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Sombor Index, Randic Index, and Forgotten Index of Fuzzy Coprime Graph on Dihedral Group

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Abstract

This research investigates the topological indices—Sombor, Randic, and Forgotten—of the fuzzy coprime graph constructed from dihedral groups. These indices, which quantify graph structural properties, have important applications in mathematical chemistry and algebraic graph theory. The study begins by defining fuzzy graphs and coprime graphs, and then introduces the fuzzy coprime graph as a combination of both. Focusing on the dihedral group D_{2n}, with $n = p$ where p is an odd prime, the paper classifies vertices and edges based on their membership values and determines the degree of each vertex. Using these classifications, general formulas for each index are derived. The results show that the fuzzy prime-coprime graph of the dihedral group forms a complete tripartite graph, enabling explicit computation of the Sombor index, Randic index, and Forgotten index. This work extends previous studies by integrating fuzziness into group-based graph representations, offering a new approach to analyzing algebraic structures through topological indices. The findings provide a foundation for future exploration of fuzzy algebraic graphs and their applications in mathematical modeling.

Keywords: Fuzzy graph; Dihedral group; Coprime graph; Sombor index; Randic index; Forgotten index.

Abstrak

Penelitian ini menyelidiki indeks topologis—Sombor, Randić, dan Forgotten—pada graf koprima fuzzy yang dibangun dari grup dihedral. Indeks-indeks ini, yang mengukur properti struktural graf, memiliki aplikasi penting dalam kimia matematika dan teori graf aljabar.Studi ini dimulai dengan mendefinisikan graf fuzzy dan graf koprima, kemudian memperkenalkan graf koprima fuzzy sebagai kombinasi keduanya. Dengan fokus pada grup dihedral D_{2n}*, dengan* $n = p$ *dengan* p *adalah bilangan prima ganjil), penelitian mengklasifikasikan simpul dan sisi berdasarkan nilai keanggotaannya serta menentukan derajat setiap simpul. Berdasarkan klasifikasi ini, rumus umum untuk setiap indeks diturunkan Hasil penelitian menunjukkan bahwa graf koprima fuzzy dari grup dihedral membentuk struktur graf tripartit lengkap, sehingga memungkinkan perhitungan eksplisit untuk indeks Sombor, indeks Randic, dan indeks Forgotten. Karya ini memperluas penelitian sebelumnya dengan mengintegrasikan konsep fuzziness ke dalam representasi graf berbasis grup, menawarkan pendekatan baru untuk menganalisis struktur aljabar melalui indeks topologis. Temuan ini memberikan dasar untuk eksplorasi lebih lanjut mengenai graf aljabar fuzzy dan aplikasinya dalam pemodelan matematika.*

Kata Kunci: *Graf fuzzy; Grup dihedral; Graf koprima; Indeks Sombor; Indeks Randic; Indeks Forgotten*.

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1. INTRODUCTION

A graph is a mathematical structure consisting of a non-empty set of vertices and a set of edges connecting those vertices. Research on graphs has been extensively conducted in recent years, with mathematicians studying various representations of graphs, such as commuting and non-commuting graphs, cycle graphs, identity graphs, and zero-divisor graphs [1]. This research is a study on graph representations from various fields, one of which is discussed in this research is the graph representation in the field of algebraic structures, specifically focusing on dihedral groups. A dihedral group is a group consisting of a set of elements that includes rotation elements $\{a\}$ and reflection elements $\{b\}$ of a regular polygon with n sides [2] [3].

The graph representation of dihedral groups includes various types of graphs, such as coprime graphs, non-coprime graphs, power graphs, and unit graphs. Specifically, this research focuses on studying fuzzy coprime graphs. The concept of a fuzzy graph was first introduced by [4]. A fuzzy graph is defined as a graph consisting of a set of vertices and a set of edges. A fuzzy graph is a pair of functions denoted by $G: (V, \sigma, \mu)$, where σ is a fuzzy subset of a set V and μ is a fuzzy relation on σ. It is assumed that V is finite and nonempty, and μ is reflexive and symmetric [5] [6]. Meanwhile, the coprime graph is a graph whose vertices consist of all the elements of the group, where two distinct vertices are adjacent if the orders of both vertices are either 1 [7] [8]. This research combines both graph concepts.

There have been many studies on the graphical representations of various groups, such as the prime graph characteristics of finite groups [9], the structure of coprime graphs on groups of integers modulo n [10], and the coprime graph structures of dihedral groups [11]. In particular, the research conducted by [12] and [13] focused on various indices related to the prime coprime graph of the group of integers modulo n were studied. In the same graph, [14] represented a different group, namely the dihedral group. Meanwhile, [15] research operations on fuzzy graphs. The representation of fuzzy graphs was further explored by [16] and [17], who defined their properties and structure. Research by [18] applied fuzzy graphs to the problem of exam scheduling, whereas [19] applied fuzzy graphs to classify traffic routes.

