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Metric Dimension of Graphs Djembe (Dj_n)

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Abstract

Let $G = (V, E)$ be a connected graph. For an ordered set $A \subseteq V(G)$, the representation of a vertex with respect to A is defined by its distance vector to the vertices of A. A set A is called a resolving set if every pair of distinct vertices in G has distinct representations with respect to A. The minimum cardinality of a resolving set is called the metric dimension of G. In this paper, we determine the metric dimension of the Djembe graph D_{j_n} for $n \geq 3$. By constructing appropriate resolving sets and proving their minimality, we obtain an exact formula for the metric dimension. The obtained value depends on the congruence class of n modulo 4, with a special case occurring when $n = 7$. These results provide a new contribution to the study of metric dimensions for cycle-based graph families and extend the existing literature on graph resolvability.

Keywords: Metric dimension; Resolving set; Distance representation; Djembe graph; Cycle-Based graph.

Abstrak

Misalkan $G = (V, E)$ adalah suatu graf terhubung. Untuk suatu himpunan terurut $A \subseteq V(G)$, representasi sebuah simpul terhadap A didefinisikan sebagai vektor jarak simpul tersebut ke setiap simpul dalam A. Himpunan A disebut himpunan pembeda (resolving set) jika setiap pasangan simpul yang berbeda dalam G memiliki representasi yang berbeda terhadap A. Kardinalitas minimum dari himpunan pembeda disebut dimensi metrik (metric dimension) dari graf G. Pada artikel ini ditentukan dimensi metrik dari graf Djembe D_{j_n} untuk $n \geq 3$. Metode yang digunakan adalah konstruksi himpunan pembeda dan pembuktian minimalitasnya untuk memperoleh batas atas dan batas bawah yang berimpit. Hasil penelitian menunjukkan bahwa dimensi metrik graf Djembe dapat dinyatakan secara eksak dalam bentuk formula tertutup yang bergantung pada kelas kongruensi n modulo 4, dengan satu kasus khusus ketika $n = 7$. Hasil ini memberikan kontribusi baru dalam kajian dimensi metrik pada keluarga graf berbasis siklus yang belum pernah diteliti sebelumnya serta memperkaya perkembangan teori resolvabilitas graf.

Kata Kunci: *Dimensi metrik, Himpunan pembeda, Representasi simpul, Graf Djembe, Resolvabilitas graf.*

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1. INTRODUCTION

With the rapid development of graph theory, various new concepts have emerged, including the metric dimension of a graph, introduced by Slater in 1975. In 1976, Harary and Melter independently introduced the same concept. Suppose G is a nontrivial connected graph with a vertex set $V(G)$. A set $W \subseteq V(G)$ is a resolving set of G if $d(u, x) \neq d(v, x)$ for any vertex $u, v \in (G)$ and some vertex $x \in W$, where $d(u, x)$ denotes the distance, i.e., the length of the shortest path from vertex x. The resolving

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set with the smallest cardinality is called the minimum resolving set or basis, and the number of elements in the basis is called the metric dimension. Several variations of resolving sets have emerged, one of which is the local resolving set introduced by Okamoto et al. [1].

The metric dimension is a concept related to graph theory. Several concepts are used to determine the metric dimension of a graph. The first is the distance between two vertices on a graph, and the other is the resolving set. F. Harary and R.A. Melter were the mathematicians who first introduced this research on the concept of metric dimension [2].

