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HOW THE ASIAN AND EUROPEAN COMMUNITIES ACCEPT NEGATIVE NUMBERS

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Abstract

This research is based on several inquiries from mathematics educators on the origins of negative numbers and their acceptance in the community. This study used library research, or more precisely, bibliographical studies, as a method. The research examined how the Asian and European communities accept negative numbers. The study's findings indicated that the Asian and European communities accept negative numbers differently. In comparison to the Asian population, the European populace takes a long time to accept negative figures. It resulted in numerous contrasts between the Asian and European Communities, one of which is in terms of religiosity. This study established that religiosity has an effect on the Asian and European communities' acceptance of negative numbers.

Keywords: history of numbers; negative numbers; religiosity

Abstrak

Penelitian ini didasarkan pada beberapa pertanyaan dari pendidik matematika tentang asal-usul bilangan negatif dan penerimaannya di masyarakat. Penelitian ini menggunakan metode penelitian kepustakaan atau lebih tepatnya studi kepustakaan. Penelitian tersebut mengkaji bagaimana masyarakat Asia dan Eropa menerima angka negatif. Temuan studi menunjukkan bahwa komunitas Asia dan Eropa menerima angka negatif secara berbeda. Dibandingkan dengan penduduk Asia, penduduk Eropa membutuhkan waktu lama untuk menerima angka negatif. Hal ini mengakibatkan banyak perbedaan antara Komunitas Asia dan Eropa, salah satunya dalam hal religiositas. Studi ini menetapkan bahwa religiusitas berpengaruh pada penerimaan masyarakat Asia dan Eropa terhadap angka negatif.

***Kata kunci**: sejarah bilangan; angka negatif; religiusitas*

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Introduction

Calculating ability is a critical foundational skill in the learning process, particularly in the field of mathematics. Without a strong calculating aptitude, it is difficult for a person to comprehend the items that follow. Even if a person comprehends the topic, he or she may provide an inaccurate answer if they lack adequate calculating ability. The activity count involves numbers real or unreal, including negative numbers. Calculations on negative numbers many used at this time as in the measurement of the temperature, finance, score calculation on the Olympics, the measurement of the depth of the sea, etc.

In the year 2014 Jessica Pierson Bishop et al. do research about the problem and the ability of students age 6 – 10 years old in operating the whole numbers, especially negative numbers. By the National Council of Teachers of Mathematics (NCTM), the research published in the journal titled 'Obstacles and Affordances for an integer reasoning: An Analysis of Children Thinking and the History of Mathematics. Bishop et al. explained that in the world of education is not a few students who have difficulties in understanding the concept of negative numbers, first understand the rules of calculation operation on negative numbers. This is not a few students who assume that the negative numbers is something absurd. This can be understood considering the position of negative numbers that 'less than nothing'. Bishop et al. also showed that the reasons advanced by the students subject they cannot accept negative numbers 'similar' with the reasons put forward by the community and are scientists earlier (Bishop, et al., 2014). Based on those problems, arised questions from students about how negative numbers are acceptable by the community which led to the emergence of similar questions from the teachers in order to give an answer to the question of their students. Based on this research concluded that is important for teachers to know the history of negative numbers to answer the question of students about how negative numbers can acceptable by the community. In addition, the history of negative numbers is important to known by teachers in order to provide the proper perception to students about negative numbers so that students can use them in daily life.

Method

To examine the reason of acceptable negative numbers by the community, we done libraries research with bibliographical studies press point. The work was undertaken by collecting data from the literature, either in the library or in other places. Bibliographical study press point aimed to understanding the documents from the situation and the time the document appears (Mahmud, 2011).

Resources which were inventoried are resources that provide information about history of mathematics, history of numbers, history of counting system, history of mathematicians, and also any other information that is related with exploration of negative numbers the perspective of mathematics history.

The study is created by providing chronological plots about history of negative numbers reviewed from the perspective of mathematics history. The plots are primarily focused on information about history of negative numbers since its existence until how they are accepted by

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Asians and Europeans based on resources that is reviewed by scientists. Other than that, reflection were made towards the conclusions.