In graph theory, a concept known as topological indices describes numerical values that represent the structural properties and connectivity of graphs [20]. Topological indices are widely used across various fields, particularly in chemistry, where they serve to numerically represent chemical structures and to predict chemical properties, molecular physical structures, and chemical reactions [21] [22]. This research focuses on determining the topological indices of graphs, specifically the Sombor index, the Randic index, and the Forgotten index. The Randić index is one of the concepts frequently used in graph theory [23]. This index was first introduced by Milan Randić in 1975 as the "branching index," which was used to study the degree of branching of carbon atoms in saturated hydrocarbon compounds. The Sombor index is a degree-based topological index introduced to describe molecular structures. This topological index is motivated by the geometric interpretation of the degree radius of an edge, defined as the distance from the origin to the corresponding ordered pair [24] [25]. The forgotten index has been determined for various graphs, including molecular graphs, star graphs, tree graphs, complete graphs, and several other special classes of graphs. This index is used to measure the properties of molecular structures based on the degrees of their vertices, particularly by taking into account the sum of the cubes of these degrees [26].

2. DEFINITIONS

This research is a quantitative study that uses a literature review of prior studies. The research begins with a literature review, followed by deriving the general formula for the Sombor index, the Randic index, and the Forgotten index of fuzzy coprime graphs on the dihedral group, generalized for several cases of n. In exploring the properties of the dihedral group, D_{2n} as discussed by [3], the group is composed of a regular polygon with n sides, where $\langle x \rangle$ denotes a rotation by an angle of $\frac{360^0}{n}$ and $\langle y \rangle$ represents a reflection. The group representation of the dihedral groups is expressed as:

$$D_{2n} = \{\langle x, y \rangle | x^n = y^2 = e, x^{-1} = bab^{-1}\}, n \in \mathbb{N}, n \neq 1,2. \quad (1)$$

In group theory, there is a concept called the order of an element, which is defined as follows.

Definition 1. [10] If G is a group with identity e and $x \in G$, the order of x is the power of natural number such that $x^k = e$ is denoted by $|x| = k$.

The fuzzy coprime graph of a finite group is constructed based on the definitions of fuzzy graphs and coprime graphs, which are defined as follows.

Definition 2. [7] Let G be a finite group such that $|G| > 2$. The coprime graph $\Gamma_G = (V, E)$ is defined as a graph where the vertex set V consists of all elements of the groups G. Two distinct vertices x and y in G are said to be adjacent if and only if $gcd(|x|, |y|) = 1$.

Definition 3. [5] A fuzzy graph is a pair of functions such that denoted by $G: (V, \sigma, \mu)$, where σ is a fuzzy subset of a set V and μ is a fuzzy relation on σ. It is assumed that V is finite and nonempty, and μ is reflexive and symmetric. Thus, if $G: (V, \sigma, \mu)$ is a fuzzy graph, then $\sigma: V \to [0,1]$, and $\mu: V \times V \to [0,1]$ is such that $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$, for all $u, v \in V$, where $\wedge$ denotes the minimum and the degree of a vertex u is defined as $d(v) = \sum_{u \neq v, u \in V} \mu(u, v)$.

Based on Definitions 2, Definition 3, and their relevance to social systems, the fuzzy coprime graph is defined as follows.

Definition 4. Let G be a finite group. The fuzzy coprime graph $\Gamma_G = (V, E)$ is defined as a graph where the vertex set V consists of all elements of the groups G. Two distinct vertices x and y in G are said to be adjacent if and only if $gcd(|x|, |y|) = 1$, where p is a prime number. Thus $\Gamma_G: (V, \sigma, \mu)$ is a fuzzy coprime graph, then $\sigma: V \to [0,1]$, and $\mu: V \times V \to [0,1]$. The membership value of a vertex is defined as follows:

$$\sigma(u) = \frac{1}{|u|}, \quad u \in V, \quad (2)$$

where, if u and v are adjacent, then

$$\mu(u, v) = \begin{cases} \frac{1}{lcm(|u|,|v|)} & , gcd(|u|, |v|) = 1 \\ 0 & , else \end{cases}, \quad (3)$$

for all $u, v \in V$, where $\wedge$ denotes the minimum and the degree of a vertex u is defined as

$$d(v) = \sum_{u \neq v, u \in V} \mu(u, v). \tag{4}$$

Example 1. Let Γ_{D_6} be the fuzzy coprime graph of D_6 and the membership value of the vertices is $\sigma(e) = 1$, $\sigma(a) = \sigma(a^2) = \frac{1}{3}$, and $\sigma(b) = \sigma(ab) = \sigma(a^2b) = \frac{1}{2}$. In addition, the membership value of the edge is $\mu(u, v) = \frac{1}{3}$ for all $u, v \in V(\Gamma_{D_6})$. Thus, the fuzzy coprime graph is presented in the figure 1.