In 2012, Permana and Darmaji [3] studied the metric dimension of the regular banana tree graph B_m, the regular caterpillar graph $C_{m,n}$ and the regular fireworks graph $F_{m,n}$. In their research, the metric dimension of the regular banana tree graph B_m, were $(n-1), m=1, n \geq 3$ and $(n-1), m \geq 2, n \geq 3$. Furthermore, it was shown that the metric dimension of the regular caterpillar graph $C_{m,n}$ a were $m(n-1), m \geq 1, n \geq 2$ the metric dimension of the regular fireworks graph $F_{m,n}$ were n, for $m=1$, $n \geq 2$ and $m(n-1)$, for $m \geq 2, n \geq 2$. In the same year, Saputro et al. [4] examined the metric dimension of the graph $S_m + K_1, m \geq 3$ where the metric dimension of the graph $S_m + K_1, m \geq 3$ is m. Next, Bahri et al. [5] found that the metric dimension of the cross-path product graph $P_m \times P_2 \times P_2$ for $m \geq 2$ is 3. Next, Widodo [6] developed the metric dimension of the sun graph (S_n), the helmet graph (H_n), for $n \geq 3$ and the double cone graph DC_n.

Saputro et al. [7] discussed the metric dimension of graphs resulting from *lexicographic* multiplication operations. In addition, Ali et al. [8] discussed the local metric dimension of graphs with m-penandts. Tomescu et al. discussed the metric dimension of several infinite regular graphs [9]. M. Ali et al. discussed graphs containing cycles and those with constant metric dimension [10], and S Wahyudi et al. discussed the metric dimension of the development of the pinwheel graph [9]. Several graph classes that have been studied include the Johnson and Kneser graphs [10], the Fullerence graph [11], the unicyclic graph [12], the flower graph $\mathrm{f}_{\mathrm{m} \times \mathrm{n}}$ [13], the dragon graph [14], the Buckminsterfullerene graph, and the dumbbell graph [15]. Furthermore, the metric dimension of several graphs resulting from amalgamation operations [16][17][18], Cartesian operations [19][20], comb product operations [21], and subdivision operations [22] has been obtained.

In [21], the determination of the metric dimension of graphs $K_s\ (s \geq 1)$, $K_s + K_t\ (s \geq 1, t \geq 2)$, and $K_s + (K_1 \cup K_t)(s, t \geq 1)$ was discussed. Furthermore, in [23], the metric dimension of the graph $P_m + P_n$, $K_m + G$ for any connected graph G, and $P_m + K_n$. is discussed. Based on the author's observation, no one has studied the metric dimension of the Djembe graph (Dj_n) for $n = 3, 4$. So we are interested in studying the metric dimension of the Djembe graph (Dj_n). Ponraj et al. [24] determined the labeling of several graphs containing cycles. One of the graphs determined is the Djembe graph, which is denoted (Dj_n). Meanwhile, the wheel graph W_n is a graph formed from the cycles (C_{n-1}) by adding one vertex w which is connected to all vertices in the cycle [24].

A subset W of $V(G)$ is called a local resolving set of G if $r(u|W) \neq r(v|W)$ for every two adjacent vertices $u, v \in V(G)$. The smallest cardinality of all local resolving sets in G is called the local metric dimension of G, denoted by $lmd(G)$. The local resolving set of G with cardinality $lmd(G)$ is called a local basis of G. In this paper, we present a novel study, a topic that has not been extensively explored in previous research, on the local metric dimension of certain operations of the generalized Petersen graph $sP_{n,m}$ and determine the lower and upper bounds of $lmd(sP_{n,m})$ dengan $n \geq 3, s \geq 1$, and $1 \leq m \leq \left\lfloor \frac{n-1}{2} \right\rfloor$. We also show that the lower bound is sharp [25].

Graph theory combines the concept of graph coloring, measured by the chromatic number $X_l(G)$,with the concept of metric dimension, which is used to determine the position of vertices based on their distances to other vertices. One development that integrates these two concepts is the locating chromatic number, introduced by Chartrand et al. in 2002 [26]. This concept merges graph coloring with the ability to distinguish each vertex through its distance representation with respect to color classes. The locating chromatic number aims to determine the minimum number of colors in a partition such that every vertex in the graph has a unique location code based on its distances to each color class. In this way, each vertex can be uniquely identified using only its distance information to the color classes. Several studies have been conducted to determine the locating chromatic number for various classes of graphs. Among them are studies on the zigzag graph [25] and the Pentagonal Circular Ladder Graph [27].