Results and Discussion

The History of The Acceptance of Negative Numbers in Asia

The Meaning of Black Rods at The Time of Dynasty Han in China

The idea of negative numbers appeared at the time of the Han dynasty (202 BCE - 220 CE) in China. This dynasty produced a book entitled *Jiuzhang Suanshu* which contains negative numbers as one of the topics of discussion. Positive numbers *(zheng)* is used to represent the having money while negative numbers *(fu)* is used to represent the having debt. At that time the Chinese people using two types of rods in the calculation. The red rods which symbolises the positive numbers and black rods which symbolises the negative numbers (Merzbach and Boyer, 2011). At that time, negative numbers is also used to represent the equations (Hodgkin, 2005).

The Counting Operation in Brahmagupta's Book Titled 'The Revised System of Brahma'

The rules of the calculation operation on the negative numbers first induced by an astronomer from India named Brahmagupta in his book titled *Brahma-Sphutta-Siddhanta* (The Revised System of Brahma) around 628 CE. In that book, Brahmagupta provide operating rules about a quantity called 'fortune' (*dhana*), 'debt' (*nina*), and 'zero' (*kha*). Operating rules that induced by Brahmagupta covers the operation of addition, reduction and multiplication as follows.

"*Kha* subtracted from *nina* is *nina*. *Kha* subtracted from *dhana* is *dhana*. *Nina* subtracted from *kha* is *dhana*. *Kha* subtracted from *kha* is *kha*. So, *dhana* subtracted from *kha* is nina. The results from the *kha* multiplied by *nina* or *dhana* is *kha*. The results from the *kha* multiplied by *kha* is *kha*. The results from the addition of two *dhana* is *dhana*. The results from the addition of two *nina* is *nina*. The results from *nina* multiplied by *dhana* is *nina*. The results from *dhana* multiplied by *nina* is *nina*." (Ifrah, 2000)

In the next development, mathematicians began to formulate a general form by changing the term 'fortune' with 'positive numbers' and the term 'debt' with 'negative numbers' (Ifrah, 2000). Brahmagupta also used symbols to represent negative numbers. Brahmagupta put a dot at the top of a number to indicate that the number is negative numbers (Tabak, 2011). With Bháskara, Brahmagupta provided operation rules about positive and negative numbers which are included with example problems and their solution. (Brahmagupta and Bháskara, 1817).

About *'Greater Numbers'* and *'Units'* from Al-Khawarizmi

There is a famous place as a center for research and science in the city of Bahgdad, namely temple of al-Hikma. Many great scientists who work in the Temple of al-Hikma, one of them is Al-Khawarizmi who lived in around 8th century – 9th century (Ifrah, 2000). Al-Khawarizmi wrote a book titled *Addshebr Walmukabala* (Cajori, 1894). This book is one of the book that made the guidelines by the students of Europe. The book played a big part in introduce operating

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rules calculation of negative numbers to the European Community. In the book, Al-Khawarizmi poured the idea about the procedures for the operation of the $(x \pm a) \times (y \pm b)$ as follows. "If there are greater numbers combined with units to be added to or subtracted from them, then four multiplications are necessary; namely, the greater numbers by the greater numbers, the greater numbers by the units, the units by the greater numbers, and the units by the units" (Mohammed ben Musa, 1831).

Al-Khawarizmi used the rules of operation of the calculation on the negative numbers that have been induced by Brahmagupta. The 'greater numbers' represented by x and y while the 'units' represented by a and b. There are four possible operating procedures $(x \pm a) \times (y \pm b)$.

1) $(x + a) \times (x + b)$
$= (x \times x) + (x \times b) + (a \times x) + (a \times b)$
$= x^2 + bx + ax + ab$
2) $(x + a) \times (x - b)$
$= (x \times x) + (x \times (-b)) + (a \times x) + (a \times (-b))$
$= x^2 + (-bx) + ax + (-ab)$
$= x^2 - bx + ax - ab$
3) $(x - a) \times (x + b)$
$= (x \times x) + (x \times b) + ((-a) \times x) + ((-a) \times b)$
$= x^2 + bx + (-ax) + (-ab)$
$= x^2 + bx - ax - ab$
4) $(x - a) \times (x - b)$
$= (x \times x) + (x \times (-b)) + ((-a) \times x) + ((-a) \times (-b))$
$= x^2 + (-bx) + (-ax) + (ab)$
$= x^2 - bx - ax + ab$

Diagram of negative numbers development in Asia can be seen in Figure 1.