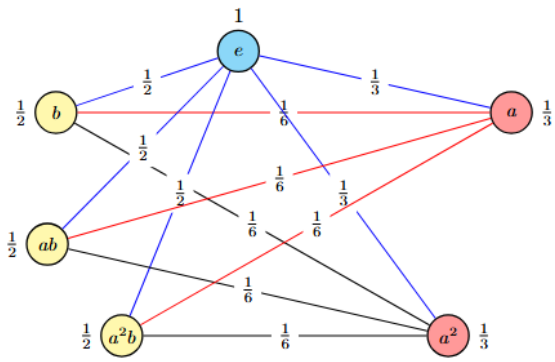

Figure 1. Fuzzy coprime graph of D_6

Moreover, the degree of every vertex in Γ_{D_6} is $d(e) = \frac{13}{6}$, $d(a) = d(a^2) = \frac{5}{6}$, and $d(b) = d(ab) = d(a^2b) = \frac{5}{6}$.

Theorem 1. [8] Let if n is an odd prime number, then the coprime graph of D_{2n} is a complete tripartite.

Proof.
Let n is an odd prime number, define three partition by $V_1 = \{e\}$,$V_2 = \{a, a^2, \cdots, a^{p-1}\}$, and $V_3 = \{b, ab, a^2b, \cdots, a^{p-1}b\}$. Clearly $|e| = 1$, as n is prime number. Since the order of a is n and the order of b is 2 then we have $|a| = |a^2| = \cdots = |a^{p-1}| = p$, and $|b| = |ab| = |a^2b| = \cdots = |a^{p-1}b| = 2$. Hence for each $x, y \in V_i$ then $\gcd(|x|, |y|) = p \neq 1$ or $\gcd(|x|, |y|) = 2 \neq 1$, $i \in \{2,3\}$. Then for any $u \in V_i$ and $v \in V_j$ where $i \neq j$ we have $\gcd(|u|, |v|) = 1$, thus u and v are adjacent so the coprime graph of the dihedral group is complete tripartite graph.

3. RESULTS

In this section, we determine the Sombor index, Randic index, and Forgotten index of the fuzzy coprime graph on the dihedral group of order p, where p prime number (except 2). The fuzzy membership values assigned to the vertices in the fuzzy coprime graph are classified into three distinct categories. This classification is formalized in the following theorem.

Theorem 2. Let D_{2n} be a dihedral group with $n = p$, where p is a prime number (except 2). Define the identity element e, $u \in \{a, a^2, a^3, \ldots, a^{p-1}\}$ and $v \in \{b, ab, a^2b, a^3b, \ldots, a^{p-1}b\}$. $\Gamma_{D_{2p^k}} = (V, \sigma, \mu)$ be the fuzzy coprime graph with V is the vertex set of the graph $\Gamma_{D_{2p^k}}$ and $e, u, v \in V$, then the membership value of the vertex in $\Gamma_{D_{2p^k}}$ is as follows:

1. $\sigma(e) = 1$;
2. $\sigma(u) = \frac{1}{p}$;
3. $\sigma(v) = \frac{1}{2}$.

Proof.
Based on Definition 1, the identity element $e \in D_{2p}$ has order $|e| = 1$, for any element rotations(a) denoted by $u \in \{e, a, a^2, a^3, \dots, a^{p-1}\} \backslash \{e\}$ has order $|u| = p$, and for any element reflection(b) denoted by $v \in \{b, ab, a^2b, a^3b, \dots, a^{p-1}b\}$ has order $|v| = 2$, with $e, u, v \in V$ then based on Definition 4 the membership value of the vertex in $\Gamma_{D_{2p}}$ is as follows:

1. For $e \in V$, the order of e is 1, then $\sigma(e) = \frac{1}{|e|} = 1$;
2. For $u \in V$, the order of u is p, then $\sigma(u) = \frac{1}{|u|} = \frac{1}{p}$;
3. For $v \in V$, the order of v is 2, then $\sigma(v) = \frac{1}{|v|} = \frac{1}{2}$. ■

Theorem 3. Let D_{2n} be a dihedral group with $n = p$, where p is a prime number (except 2). Define $u \in \{a, a^2, a^3, \dots, a^{p-1}\}$ and $v \in \{b, ab, a^2b, a^3b, \dots, a^{p-1}b\}$. Let $\Gamma_{D_{2p^k}} = (V, \sigma, \mu)$ be the fuzzy coprime graph the membership value of an edge in $\Gamma_{D_{2p^k}}$ is as follows:

1. $\mu(e, v) = \frac{1}{2}$;
2. $\mu(e, u) = \frac{1}{p}$;
3. $\mu(u, v) = \frac{1}{2p}$.