Although the metric dimension has been studied for many graph classes and graph operations, the metric dimension of the Djembe graph has not yet been investigated. The Djembe graph (Dj_n) has appeared in previous work in the context of graph labeling, but its resolving sets and metric dimension have not been considered. This gap may be due to the fact that the Djembe graph is a relatively specific cycle-based graph family, and previous studies have mainly focused on more common graph classes, such as paths, cycles, wheel trees, and graphs obtained from standard graph operations. However, the Djembe graph has a distinctive structure derived from a cycle and additional adjacency relations, so its metric dimension cannot be obtained directly from known results on wheel graphs or other related graph families. Therefore, analyzing its metric dimension is important for extending the known results on resolving sets to a new graph family and for understanding how the structural modification of a cycle-based graph affects the uniqueness of vertex representations.

The objective of this study is to determine the exact metric dimension of the Djembe graph $D_j(n)$ for all $n \geq 3$. The main contribution of this paper is the construction of minimum resolving sets and the proof of their minimality for the Djembe graph. As a consequence, an explicit formula for its metric dimension is obtained. This result extends the current literature on metric dimension by introducing a new cycle-based graph family whose resolvability properties have not previously been investigated.

2. DEFINITIONS

In this section, we present several definitions and preliminary results that will be used todetermine the metric dimension of the Djembe graph. Throughout this paper, all graphs considered are finite, simple, and connected.

Definition 1. [2][24] Let $G = (V(G), E\ (G))$ be a connected graph for two vertices $u, v\ \in V(G)$, the distance between u and v, denoted by $d(u, v\ )$, is the length of a shortest path joining u and v in G.

Definition 2. [2][24] Let $G = (V(G), E\ (G))$ be a connected graph and let $A = \{a_1, a_2, \dots, a_k\}$ be an ordered subset of $V(G)$. For a vertex $v \in V(G)$, the representation of v with respect to A is defined by $r(v|A) = \big(d(v, a_1), d(v, a_2), \dots, d(v, a_k)\big)$.

Definition 3. [2][24] Let $G = (V(G), E\ (G))$ be a connected graph. A subset $A \subseteq V(G)$ is called a resolving set of G if every two distinct vertices of G have distinct representations with respect to A. A resolving set with minimum cardinality is called a basis of G. The cardinality of a basis of G is called the metric dimension of G and is denoted by $dim(G)$.

Definition 4. [24] Let $n \geq 3$ be an integer. The Djembe graph (Dj_n) is a graph obtained from a cycle-based structure with a central vertex and additional adjacency relations forming a Djembe-like structure. More precisely, the Djembe graph (Dj_n) has a vertex set

$$V\,(Dj_n) = \{w\} \cup \{u_i, v_i : 1 \leq i \leq n\},$$

where w is the central vertex and the vertices u_i and v_i from the outer and inner parts of the graph, respectively.

3. RESULTS

The Djembe graph (Dj_n) is a modification of the wheel graph. It is constructed from a cycle together with a central vertex that is connected to every vertex of the cycle. More precisely, the wheel graph Wn is formed by taking a cycle C_n with n vertices and adding one additional central vertex w, which is adjacent to all vertices of the cycle. The following is the definition of the Djembe graph (Dj_n). See figure 1.

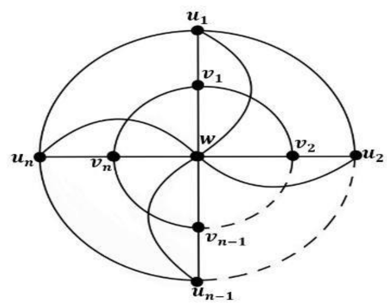

Figure 1. Graph Djembe (Dj_n)