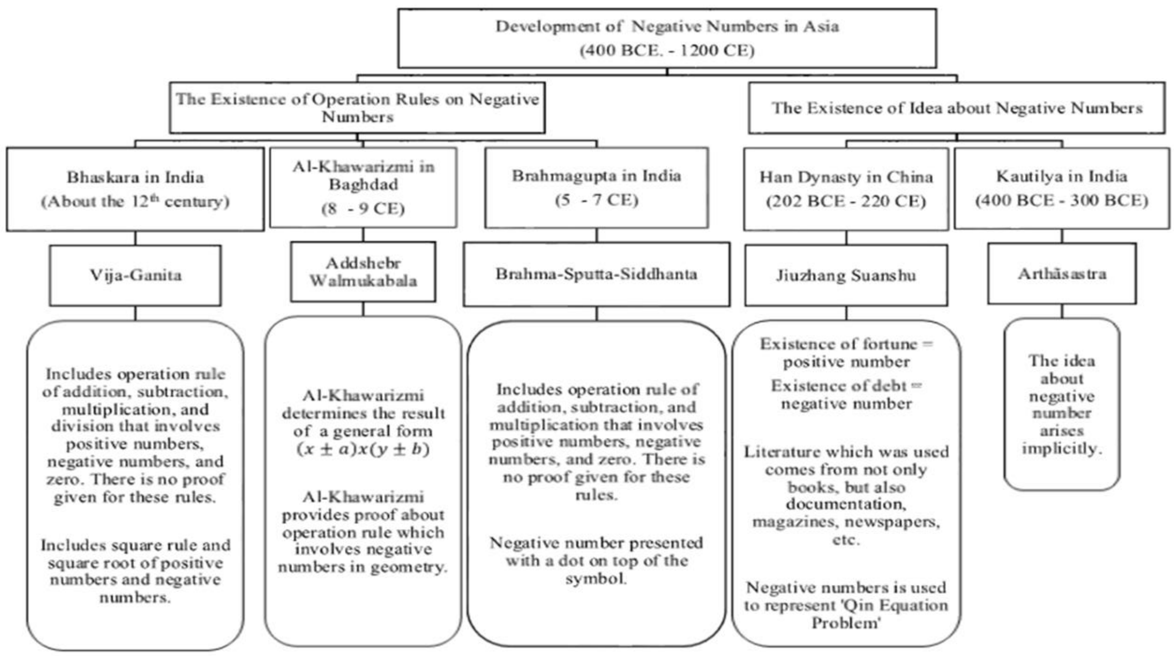

Figure 1. Diagram of Negative Numbers Development in Asia

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The History of The Acceptance of Negative Numbers in Europe

The influence of Islamic civilization on Italian scientists in the development of negative numbers in Europe (15th - 16th century)

Until the beginning of the 12th century, there is no European who are able to be a mathematician or astronomist without having well knowledge about Arabs, especially in Arabic language (Boyer, 1968). The condition caused by a lot of literatures which includes mathematics and astronomy in libraries under Islamic authority, where Arabic is the language used by the society in that Islamic territory at that time.

Leonardo of Pisa is an Italian scientist who gets academic advantages from a relatively harmonic relationship between Islamic civilization and Europeans. The relationship caused Leonardo of Pisa to learn society in Islam civilization, their language, and their developed knowledge. This condition made Italy as a starting point for the spreadness of knowledge that is learned and developed by the scientists in Islamic territory to mainland Europe. Leonardo of Pisa is the first European who introduce negative numbers to the mainland Europe around the twentieth century (Ariani, 2008).