Proof.
Based on Definition 1, the identity element $e \in D_{2p}$ has order $|e| = 1$, for any element rotations(a) denoted by $u \in \{a, a^2, a^3, \dots, a^{p-1}\}$ has order $|u| = p$, and for any element reflection(b) denoted by $v \in \{b, ab, a^2b, a^3b, \dots, a^{p-1}b\}$ has order $|v| = 2$. Since only the element $e \in D_{2p}$ has order 1 then based on Definition 4, the membership value of an edge in $\Gamma_{D_{2p}}$ is as follows:

1. $\mu(e, v) = \frac{1}{2}$;
2. $\mu(e, u) = \frac{1}{p}$;
3. $\mu(u, v) = \frac{1}{2p}$. ■

Theorem 4. Let D_{2n} be a dihedral group with $n = p$, where p is a prime number (except 2). Define $u \in \{a, a^2, a^3, \dots, a^{p-1}\}$ and $v \in \{b, ab, a^2b, a^3b, \dots, a^{p-1}b\}$. Let $\Gamma_{D_{2p}} = (V, \sigma, \mu)$ be the fuzzy coprime graph, the membership value of the vertex in $\Gamma_{D_{2p}}$ is as follows:

1. $d(e) = \frac{p^2+2p-2}{2p}$;
2. $d(u) = \frac{p+2}{2p}$ for $u \in \left\{a, a^2, \dots, a^{p^k-1}\right\} \backslash \{e\}$;

3. $d(v) = \frac{2p-1}{2p}$ for $v \in \left\{b, ab, a^2b, \dots, a^{p^k-1}b\right\}$.

Proof.
Based on Theorem 1, $\Gamma_{D_{2p}}$ is a complete tripartite graph with three partitions defined by $V_1 = \{e\}$, $V_2 = \{a, a^2, \dots, a^{p-1}\}$, and $V_3 = \{b, ab, a^2b, \dots, a^{p-1}b\}$. Based on the definition of the degree of a vertex in a coprime fuzzy graph, it follows that:

1. For elements $e \in V_1$,

$$d(e) = \underbrace{\frac{1}{p} + \frac{1}{p} + \cdots + \frac{1}{p}}_{p-1} + \underbrace{\frac{1}{2} + \frac{1}{2} + \cdots + \frac{1}{2}}_{p} = \frac{p-1}{p} + \frac{p}{2} = \frac{p^2+2p-2}{2p}.$$

2. For elements $u \in V_2$,

$$d(u) = \frac{1}{p} + \underbrace{\frac{1}{2p} + \frac{1}{2p} + \cdots + \frac{1}{2p}}_{p} = \frac{1}{p} + \frac{1}{2} = \frac{p+2}{2p}.$$

3. For elements $v \in V_3$,

$$d(v) = \frac{1}{2} + \underbrace{\frac{1}{2p} + \frac{1}{2p} + \cdots + \frac{1}{2p}}_{p-1} = \frac{1}{2} + \frac{p-1}{2p} = \frac{2p-1}{2p}.$$ ■

3.1. Randic Index

The Randić index of a graph is used to analyze the relationships among its vertices and can be particularly useful in mathematical chemistry for studying molecular structures. The Randić index is defined as:

Definition 5. [27] Suppose we are given a graph Γ with a set of vertices $V(\Gamma)$ and a set of edges $E(\Gamma)$. Then the Randic index of Γ, which is denoted by $R(\Gamma)$ is:

$$R(\Gamma) = \sum_{u,v \in E(\Gamma)} \frac{1}{\sqrt{d(u)d(v)}}. \tag{5}$$

Based on Definition 5, the Randić index for a fuzzy coprime graph is defined as follows.