Definition 5. $V(Dj_n) = \{w\} \cup \{v_i \mid 1 \leq i \leq n\} \cup \{u_i \mid 1 \leq i \leq n\} \cup \{w_i \mid 1 \leq i \leq n\}$

$$E(Dj_n) = \{(u_i u_{i+1}), (v_i v_{i+1}), (w v_i), (w u_i), (u_i v_i) \mid 1 \leq i \leq n, u_{n+1} = u_i, v_{n+1} = v_1\}.$$

In this section, we determine the metric dimension of the Djembe graph Dj_n for $n \geq 3$. The proof is organized into two main parts. First, we construct a resolving set to obtain an upper bound. Second, we prove that no resolving set with a smaller cardinality exists, which gives the corresponding lower bound. This approach avoids repetitive case-by-case arguments and emphasizes the general pattern of the construction

Theorem 1. Let (Dj_n) be the Djembe graph with $n \geq 3$. Then, the metric dimension of (Dj_n), denoted by dim (Dj_n), is given by:

$$dim(Dj_n) = \begin{cases} \dfrac{3n}{4}, & n \equiv 0\ (mod\ 4) \\ 1 + 3\left\lceil \dfrac{n}{4} \right\rceil, & n \equiv 1\ (mod\ 4) \\ 2 + 3\left\lceil \dfrac{n}{4} \right\rceil, & n \equiv 2\ (mod\ 4) \\ 3 + 3\left\lceil \dfrac{n}{4} \right\rceil, & n \equiv 3\ (mod\ 4), n \neq 7 \\ 5 & n \equiv\ 7. \end{cases}$$

Proof.
The set of vertices and edges of the Djembe graph (Dj_n) is given in Definition 5. First, we determine the upper bound of $dim(Dj_n)$ by constructing a resolving set W such that every vertex in the Djembe graph (Dj_n) has a different representation with respect to W. Next, we determine the lower bound by showing that any resolving set W^* with cardinality less than that of W cause some vertices in the Djembe graph (Dj_n) to have the same representation.

To provide a clearer proof of the theorem, several cases related to the structure of the Djembe graph (Dj_n) are considered. By analyzing these cases, it can be shown that each vertex has a unique representation with respect to the chosen resolving set.

Therefore, the proof of the metric dimension of the Djembe graph (Dj_n) is divided into the following two cases:

Case 1: $3 \leq n \leq 8$.

For the small cases $3 \leq n \leq 8$, the same construction and contradiction argument applies. Hence, we combine them into one general statement. For the upper bound, consider the following sets:

$$W_n = \begin{cases} \{u_1, u_2, u_3\}, & n = 3,4 \\ \{u_1, u_2, u_3, u_4\}, & n = 5 \\ \{u_1, u_2, u_3, u_4, u_5\}, & n = 6,7 \\ \{u_1, u_2, u_3, u_4, u_5, u_7\}, & n = 8. \end{cases}$$

Table 1 shows the representation of the vertex sets for $n = 3$ and $n = 8$.

By direct computation, every vertex of Dj_n has a distinct representation with respect to W_n. Therefore $Dim(Dj_n) \leq W_n| - 1|$. Next, we show that this bound is sharp. Suppose, to the contrary, that there exists a resolving set W^* of Dj_n such that $|W^*| = |W_n| - 1$. By considering the representation choices of W^*, one always finds two distinct vertices of Dj_n having the same representation with respect to W^*. Thus, W^* cannot be a resolving set. More precisely, the repeated pattern is as follows.
For $n = 3$, there exist two vertices, for example v_2 and v_3, such that $r(v_2|W^*) = r(v_3|W^*)$.
For $n = 4$, one obtains either $r(w|W^*) = r(v_3|W^*)$ or $r(u_3|W^*) = r(u_4|W^*)$.
For $n = 5$, one obtains either $r(v_3|W^*) = r(v_4|W^*)$ or $r(u_4|W^*) = r(u_5|W^*)$.
For $n = 6$, one obtains either $r(v_4|W^*) = r(v_5|W^*)$ or $r(u_5|W^*) = r(u_6|W^*)$.
For $n = 7$, one again obtains two distinct vertices with identical representations, for instance $r(v_4|W^*) = r(v_5|W^*)$ or $r(u_5|W^*) = r(u_6|W^*)$.
Finally, for $n = 8$, there exist two vertices, for example u_6 and u_7, such that $r(u_6|W^*) =$