Leonardo of Pisa is well known as 'Fibonacci'. Fibonacci acquired knowledge on the negative numbers from his journey in Islamic territory (Egypt, Syiria, Greece, Sicily) and from the Al-Khawarizmi's book titled *Addshebr Walmukabala* (Cajori, 1894). Fibonacci introduced negative numbers in his work entitled *Liber Abaci* which published in the year 1202 CE (Ball, 2010). In that paper, there is a quotation from the word problems as follows. "*… and from the* 240 *you subtract the* 288 *leaving minus* 48*, and this I say because the* 288 *cannot be subtracted from the* 240*; from this* 48 *you take* $\frac{1}{3}$ *for the* $\frac{1}{3}$ *of the second position; there will be minus* 16 *that you save and you subtract the* 72 *from the* 120 *leaving plus* 48 *for which* $\frac{1}{4}$ *is had similarly plus* 12." (Leonardo of Pisa, 2003).

Adapted from Islamic civilization, such as bank, letter of credit, and bill of exchange, on 15th century, there is a developed accounting methods which was began by the famous invention of double entry bookkeeping. The system was developed by an Italian scientist Luca Pacioli (Hodgkin, 2005). Double entry bookkeeping system tracks calculation of income and outcome values in commerce. Hence, someone may find out profit or loss result with comparing income and outcome values noted in two sides of the recordings. The case indicates that double entry bookkeeping system includes implicit implementation of negative numbers.

Furthermore, inspired by Al Khawarizmi, Italian scientist Girolamo Cardano, wrote his thoughts about negative numbers on a set of articles which is known as Opera Omnia (Cardano, 1663). In Opera Omnia, there are demonstrations of truth from the multiplication rule on negative numbers in geometry, as well as demonstrations of roots with a power of even numbers having two possibilities, namely positive and negative roots with the same value.

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The Contradictions of The Acceptance of Negative numbers by European Scientists (16^{th} – 19^{th} century CE)

Not like in the Asian region, in the European region the existence of negative numbers had contradiction. The existence of negative numbers be a doubt not only among the public in general, but also among scientists. The idea contradictory scientists Europe about the existence of negative numbers is as follows.

Negative Root is 'False Root'

Around the sixteenth century CE, Déscartes, a French scientist, released a book titled Le Geométrié. In the book, Déscartes states that the root of negative value was the 'false root' as in the following quote. "*It often happens, however, that some of the roots are false or less than nothing. Thus, if we suppose x to represent the defect of a quantity 5, we have* $x + 5 = 0$ *which, multiplied by* $x^3 - 9x^2 + 26x - 24 = 0$, *yields* $x^4 - 4x^3 - 19x^2 + 106x - 120 = 0$, *an equation having four roots, namely three true roots* 2, 3, *and* 4, *and one false root* 5" (Cartesius, 1925).

Negative Numbers Cause an Equation Become 'False'

In the year 1751, a French scientist named Joan d'Alembert published a encyclopedia titled *Encyclopédie, ou Dictionnaire Raisonné des Sciences, des Arts et des Métiers, par Une Sociéte de Gens de Lettres*. In the encyclopedia states an article titled *negative*. This article will discuss the view of most of the European community and the views of Jean le Rond d'Alembert himself about negative numbers.

In the article written that it is not easy to understand the concept of negative numbers. Even some scientists contribute in providing accurate data to show that the concept of negative numbers is a concept that 'confused.' In the article d'Alembert states that equation $x + 100 = 50$ is the '*false*' equation.

The article states that equation $x + 100 = 50$ is the '*false*' equation. That caused the value of x from the equation is negative numbers or less than nothing, which is $x = -50$. According to d'Alembert, the equation should be written $100 - x = 50$. So the value of x from the equation is positive numbers, which is $x = 50$ (D'Alembert, 1751).

The 'Ridiculous' Number

In the year 1796, a British scientist named William Frend published a book titled *The Principles of Algebra*. In that book, William Frend stated that the negative numbers is 'ridiculous'. This can be seen from the quotation taken from the paper as follow, "It submits to be taken away from another number greater than itself, but to attempt to take it away from a number less than itself is ridiculous" (Frend, 1796). Reduce a number from a smaller number producing negative numbers. That's means William Frend stated the negative numbers is ridiculous.

