatic number of the zigzag graph Z_3. Define $c : V(Z_3) \rightarrow \{1, 2, 3\}$ such that:

$$c(v_1) = 1,$$
$$c(v_2) = 3,$$
$$c(v_3) = 2.$$

And partition class $\Pi = \{Q_1, Q_2, Q_3\}$ with

$$Q_1 = \{v_1\},$$
$$Q_2 = \{v_3\},$$
$$Q_3 = \{v_2\}.$$

The color code of vertices in Z_3 graph are:

$$
\begin{aligned}
c_\Pi(v_1) &= (0,1,1), \\
c_\Pi(v_2) &= (1,0,1), \\
c_\Pi(v_3) &= (1,1,0).
\end{aligned}
$$

Because every vertex on the graph has a different color code, so, upper bound of $\chi_L(Z_3) \leq 3$. Next, we will determine the lower bound of the location chromatic number of graph Z_3, namely $\chi_L(Z_3) \geq 3$. If Z_3 has a $2-$location coloring, then there will be two dominant vertices with the same color and two adjacent vertices. $c(v_1) = c(v_3) = 1$, thus obtaining $c_\Pi(v_1) = c_\Pi(v_2) = (0,1)$. Consequently, $\chi_L(Z_3) \geq 3$. Therefore, the location chromatic number of graph Z_3 is $\chi_L(Z_3) = 3$.

Case 2. $\chi_L(Z_n) = 4$ for $n \geq 4$.

We will find the upper bound of the locating chromatic number of the zigzag graph Z_n for $n \geq 4$. Define $c : V(Z_3) \to \{1,2,3,4\}$ such that:

1. For $n = 4$,

$$
\begin{aligned}
c(v_1) &= 1 \\
c(v_2) &= 4 \\
c(v_3) &= 3 \\
c(v_4) &= 2
\end{aligned}
$$

2. For $n \geq 5$,

$$
c(v_i) = \begin{cases}
1, \ i = 3a-2, a = 1, for \ \ 4 \leq n \leq 8 \\
1, \ i = 3a-2, a = 2 + \left\lfloor \frac{n-9}{6} \right\rfloor, for \ \ n \geq 9 \\
1, \ i = n-(3a-1), a = \left\lfloor \frac{n}{6} \right\rfloor, for \ \ n \geq 6 \\
2, \ i = n, i = 3a-1, a = \left\lfloor \frac{n+1}{6} \right\rfloor, for \ \ n \geq 5 \\
2, \ i = n-3a, a = \left\lfloor \frac{n-2}{6} \right\rfloor, for \ \ n \geq 8 \\
3, \ \ i = 3a, a = \left\lfloor \frac{n-1}{6} \right\rfloor, for \ \ n \geq 7 \\
3, \ i = n-(3a-2), a = \left\lfloor \frac{n+2}{6} \right\rfloor, for \ \ n \geq 4 \\
4, \ i = \left\lceil \frac{n}{2} \right\rceil.
\end{cases}
$$

From the description of the vertex coloring above, the color codes of the zigzag graph Z_n with respect to the partition $\Pi = \{Q_1, Q_2, Q_3, Q_4\}$ are obtained as follows.

1. For $n = 4$,

$$
\begin{aligned}
c_\Pi(v_1) &= (0,1,1,1) \\
c_\Pi(v_2) &= (1,2,1,0) \\
c_\Pi(v_3) &= (1,0,1,1) \\
c_\Pi(v_4) &= (1,0,1,2).
\end{aligned}
$$