$r(u_7|W^*)$.
Hence, no set of cardinality $|W_n|-1$ resolves Dj_n. It follows that $\dim(Dj_n) \geq |W_n|$. Combining this with the upper bound, we conclude that

$$\dim(Dj_3) = \dim(Dj_4) = 3, \dim(Dj_5) = 4, \dim(Dj_6) = \dim(Dj_7) = 5, \dim(Dj_8) = 6.$$

Table 1. Representation of vertex sets for $n = 3$ until $n = 8$.

N	$v \in V(G)$	$r(v \in \|W_n)$	N	$v \in V(G)$	$r(v \in \|W_n)$
3	u_1	(0,1,1)	4	u_1	(0,1,2)
	u_2	(1,0,1)		u_2	(1,0,1)
	u_3	(1,1,0)		u_3	(2,1,0)
	v_1	(1,2,2)		u_4	(1,2,1)
	v_2	(2,1,2)		v_1	(1,2,2)
	v_3	(2,2,1)		v_2	(2,1,2)
	w	(1,1,1)		v_3	(2,2,1)
				v_4	(2,2,1)
5	u_1	(0,1,2,2)		w	(1,2,1)
	u_2	(1,0,1,2)			
	u_3	(2,1,0,1)	6	u_1	(0,1,2,2,2)
	u_4	(1,2,1,0)		u_2	(1,0,1,2,2)
	u_5	(1,2,2,1)		u_3	(2,1,0,1,2)
	v_1	(1,2,2,2)		u_4	(2,2,1,0,1)
	v_2	(2,1,2,2)		u_5	(2,2,2,1,0)
	v_3	(2,2,1,2)		u_6	(1,2,2,2,1)
	v_4	(2,2,2,1)		v_1	(1,2,2,2,2)
	v_5	(2,2,2,2)		v_2	(2,1,2,2,2)
	w	(1,1,1,1)		v_3	(2,2,1,2,2)
				v_4	(2,2,2,1,2)
7	u_1	(0,2,2,2,2)		v_5	(2,2,2,2,1)
	u_2	(1,1,2,2,2)		v_6	(2,2,2,2,2)
	u_3	(2,0,2,2,2)		w	(1,1,1,1,1)
	u_4	(2,1,1,2,2)			
	u_5	(2,2,0,1,2)	8	u_1	(0,1,2,2,2,2)
	u_6	(2,2,0,1,2)		u_2	(1,0,1,2,2,2)
	u_7	(1,2,2,2,1)		u_3	(2,1,0,1,2,2)
	v_1	(1,2,2,2,2)		u_4	(2,2,1,0,1,2)
	v_2	(2,2,2,2,2)		u_5	(2,2,2,1,0,2)
	v_3	(2,1,2,2,2)		u_6	(1,2,2,2,1,1)
	v_4	(2,2,2,1,2)		u_7	(2,2,2,2,2,0)
	v_5	(2,2,1,0,2)		u_8	(1,2,2,2,2,1)
	v_6	(2,2,2,1,1)		v_1	(1,2,2,2,2,2)
	v_7	(2,2,2,2,0)		v_2	(2,1,2,2,2,2)
	w	(1,1,1,1,1)		v_3	(2,2,1,2,2,2)
				v_4	(2,2,2,1,1,2)
				v_5	(2,2,2,2,0,2)
				v_6	(2,2,2,2,1,2)
				v_7	(2,2,2,2,2,1)
				v_8	(2,2,2,2,2,2)
				w	(1,1,1,1,1,1)

Case 2: For $n \geq 9$.