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The Cause of The Operation of The calculation Being 'Impossible'

In the year 1803, a French scientist named Lazare Carnot publish a book titled *Géométrie de Position*. In that paper, Lazare conveyed the idea about the 'impossible' operation. The idea could be seen from the quotation books *Géométrie de Positions* as follows, "*Pour obtenir réellement une quantité négative isoleé, il faudroit retracher une quantité effective de zéro, ôter quelque chose de rien: opèration impossible. Comment done concevoir une quantité négative isoleé?*" (Carnot, 1803). English translation from the quotation is as follows. "To get negative quantity (quantity is isolated), should reduce the number of effective (positive numbers) from zero, reduce something from which nothing: the operation is impossible. How can bear a negative quantity (quantity is isolated)?". Based on the quotation can be seen that the Lazare Carnot holds that the idea of negative numbers is difficult to be accepted. It caused impossible to reduce a number from nothing. In other words, operations $0 - a$ with $a > 0$ is 'impossible' operation.

The Absurd Thing

In his work entitled *On The Study and difficulties of Mathematics*, a British scientist named Augustus de Morgan stated that the negative numbers is 'absurd.' This can be seen from the quotation of the book On the Study and difficulties of Mathematics as follows. "For example, $8 - 3$ is easily understood; 3 can be taken from 8 and the remainder is 5; but $3 - 8$ is an impossibility, it requires you to take from 3 more than there is in 3, which is absurd. If such an expression as $3 - 8$ should be the answer to a problem, it would denote either that there was some absurdity inherent in the problem itself, or in the manner of putting it into an equation."(De Morgan, 1910)

The Recognition of The Existence of Negative Numbers by the European Community (17th – 18th century CE)

The acceptance of negative numbers by the European Community and the world does not remove from the role of the three renowned scientists at that time, namely John Wallis, Sir Isaac Newton and Leonhard Euler. They acknowledge the existence of negative numbers in their works.

The Existence of Negative Numbers Recognized in The *Treatise of Algebra*

In 1685, John Wallis published a book entitled *Treatise of Algebra*. In that paper, John Wallis acknowledge the existence of negative numbers. It can we see in the above quotation Treatise of Algebra as follows.

Yet is not that supposition (of negative quantities,) either not useful or absurd, when rightly understood. Though, as to the bare algebraic notation, it imports a quantity less than nothing. Yet, when it comes to a physical application, it denotes as real a quantity as if the sign were +; but to be interpreted in a contrary sense. (Wallis, 1685)

Wallis was one of the first to use negative line numbers. As evidenced by the following passage from Treatise on Algebra. As for instance: Supposing a man to have advanced or moved forward, (from A to B) 5 Yards, and then to retreat (from B to C) 2 Yards: If it be asked, how much he had Advanced (upon the whole march) when at C? or how many Yards he is now

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Forwarder than when he was at A? I find (by Sub ducting) 2 from 5 that he is Advanced 3 Yards (Because $+5 - 2 = +3$).

Figure 2. Number line's illustration by John Wallis

But if, having Advanced 5 Yards to B, he thence Retreat 8 Yards to D; and it be then asked, How much he is Advanced when at D, or how much Forwarder than when he was at A: I say -3 Yards. (Because $+5 - 8 = -3$). That is to say, he is advanced 3 Yards less than nothing. (Wallis, 1685)

The Existence of Negative Numbers Recognized in The *Universal Arithmetick*

Around the seventeenth century CE, a British scientist named Isaac Newton published a book entitled *Universal Arithmetic*. In that paper, Newton states that "Quantities are either affirmative, or greater than nothing; or negative, or less than nothing. Thus, in human affairs, possessions or stock may be called affirmative goods, and debts negative ones. In local motion, progression may be called affirmative motion, and regression negative motion, ..." (Newton, 1769)