2. For $\left\lceil \frac{n}{2} \right\rceil$, $n \geq 4$, n even and odd

$$d\left(v_{\left\lceil\frac{n}{2}\right\rceil}, Q_1\right) = \begin{cases} 1 & , n = 3a+1, a \geq 1 \\ 1 & , n = 3a+3, a \geq 1 \\ 2 & , n = 3a+2, a \geq 1 \end{cases}$$

$$d\left(v_{\left\lceil\frac{n}{2}\right\rceil}, Q_2\right) = \begin{cases} 2 & , n = 3a+1, a \geq 1 \\ 1 & , n = 3a+3, a \geq 1 \\ 1 & , n = 3a+2, a \geq 1 \end{cases}$$

$$d\left(v_{\left\lceil\frac{n}{2}\right\rceil}, Q_3\right) = \begin{cases} 1 & , n = 3a+1, a \geq 1 \\ 2 & , n = 3a+3, a \geq 1 \\ 1 & , n = 3a+2, a \geq 1 \end{cases}$$

$$d\left(v_{\left\lceil\frac{n}{2}\right\rceil}, Q_4\right) = 0.$$

3. For $n \geq 4$, n even and odd

$$d(v_i, Q_1) = \begin{cases} 0, & i = 3a-2, a = 1, for\ 4 \leq n \leq 8 \\ 0, & i = 3a-2, a = 2 + \left\lfloor\frac{n-9}{6}\right\rfloor, for\ n \geq 9 \\ 0, & i = n-(3a-1), a = \left\lfloor\frac{n}{6}\right\rfloor, for\ n \geq 6 \\ 1, & i = 3a-1, a = \left\lfloor\frac{n+1}{6}\right\rfloor, for\ n \geq 5 \\ 1, & i = n-3a, a = \left\lfloor\frac{n+2}{6}\right\rfloor, for\ n \geq 4 \\ 1, & i = 3a, a = \left\lfloor\frac{n-1}{6}\right\rfloor, for\ n \geq 7 \\ 1, & i = n-(3a-2), a = 1, for\ \ 4 \leq n \leq 7 \\ 1, & i = n-(3a-2), a = 2 + \left\lfloor\frac{n-8}{6}\right\rfloor, for\ \ n \geq 7 \end{cases}$$

$$d(v_i, Q_2) = \begin{cases} 1, & i = 3a-2, a = 1, for\ 4 \leq n \leq 8 \\ 1, & i = 3a-2, a = 2 + \left\lfloor\frac{n-9}{6}\right\rfloor, for\ n \geq 9 \\ 1, & i = n-(3a-1), a = \left\lfloor\frac{n}{6}\right\rfloor, for\ n \geq 5 \\ 0, & i = 3a-1, a = \left\lfloor\frac{n+1}{6}\right\rfloor, for\ n \geq 5 \\ 0, & i = n-3a, a = 1, untuk\ 4 \leq n \leq 7 \\ 0, & i = n-3a, a = 2 + \left\lfloor\frac{n-8}{6}\right\rfloor, for\ n \geq 8 \\ 1, & i = 3a, a = \left\lfloor\frac{n-1}{6}\right\rfloor, for\ n \geq 7 \\ 1, & i = n-(3a-2), a = \left\lfloor\frac{n+2}{6}\right\rfloor, for\ n \geq 4. \end{cases}$$

$$d(v_i, Q_3) = \begin{cases} 1,\ i = 3a-2, a = 1, for\ 4 \le n \le 8 \\ 1,\ i = 3a-2, a = 2 + \left\lfloor \frac{n-9}{6} \right\rfloor, for\ n \ge 9 \\ 1, i = n-(3a-1), a = \left\lfloor \frac{n}{6} \right\rfloor, for\ n \ge 6 \\ 1,\ i = 3a-1, a = \left\lfloor \frac{n+1}{6} \right\rfloor, for\ n \ge 5 \\ 1,\ i = n-3a, a = 1, for\ 4 \le n \le 7 \\ 1,\ i = n-3a, a = 2 + \left\lfloor \frac{n-8}{6} \right\rfloor, for\ n \ge 8 \\ 0,\ i = 3a, a = \left\lfloor \frac{n-1}{6} \right\rfloor, for\ n \ge 7 \\ 0,\ i = n-(3a-2), a = \left\lfloor \frac{n+1}{6} \right\rfloor, for\ n \ge 5 \end{cases}$$

4. For $n \ge 6, n$ even

$$d(v_i, Q_4) = \begin{cases} n - \left(i + \left\lceil \frac{n}{2} \right\rceil\right),\ 1 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1, \\ i - \left\lceil \frac{n}{2} \right\rceil,\ \left\lceil \frac{n}{2} \right\rceil \le i \le n, \\ 0,\ i = \left\lceil \frac{n}{2} \right\rceil. \end{cases}$$