As in the case 1, we divide the proof according to the residue class of n modulo 4. For each class, let W_n be the set constructed for Dj_n. By direct computation, all the vertices of Dj_n have distinct representations with respect to W_n. Hence, W_n is a resolving set.

Subcases 2.1. If $n \equiv 0$0 (mod 4), then $|\, W| = \frac{3n}{4}$ and therefore $\dim(Dj_n) =\leq \frac{3n}{4}$. To show the reserve inequality, suppose that there exists a resolving set W^* such that $|W^*| = \frac{3n}{4} - 1$. Since the same obstruction appears in all similar choices of W^*, it is enough to consider representative configurations. In each such configuration, there exist two distinct vertices of Dj_n having the same representation with respect to W^*; for example, one obtains either $r(v_8|W^*) = r(v_9|W^*)$ or $r(u_6|W^*) = r(u_7|W^*)$. Thus, W^* is not a resolving set. Hence $\dim(Dj_n) =\geq \frac{3n}{4}$. Therefore $\dim(Dj_n) = \geq \frac{3n}{4}$ for $n \equiv 0$ (mod 4).

Subcases 2.2. If $n \equiv 1$ (mod 4), then $|W_n| = 1 + 3\left\lfloor \frac{n}{4} \right\rfloor$, so $\dim(Dj_n) \leq 1 + 3\left\lfloor \frac{n}{4} \right\rfloor$. Now, suppose that there exists a resolving set W^* such that $|W^*| = 3\left\lfloor \frac{n}{4} \right\rfloor$. Again, all similar choices of W^* lead to the same type of contradiction, so it suffices to consider representative ones. In each case, there are two distinct vertices with identical representations with respect to W^*; for instance, one obtains either $r(v_3|W^*) = r(v_4|W^*)$ or $r(u_6|W^*) = r(u_7|W^*)$. Hence, W^* is not a resolving set. Therefore $\dim(Dj_n) \geq 1 + 3\left\lfloor \frac{n}{4} \right\rfloor$ and so $\dim(Dj_n) = 1 + 3\left\lfloor \frac{n}{4} \right\rfloor$ for $n \equiv 1$(mod 4).

Sub case 2.3. If $n \equiv 2$(mod 4), then $|W_n| = 2 + 3\left\lfloor \frac{n}{4} \right\rfloor$, which gives $\dim(Dj_n) \leq 2 + 3\left\lfloor \frac{n}{4} \right\rfloor$. For the lower bound, suppose that there exists a resolving set W^* such that $|W^*| = 1 + 3\left\lfloor \frac{n}{4} \right\rfloor$. As in the previous sub-case, all similar possibilities for W^* yield the same contradiction. Indeed, there always exist two distinct vertices of Dj_n having the same representation with respect to W^*; for example $r(v_3|W^*) = r(v_4|W^*)$ or equivalently, another representative pair with identical coordinates. Thus, W^* is not a resolving set. It follows that $\dim(Dj_n) \geq 2 + 3\left\lfloor \frac{n}{4} \right\rfloor$. Consequently $\dim(Dj_n) = 2 + 3\left\lfloor \frac{n}{4} \right\rfloor$ for If $n \equiv 2$(mod 4).

Subcases 2.4. If $n \equiv 3$(mod 4), then $|W_n| = 3 + 3\left\lfloor \frac{n}{4} \right\rfloor$ and therefore $\dim(Dj_n) \leq 3 + 3\left\lfloor \frac{n}{4} \right\rfloor$. Assume, to the contrary, that there exists a resolving set W^* satisfying $|W^*| = 2 + 3\left\lfloor \frac{n}{4} \right\rfloor$. Using the same argument as above, it is enough to examine representative configurations of W^*, since the remaining choices produce the same pattern. In every such configurations, two distinct vertices have identical representations with respect to W^*; for instance $r(v_3|W^*) = r(v_4|W^*)$. Hence, W^*fails to resolve Dj_n. Thus $\dim(Dj_n) \geq 3 + 3\left\lfloor \frac{n}{4} \right\rfloor$. Therefore $\dim(Dj_n) = 3 + 3\left\lfloor \frac{n}{4} \right\rfloor$ for $n \equiv 3$(mod 4). ∎