The Existence of Negative Numbers Recognized in The *Elementary of Algebra*

In Elementary of Algebra, a scientist named Leonhard Euler stated that a number very influenced by the accompanying sign. There are two signs that can accompany a numbers, which is the sign + that shows the quantity of positive and sign − that shows the quantity of negative (Euler, 1828). In the real application, Euler describes the use of negative numbers in the field of properties. It can be seen from the quotation on Elementary of Algebra as follows. The manner in which we generally calculate a person's property, in an apt illustration of what has just been said. For we denote what a man really possesses by positive numbers using, or understanding the sign +, whereas his debts are represented by negative numbers, or by using the sign − (Euler, 1828). Illustration of negative numbers development in Europe can be seen in Figure 3.

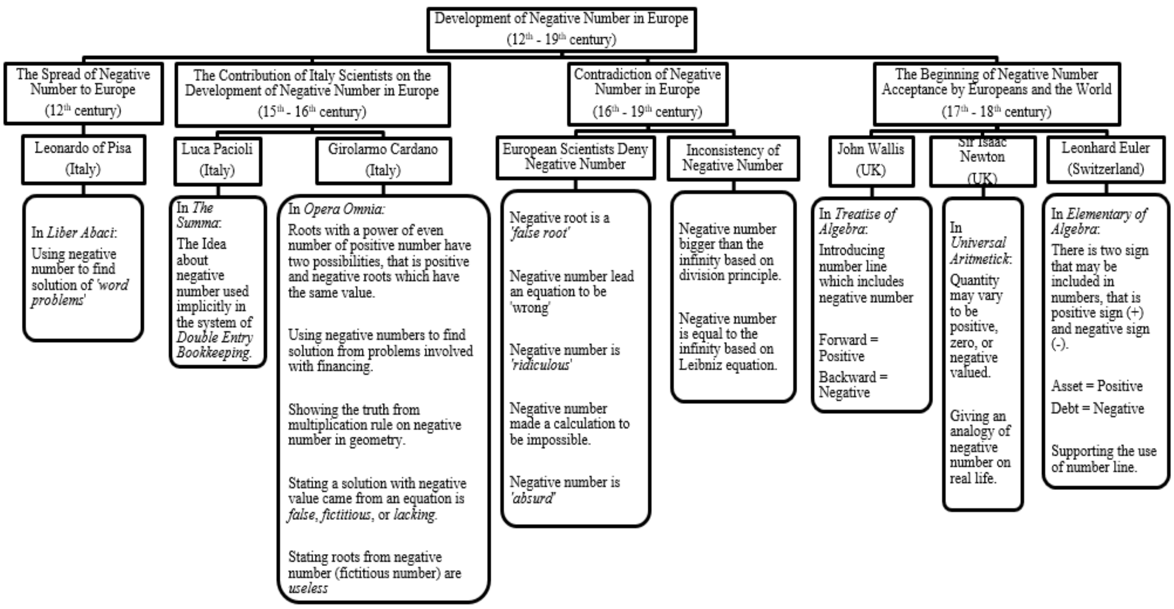

Figure 3. Diagram of Negative Numbers Development in Europe

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The Cause of Difference of Negative Numbers Acceptance Between Asian and European Community

Asian and European societies are fundamentally different in terms of history, social life, culture, and religious. Asian nations place a high premium on religiosity in all spheres of life, including science. Due to the widespread usage of religiosity in all aspects of life, Asian societies have developed an affinity for the abstract, mystical, and religious. This enhances their ability to think abstractly and to embrace the concept of negative numbers.

Asian Societies with Their Religion

The community of China, India, and Baghdad is a religious society. They use religion as the basis in every aspect of life, including in the aspect of science. According to Martzloff, Chinese society to accept negative numbers as opponents of the positive numbers are backed by the philosophy of dualism which they assume, namely *Yin* and *Yang* (Hodgkin, 2005). The philosophy is symbolized by a circle symbol which has black and white with a balanced quantity, can be seen in Figure 4. They believe that all things have enemy; hot opponents of cold, evil opponents of goodness, having debt opponents of having money. The Since the Han dynasty (202 BCE – 220 CE), philosophy has conditioned Chinese society to accept negative numbers, which are the inverse of positive numbers.