Definition 6 [28] [29] Let $\Gamma = (V, \sigma, \mu)$ be a fuzzy coprime graph with a set of vertices $V(\Gamma)$ and a set of edges $E(\Gamma)$. Then the Randic index of Γ, which is denoted by $R(\Gamma)$ is:

$$R(\Gamma) = \sum_{u,v \in E(\Gamma)} \frac{1}{\sqrt{\sigma(u)d(u)\sigma(v)d(v)}}. \tag{6}$$

Example 2. From Example 1, we get the Randic index of the fuzzy coprime graph Γ_{D_6} as follows.

$$R(\Gamma_{D_6}) = \frac{1}{\sqrt{\big(\sigma(e)d(e)\big)\big(\sigma(a)d(a)\big)}} + \frac{1}{\sqrt{\big(\sigma(e)d(e)\big)\big(\sigma(a^2)d(a^2)\big)}}$$

$$+\frac{1}{\sqrt{(\sigma(e)d(e))(\sigma(b)d(b))}}+\frac{1}{\sqrt{(\sigma(e)d(e))(\sigma(ab)d(ab))}}$$
$$+\frac{1}{\sqrt{(\sigma(e)d(e))(\sigma(a^2b)d(a^2b))}}+\frac{1}{\sqrt{(\sigma(a)d(a))(\sigma(b)d(b))}}$$
$$+\frac{1}{\sqrt{(\sigma(a)d(a))(\sigma(ab)d(ab))}}+\frac{1}{\sqrt{(\sigma(a)d(a))(\sigma(a^2b)d(a^2b))}}$$
$$+\frac{1}{\sqrt{(\sigma(a^2)d(a^2))(\sigma(b)d(b))}}+\frac{1}{\sqrt{(\sigma(a^2)d(a^2))(\sigma(ab)d(ab))}}$$
$$+\frac{1}{\sqrt{(\sigma(a^2)d(a^2))(\sigma(a^2b)d(a^2b))}}$$
$$=\frac{1}{\sqrt{\left(\frac{13}{6}\right)\left(\frac{5}{18}\right)}}+\frac{1}{\sqrt{\left(\frac{13}{6}\right)\left(\frac{5}{18}\right)}}+\frac{1}{\sqrt{\left(\frac{13}{6}\right)\left(\frac{5}{12}\right)}}+\frac{1}{\sqrt{\left(\frac{13}{6}\right)\left(\frac{5}{12}\right)}}$$
$$+\frac{1}{\sqrt{\left(\frac{13}{6}\right)\left(\frac{5}{12}\right)}}+\frac{1}{\sqrt{\left(\frac{5}{18}\right)\left(\frac{5}{12}\right)}}+\frac{1}{\sqrt{\left(\frac{5}{18}\right)\left(\frac{5}{12}\right)}}+\frac{1}{\sqrt{\left(\frac{5}{18}\right)\left(\frac{5}{12}\right)}}$$
$$+\frac{1}{\sqrt{\left(\frac{5}{18}\right)\left(\frac{5}{12}\right)}}+\frac{1}{\sqrt{\left(\frac{5}{18}\right)\left(\frac{5}{12}\right)}}+\frac{1}{\sqrt{\left(\frac{5}{18}\right)\left(\frac{5}{12}\right)}}$$
$$=\frac{2}{\sqrt{\left(\frac{65}{108}\right)}}+\frac{3}{\sqrt{\left(\frac{65}{72}\right)}}+\frac{6}{\sqrt{\left(\frac{25}{216}\right)}}$$
$$=\frac{36}{65}\sqrt{\frac{65}{3}}+\frac{36}{65}\sqrt{\frac{65}{2}}+\frac{216}{5}\sqrt{\frac{1}{6}}.$$

Theorem 5. Let D_{2n} be a dihedral group with $n = p$, where p is a prime number (except 2), then the Randic index of $\Gamma_{D_{2p}}$ is

$$R\left(\Gamma_{D_{2p}}\right)=\frac{2p^3-2p^2}{p^3+4p^2+2p-4}\sqrt{\frac{p^3+4p^2+2p-4}{p}}+\frac{4p^2}{2p^3+3p^2-6p+2}\sqrt{\frac{2p^3+3p^2-6p+2}{2}}+\frac{4p^4-4p^3}{2p^2+3p-2}\sqrt{\frac{2p^2+3p-2}{2p}}. \tag{7}$$

Proof.
Based on Theorem 1, $\Gamma_{D_{2p}}$ is a complete tripartite graph with define three partition by $V_1 = \{e\}$, $V_2 = \{a, a^2, \dots, a^{p-1}\}$, and $V_3 = \{b, ab, a^2b, \dots, a^{p-1}b\}$. The number of vertex pair in $\Gamma_{D_{2p}}$ is as follows:

1. For $u \in V_1$ and $v \in V_2$. The number of vertex pairs (u, v) is $p - 1$.
2. For $u \in V_1$ and $v \in V_3$. The number of vertex pairs (u, v) is p.
3. For $u \in V_2$ and $v \in V_3$. The number of vertex pairs (u, v) is $p^2 - p$.