5. For $n \ge 6, n$ odd

$$d(v_i, Q_4) = \begin{cases} n - \left(i + \left\lceil \frac{n}{2} \right\rceil - 1\right),\ 1 \le i \le \left\lceil \frac{n}{2} \right\rceil - 1, \\ i - \left\lceil \frac{n}{2} \right\rceil, \left\lceil \frac{n}{2} \right\rceil \le i \le n, \\ 0,\ i = \left\lceil \frac{n}{2} \right\rceil. \end{cases}$$

Because every vertex on the graph has a different color code, so, upper bound of $\chi_L(Z_n) \le 4$, for $n \ge 4$.

Next, we will determine the lower bound of the location chromatic number of the graph Z_n for $n \ge 4$, namely $\chi_L(Z_n) \ge 4$. Suppose Z_n for $n \ge 4$ has a $3-$location coloring. Without loss of generality, there will be two dominant vertices with the same color, namely $c(v_2) = c(v_4) = 2$, thus obtaining $c_\Pi(v_2) = c_\Pi(v_4) = (1, 0, 1)$. Consequently, v_2 and v_4 are two dominant vertices with the same color. This shows that a $3-$location coloring is not possible. Consequently, $\chi_L(Z_n) \ge 4$ for $n \ge 4$. Therefore, the location chromatic number of the graph Z_n for $n \ge 4$ is $\chi_L(Z_n) = 4$. ∎

4. DISCUSSIONS

The results obtained in this study indicate that the structural properties of zigzag graphs exhibit similarities to other graph families that contain cycles and paths as subgraphs, such as those investigated by Asmiati et al. [11][12] and Apriliza et al. [13]. The main distinction lies in the interconnected pattern of vertices in zigzag graphs, which form a serrated structure and require more meticulous analysis of distances to determine each vertex's color code. Therefore, the findings that the locating chromatic number equals 3 for $n = 3$ and equals 4 for all $n \ge 4$ 4 are consistent with the general trend that more complex graph structures tend to require a larger locating chromatic number.

The determination of both the lower and upper bounds of the locating chromatic number of zigzag graphs also highlights a strong relationship with the concept of dominant vertices, as studied by Welyyanti et al. [10]. The presence of two dominant vertices in the construction of zigzag graphs necessitates stricter color differentiation to avoid identical color codes. Furthermore, comparison with other layered or chain-like graphs, such as firecracker graphs [11] and fibonacene graphs [14], indicates that more irregular connection patterns often allow greater flexibility in vertex coloring, while structured patterns like those in zigzag graphs generally limit the range of feasible color configurations.

The findings of this study also open opportunities for future investigations, particularly regarding how slight modifications to the structure of zigzag graphs may influence their locating chromatic number. For instance, if additional paths are attached to specific vertices as in pizza graphs [24], or if certain vertices are amalgamated as examined by Hartiansyah and Darmaji [16], significant changes in the locating chromatic number may arise. Moreover, similar approaches may be applied to extended zigzag graphs in higher dimensions or to irregular zigzag graphs, thereby broadening the scope of research on location-based graph parameters.

5. CONCLUSIONS

In this study, we determined the locating chromatic number of the zigzag graph G_z for all $n \geq 3$. By analyzing both the upper and lower bounds through explicit locating colorings, we established that the zigzag graph exhibits two distinct behaviors depending on its order. Specifically, we proved that the locating chromatic number is equal to 3 when $n = 3$, whereas for all $n \geq 4$, the locating chromatic number increases to 4. These results provide the first complete characterization of locating colorings on zigzag graphs. The structural arrangement of alternating cycle–path components in zigzag graphs contributes significantly to the uniqueness of color codes, especially for larger values of n, where dominant vertices impose stricter requirements on feasible color assignments. Overall, the findings of this work not only enrich the study of locating chromatic numbers on graphs with structured topologies but also create opportunities for further extensions, such as exploring generalized zigzag graphs, analyzing structural modifications, or determining relationships with other location-based graph parameters.

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