Figure 4. *Yin* and *Yang* Symbol

Hinduism became a source of guidance for the Indian people, including in the field of science (628 BCE). The Indian community cloaks their discoveries and conclusions in the realm of mathematics in mystical and holy terminology (Cajori, 1894). They used mathematics to examine the heavenly bodies (Tabak, 2011). They believe that heaven is the place of the gods and goddesses lived. Examine the heavenly bodies is a very special work done by the special person (the Brahmana). The Brahmana (the saints) was highly respected by the community India and appreciated the results of his thoughts. In addition, Indian Hindus believe in the law of Karma. In a concept of Karma, it is believed that what one's does towards people, is what people do towards them. This belief provided an image that society of India believed for positive purpose in doing good deeds, and negative purpose in doing bad deeds. Therefore, Brahmagupta's ideas about negative numbers and operating rules on the negative numbers easily accepted by the community of India.

The majority of Baghdad's population is Muslim. In Islam, sin is the polar opposite of reward, and the afterlife is the polar opposite of the world's nature. Baghdad's community trusts something that seems abstract and intangible, yet is genuine. Therefore, the community of Baghdad accept the concept of negative numbers easily (the 8th century – 9th century).

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European Community with Dichotomy of Sciences

Before the 15th century CE (called with the dark ages) churches played a very central to the life of the European Community, including in the field of science. All the findings must be in accordance with the doctrine of the church. The scientists who published their findings which contrary to the church will receive the sanction from the church, including Copernicus and Galileo that triggered the idea of heliocentric (Hadiwijoyo, 2011). On the next development, many European scientists and European societies do not agree about that having done by the church to that scientists. The situation caused the emergence of the Renaissance where science dichotomy. False dichotomies science is the separation between the sciences of religion with science certainly (Qomar, 2006). False dichotomies science bare scientists that separates the results of their findings with all the things that smell of religion. All things must be proved in a rational and can be proved by the senses come. In this case, they believe in something that can be seen and heard, whatsoever. They do not believe in something that looks abstract. Therefore the concept of negative numbers and reap the contradictions of scientists Europe up to three major European scientists; Euler, Wallis, and Newton, start the acceptance of negative numbers ahead of the 17th century CE.

Conclusions

The development of negative number acceptance is begun earlier by implicit implementation than explicit implementation. The first time implicit implementation of negative number is used in 400 – 300 BCE with showing an analogy that positive number represent as an asset whereas negative number as a debt. The development of implicit negative number implementation tends to be stagnant until 16th century. However, negative number was implicitly implemented into mathematical equations in 8th to 9th century. Implementation of negative number has developed significantly starting from 16th to date (21th century).

The development of negative number acceptance, which is used explicitly, started in 2 BCE – 2th century in the form of symbolization, designation, and utilization of negative number to represent an equation. Explicit implementation of negative number developed dynamically from 200 BCE to 20th century. The development of explicit implementation of negative number, which is achieved one by one, are calculation rule, utilization to represent an equation, and utilization in number line. In 20th to 21th century, the development of explicit implementation of negative number tends to be stagnant.

Asian community accepted negative numbers easier than European Community. This can be seen from the contradictions about the existence of the negative numbers in the 16th – 19th century CE in Europe, an incident which did not happen in Asia. The influence of religiosity in the Asian community makes it easier for them to accept the concept of negative numbers than the European community.

From the results of this research, one may also show how mathematics and sciences learning are able to be integrated with religion aspects nicely. 'Attitude' and 'character' educations are not only chosen to be subjects of teaching but also integrated with mathematics. One may observe the elaboration of *Yin* and *Yang* in China, Hinduism in India, and Islam in Baghdad

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which represent positive and negative matters from each belief or religion. For instance, a belief of evil being the inverse of goodness, hereafter being the inverse of the nature of the world, sins being the inverse of rewards, and concepts of heaven and hell. These propositions are relevant with positive and negative numbers representation.

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