Based on Theorem 2, Theorem 3, Theorem 4, and Definition 6 then Randic index of $\Gamma_{D_{2p}}$ is as follows.

$$
\begin{aligned}
R\left(\Gamma_{D_{2p}}\right) &= \sum_{(u,v)\in E_{v_1v_2}} \frac{1}{\sqrt{\left(\frac{p^2+2p-2}{2p}\right)\left(\frac{p+2}{2p^2}\right)}} + \sum_{(u,v)\in E_{v_1v_3}} \frac{1}{\sqrt{\left(\frac{p^2+2p-2}{2p}\right)\left(\frac{2p-1}{4p}\right)}} \\
&\quad + \sum_{(u,v)\in E_{v_1v_3}} \frac{1}{\sqrt{\left(\frac{p+2}{2p^2}\right)+\left(\frac{2p-1}{4p}\right)}} \\
&= \frac{p-1}{\sqrt{\frac{p^3+4p^2+2p-4}{4p^3}}} + \frac{p}{\sqrt{\frac{2p^3+3p^2-6p+2}{8p^2}}} + \frac{p^2-p}{\sqrt{\frac{2p^2+3p-2}{8p^3}}} \\
&= \frac{2p^3-2p^2}{p^3+4p^2+2p-4}\sqrt{\frac{p^3+4p^2+2p-4}{p}} \\
&\quad + \frac{4p^2}{2p^3+3p^2-6p+2}\sqrt{\frac{2p^3+3p^2-6p+2}{2}} + \frac{4p^4-4p^3}{2p^2+3p-2}\sqrt{\frac{2p^2+3p-2}{2p}}. \quad \blacksquare
\end{aligned}
$$

3.2. Forgotten Index

Definition 7. [26] Let Γ be a graph with a set of vertices $V(\Gamma)$ and a set of edges $E(\Gamma)$. Then the Forgotten index of Γ, which is denoted by $F(\Gamma)$ is:

$$F(\Gamma) = \sum_{u\in V(G)} d(u)^3. \tag{8}$$

Definition 8. [30] Let Γ be a graph with a set of vertices $V(\Gamma)$ and a set of edges $E(\Gamma)$. Then the Forgotten index of Γ, which is denoted by $F(\Gamma)$ is:

$$F(\Gamma) = \sum_{u\in V(G)} \big(\sigma(u)d(u)\big)^3. \tag{9}$$

Example 3 From **Example 1**, we get the Forgotten index of the fuzzy coprime graph Γ_{D_6} as follows.

$$
\begin{aligned}
F\left(\Gamma_{D_{2p}}\right) &= \big(\sigma(e)d(e)\big)^3 + \big(\sigma(a)d(a)\big)^3 + \big(\sigma(a^2)d(a^2)\big)^3 + \big(\sigma(b)d(b)\big)^3 + \big(\sigma(ab)d(ab)\big)^3 \\
&= \big(\sigma(a^2b)d(a^b)\big)^3 \\
&= \left(\tfrac{13}{6}\right)^3 + \left(\tfrac{5}{18}\right)^3 + \left(\tfrac{5}{18}\right)^3 + \left(\tfrac{5}{12}\right)^3 + \left(\tfrac{5}{12}\right)^3 + \left(\tfrac{5}{12}\right)^3 \\
&= \tfrac{2197}{216} + \tfrac{250}{5832} + \tfrac{375}{1728}
\end{aligned}
$$

$$= \frac{474552+2000+10125}{46656}$$
$$= \frac{486677}{46656}.$$

Theorem 6. Let D_{2n} be a dihedral group with $n = p$, where p is a prime number (except 2), then the Forgotten indeks of $\Gamma_{D_{2p}}$ is

$$F\left(\Gamma_{D_{2p}}\right) = \frac{8p^9+48p^8+56p^7-140p^6-90p^5+199p^4-24p^3+48p^2-32p-64}{64p^6}. \quad (10)$$

Proof.
Based on Theorem 1, $\Gamma_{D_{2p}}$ is a complete tripartite graph with define three partition by $V_1 = \{e\}$, $V_2 = \{a, a^2, \dots, a^{p-1}\}$, and $V_3 = \{b, ab, a^2b, \dots, a^{p-1}b\}$, with $|V_1| = 1$, $|V_2| = p - 1$, and $|V_3| = p$. Based on Theorem 2, Theorem 3, Theorem 4, and Definition 8 the Forgotten index of $\Gamma_{D_{2p}}$ is as follows.