4. DISCUSSIONS

The metric dimension of a graph reflects the minimum number of reference vertices required to uniquely identify all vertices through their distance representations. In this study, the metric dimension of the Djembe graph was determined exactly by constructing resolving sets and proving their minimality. The obtained formula shows that the metric dimension depends on the residue class of (n) modulo (4), indicating that the arrangement of vertices in the Djembe graph influences the resolving requirements periodically.

The result is consistent with previous studies showing that graph structure plays a crucial role in determining metric dimension. For example, Chartrand et al. [28] established the foundational theory of metric dimension and demonstrated that different graph structures may require significantly different numbers of resolving vertices. Similar structural dependence has also been observed in cycle-related graphs studied by Ali et al. [8], where the presence of cyclic symmetries affects the uniqueness of vertex representations. The Djembe graph exhibits a comparable phenomenon because its construction combines a cycle-based structure with additional vertices attached to the cycle, creating several groups of vertices with similar distance patterns.

Compared with graph families such as the regular banana tree, regular caterpillar, and regular fireworks graphs studied by Permana and Darmaji [3], the Djembe graph possesses a higher degree of symmetry. Consequently, additional vertices are required in the resolving set to distinguish vertices that would otherwise share identical distance representations. A similar effect has been reported for wheel-related and cycle-based graph classes investigated by Widodo [6] and Saputro et al. [7]. However, the Djembe graph differs from these graphs because the additional layers of vertices generate more complex distance relationships, leading to a distinct formula for its metric dimension.

The obtained result also complements studies on special graph classes, such as Johnson and Kneser graphs [10], fullerene graphs [11], unicyclic graphs [12], flower graphs [13], and dragon graphs [14]. While the metric dimensions of those graph families are determined by their own structural characteristics, the present work shows that the Djembe graph exhibits a regular growth pattern in which the metric dimension increases approximately linearly with the graph order. This suggests that the additional structural components introduced in the Djembe graph contribute systematically to the complexity of vertex resolution.

Furthermore, the methodology employed in this paper follows a common approach in metric dimension research, namely, establishing an upper bound by explicitly constructing resolving sets and proving a matching lower bound by contradiction. Similar proof techniques have been successfully applied in studies of graph amalgamations [17][18], graph products [7], and subdivision-related graph constructions [22]. The agreement between the upper and lower bounds confirms the optimality of the proposed resolving sets and guarantees the exactness of the obtained formula.

Overall, this study extends the literature on metric dimension by introducing the Djembe graph as a new graph family whose resolvability properties have been completely characterized. The results provide additional insight into how structural modifications of cycle-based graphs affect distance representations and resolving sets. Moreover, they may serve as a foundation for future investigations on related parameters, including local metric dimension, strong metric dimension, partition dimension, and locating chromatic number of Djembe-type graphs.

5. CONCLUSIONS

In this paper, the metric dimension of the Djembe graph (DJ_n) was determined for all integers ($n \geq 3$). By constructing suitable resolving sets and proving their minimality, an exact formula for the metric dimension was obtained. The result shows that the metric dimension depends on the residue class of n modulo 4, with a special case occurring when $n = 7$. This study provides a complete characterization of the metric dimension of the Djembe graph and extends the existing literature on metric dimension to a graph family that has not previously been investigated from the perspective of graph resolvability. The obtained formula also demonstrates how the structural properties of the Djembe graph influence the size of a minimum resolving set. Future research may consider other distance-based parameters of the Djembe graph, such as the local metric dimension, strong metric dimension, partition dimension, and locating chromatic number. In addition, the investigation of generalized Djembe graphs and Djembe-type graphs arising from graph operations may provide further insights into the relationship between graph structure and resolvability.

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