$$F\left(\Gamma_{D_{2p^k}}\right) = \sum_{u\in V(G)} \left(\sigma(u)d(u)\right)^3$$
$$= \left(\frac{p^2+2p-2}{2p}\right)^3 + (p-1)\left(\frac{1}{p}\left(\frac{p+2}{2p}\right)\right)^3 + p\left(\frac{1}{2}\left(\frac{2p-1}{2p}\right)\right)^3$$
$$= \frac{p^6+6p^5+6p^4-16p^3-12p^2+24p-8}{8p^3} + \frac{p^4+5p^3+6p^2-4p-8}{8p^6} + \frac{8p^3-12p^2+6p-1}{64p^2}$$
$$= \frac{8p^9+48p^8+56p^7-140p^6-90p^5+199p^4-24p^3+48p^2-32p-64}{64p^6}. \quad \blacksquare$$

3.3. Sombor Index

Definition 9. [31] [32] Suppose we are given a graph Γ with a set of vertices $V(\Gamma)$ and a set of edges $E(\Gamma)$. Then the Sombor index of Γ, which is denoted by $SO(\Gamma)$ is:

$$SO(\Gamma) = \sum_{u,v\in E(\Gamma)} \sqrt{d(u)^2 + d(v)^2}. \quad (11)$$

Definition 10. [33] Suppose we are given a graph Γ with a set of vertices $V(\Gamma)$ and a set of edges $E(\Gamma)$. Then the Sombor index of Γ, which is denoted by $SO(\Gamma)$ is:

$$SO(\Gamma) = \sum_{u,v\in E(\Gamma)} \sqrt{\left(\sigma(u)d(u)\right)^2 + \left(\sigma(v)d(v)\right)^2}. \quad (12)$$

Example 4. From Example 1, we get the Sombor index of the fuzzy coprime graph Γ_{D_6} as follows.

$$SO\left(\Gamma_{D_6}\right) = \sqrt{\left(\sigma(e)d(e)\right)^2 + \left(\sigma(a)d(a)\right)^2} + \sqrt{\left(\sigma(e)d(e)\right)^2 + \left(\sigma(a^2)d(a^2)\right)^2}$$
$$+\sqrt{\left(\sigma(e)d(e)\right)^2 + \left(\sigma(b)d(b)\right)^2} + \sqrt{\left(\sigma(e)d(e)\right)^2 + \left(\sigma(ab)d(ab)\right)^2}$$
$$+\sqrt{\left(\sigma(e)d(e)\right)^2 + \left(\sigma(a^2b)d(a^2b)\right)^2} + \sqrt{\left(\sigma(a)d(a)\right)^2 + \left(\sigma(b)d(b)\right)^2}$$
$$+\sqrt{\left(\sigma(a)d(a)\right)^2 + \left(\sigma(ab)d(ab)\right)^2} + \sqrt{\left(\sigma(a)d(a)\right)^2 + \left(\sigma(a^2b)d(a^2b)\right)^2}$$

$$+\sqrt{\left(\sigma(a^2)d(a^2)\right)^2+\left(\sigma(b)d(b)\right)^2}+\sqrt{\left(\sigma(a^2)d(a^2)\right)^2+\left(\sigma(ab)d(ab)\right)^2}$$
$$+\sqrt{\left(\sigma(a^2)d(a^2)\right)^2+\left(\sigma(a^2b)d(a^2b)\right)^2}$$
$$=\sqrt{\left(\tfrac{5}{3}\right)^2+\left(\tfrac{4}{9}\right)^2}+\sqrt{\left(\tfrac{5}{3}\right)^2+\left(\tfrac{4}{9}\right)^2}+\sqrt{\left(\tfrac{5}{3}\right)^2+\left(\tfrac{1}{3}\right)^2}+\sqrt{\left(\tfrac{5}{3}\right)^2+\left(\tfrac{1}{3}\right)^2}$$
$$+\sqrt{\left(\tfrac{5}{3}\right)^2+\left(\tfrac{1}{3}\right)^2}+\sqrt{\left(\tfrac{4}{9}\right)^2+\left(\tfrac{1}{3}\right)^2}+\sqrt{\left(\tfrac{4}{9}\right)^2+\left(\tfrac{1}{3}\right)^2}+\sqrt{\left(\tfrac{4}{9}\right)^2+\left(\tfrac{1}{3}\right)^2}$$
$$+\sqrt{\left(\tfrac{4}{9}\right)^2+\left(\tfrac{1}{3}\right)^2}+\sqrt{\left(\tfrac{4}{9}\right)^2+\left(\tfrac{1}{3}\right)^2}+\sqrt{\left(\tfrac{4}{9}\right)^2+\left(\tfrac{1}{3}\right)^2}$$
$$=\tfrac{2}{9}\sqrt{241}+\sqrt{26}+\tfrac{10}{3}$$
$$=\tfrac{2\sqrt{241}+3\sqrt{26}+30}{9}.$$

Theorem 7. Let D_{2n} be a dihedral group with $n=p$, where p is a prime number (except 2), then the Sombor indeks of $\Gamma_{D_{2p}}$ is

$$SO\left(\Gamma_{D_{2p^k}}\right)=\frac{(p-1)}{2p^2}\sqrt{p^6+4p^5-8p^3+5p^2+4p+4}+\frac{1}{4}\sqrt{4p^4+16p^3+4p^2-36p+17}+\tfrac{(p-1)}{4p}\sqrt{4p^4-4p^3+5p^2+16p+16}\,. \quad (13)$$

Proof.

Based on Theorem 1, $\Gamma_{D_{2p}}$ is a complete tripartite graph with define three partition by $V_1=\{e\}$, $V_2=\{a,a^2,\dots,a^{p-1}\}$, and $V_3=\{b,ab,a^2b,\dots,a^{p-1}b\}$. The number of vertex pair in $\Gamma_{D_{2p}}$ is as follows:

1. For $u\in V_1$ and $v\in V_2$. The number of vertex pairs (u,v) is $p-1$.
2. For $u\in V_1$ and $v\in V_2$. The number of vertex pairs (u,v) is p.
3. For $u\in V_2$ and $v\in V_3$. The number of vertex pairs (u,v) is p^2-p.

Based on Theorem 2, Theorem 3, Theorem 4, and Definition 10 then Sombor index of $\Gamma_{D_{2p}}$ is as follows.

$$SO\left(\Gamma_{D_{2p}}\right)=\underbrace{\sqrt{\left(\tfrac{p^2+2p-2}{2p}\right)^2+\left(\tfrac{p+2}{2p^2}\right)^2}+\cdots+\sqrt{\left(\tfrac{p^2+2p-2}{2p}\right)^2+\left(\tfrac{p+2}{2p^2}\right)^2}}_{p-1}$$
$$\underbrace{\sqrt{\left(\tfrac{p^2+2p-2}{2p}\right)^2+\left(\tfrac{2p-1}{4p}\right)^2}+\cdots+\sqrt{\left(\tfrac{p^2+2p-2}{2p}\right)^2+\left(\tfrac{2p-1}{4p}\right)^2}}_{p}$$
$$\underbrace{\sqrt{\left(\tfrac{p+2}{2p^2}\right)^2+\left(\tfrac{2p-1}{4p}\right)^2}+\cdots+\sqrt{\left(\tfrac{p+2}{2p^2}\right)^2+\left(\tfrac{2p-1}{4p}\right)^2}}_{p^2-p}$$

$$= (p-1)\sqrt{\frac{p^6+4p^5-8p^3+5p^2+4p+4}{4p^4}} + (p)\sqrt{\frac{4p^4+16p^3+4p^2-36p+17}{16p^2}}$$
$$= +(p^2-p)\sqrt{\frac{4p^4-4p^3+5p^2+16p+16}{16p^4}}$$
$$= \frac{(p-1)}{2p^2}\sqrt{p^6+4p^5-8p^3+5p^2+4p+4}$$
$$+\frac{1}{4}\sqrt{4p^4+16p^3+4p^2-36p+17} + \frac{(p-1)}{4p}\sqrt{4p^4-4p^3+5p^2+16p+16}\ .\blacksquare$$

4. DISCUSSION

This study derives general formulas for the Sombor, Randic, and Forgotten indices on the fuzzy coprime graph of dihedral groups, extending previous research findings. The results are consistent with earlier studies but offer a new approach by combining fuzzy graph concepts with dihedral group structures. This research opens opportunities for further exploration of topological indices within broader algebraic structures.

The integration of fuzzy relations into algebraic graphs provides a more realistic framework for modeling uncertainty in algebraic systems. Previous studies on conventional coprime graphs of finite groups, such as those by [7], established foundational properties but were limited to crisp (non-fuzzy) settings. By applying the fuzzy graph theory introduced by [4], this study captures the degree of connectivity between elements of dihedral groups based on their coprimality. Furthermore, the calculation of topological indices like the Sombor index originally proposed by [34] to describe geometric properties of graphs on this fuzzy algebraic structure shows that topological invariants can reflect complex algebraic characteristics. This aligns with recent trends where topological indices are employed to characterize the structural compactness of algebraic graph.

5. CONCLUSIONS

This research formulates general expressions for the Sombor, Randić, and Forgotten indices of the fuzzy prime coprime graph on the dihedral group D_{2p}, where p is an odd prime. By combining the structure of dihedral groups with fuzzy and coprime graph concepts, the resulting graph is shown to be a complete tripartite graph. This structure facilitates the accurate computation of the selected topological indices. The findings contribute a novel approach to studying algebraic structures through fuzzy graph theory. Future research may extend this approach to other group types or apply fuzzy coprime graphs in areas such as network analysis, optimization, and mathematical modeling.

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