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The Total Irregularity Strength of a Comb Product of Stars

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Abstract

A totally irregular total k-labeling $\lambda : V \cup E \to \{1, 2, \cdots, k\}$ of a graph G is a labeling where the weights of all distinct vertices and edges are unique. The weight $w(x)$ of a vertex x is defined as the sum of its label and the labels of all edges incident to it, while the weight $w(e)$ of an edge e is the sum of its label and the labels of its two endpoints. The minimum k for which G admits such a labeling is known as the total irregularity strength of G, denoted $ts(G)$. This study focuses on determining $ts(G)$ for specific classes of trees, including the comb product of stars, where the contact vertex is the central vertex of one star, and the triple star graph.

Keywords: comb product; star; total irregularity strength; totally irregular total labeling graph.

Abstrak

Pelabelan k-total tak teratur total $\lambda : V \cup E \to \{1, 2, \cdots, k\}$ dari suatu graf G adalah suatu pelabelan sedemikian sehingga bobot setiap titik dan sisi masing-masing berbeda. Bobot suatu titik $w(x)$ adalah jumlah label titik x dan label setiap sisi yang terkait ke x, dan bobot suatu sisi $w(e)$ adalah jumlah label sisi e dan kedua titik yang terkait ke e. Nilai minimum k sehingga suatu graf g memiliki pelabelan tersesebut dikenal sebagai nilai ketakteraturan total dari G, dinotasikan dengan $ts(G)$. Pada artikel ini, ditentukan nilai ketakteraturan total dari suatu kelas graf pohon, yaitu hasil operasi comb dari graf bintang, dimana titik tetapnya adalah titik pusat graf bintang, dan graf bintang tripel.

Kata Kunci: *hasil operasi comb; graf bintang, nilai ketakteraturan total; pelabelan total tak teratur total.*

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1. INTRODUCTIONS

Let $G = (V, E)$ be a finite, simple, undirected, and connected graph with vertex set V and edge set E. A labeling of G is a mapping that assigns numbers, typically positive or non-negative integers, to elements of the graph. Depending on the domain of the labeling, it is referred to as a vertex labeling (when the domain is V), an edge labeling (when the domain is E), or a total labeling (when the domain is $V \cup E$). A comprehensive survey of various graph labeling methods can be found in Gallian [1].

In [2], Bača et al. introduced the concept of edge irregular total labeling and vertex irregular total labeling. Under a total labeling $\lambda : V \cup E \to \{1, 2, \cdots, k\}$, the weight of an edge xyxy is defined as $w(xy) = \lambda(x) + \lambda(y) + \lambda(xy)$, and the weight of a vertex x is $w(x) = \lambda(x) + \sum_{xz \in E} \lambda(xz)$. A total labeling λ is called an edge irregular total labeling if the weights of any two distinct edges x_1y_1 and x_2y_2 in E satisfy $w(x_1y_1) \neq w(x_2y_2)$. Similarly, λ is a vertex irregular total labeling if the weights of any two distinct vertices x and y in V satisfy $w(x) \neq w(y)$. The smallest k for which G admits an edge irregular total kk-labeling is the total edge irregularity strength, denoted by $tes(G)$. Likewise, the

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smallest k for which G admits a vertex irregular total kk-labeling is the total vertex irregularity strength, denoted by $tvs(G)$. They proved that $\left\lceil\frac{|E|+2}{3}\right\rceil \leq tes(G) \leq n$ and $\left\lceil\frac{|V|+\delta}{\Delta+1}\right\rceil \leq tvs(G) \leq |V| + \Delta - 2\delta + 1$. In [3], Ivanco and Jendrol proved that for any tree T,

$$tes(T) = \max\left\{\left\lceil\frac{|E|+2}{3}\right\rceil, \left\lceil\frac{\Delta+1}{2}\right\rceil\right\}. \quad (1)$$

Later on, Nurdin et al. in [4] gave the lower bound for $tvs(T)$ where T is any tree having n_i vertices of degree i $(i = 1, 2, \cdots, \Delta)$ where Δ is the maximum degree in T.

$$tvs(T) \geq \max\left\{\left\lceil\frac{1+n_1}{2}\right\rceil, \left\lceil\frac{1+n_1+n_2}{3}\right\rceil, \cdots, \left\lceil\frac{1+n_1+n_2+\cdots+n_\Delta}{\Delta+1}\right\rceil\right\}. \quad (2)$$

Nurdin et al. [4] proved that the lower bound is sharp for any tree T_1 with no vertex of degree 2, that is

$$tvs(T_1) = \left\lceil\frac{1+n_1}{2}\right\rceil. \quad (3)$$

Marzuki et al. [5] unified the concepts of edge and vertex labeling by introducing the notion of a totally irregular total k-labeling. For a graph G with vertex set V and edge set E, a totally irregular total k-labeling $\lambda : V \cup E \to \{1, 2, \cdots, k\}$ is a total labeling where every pair of distinct edges x_1y_1 and x_2y_2 in E have distinct weights w$(x_1y_1) \neq w(x_2y_2)$, and every pair of distinct vertices x and y in V also have distinct weights $(x) \neq w(y)$. The smallest k for which G admits such a labeling is called the total irregularity strength of G, denoted by $ts(G)$. They gave the lower bound for $ts(G)$,

$$ts(G) = \max\{tes(G), tvs(G)\}. \quad (4)$$

In [5], they proved that the lower bound is exact for a path P_n when $n \neq 2$ or 5, as well as for a cycle C_n. Specifically, for a path P_n with n vertices, the lower bound holds,

$$ts(P_n) = \begin{cases} \left\lceil\frac{n+2}{3}\right\rceil, & \text{for } n = 2,5; \\ \left\lceil\frac{n+1}{3}\right\rceil, & \text{otherwise.} \end{cases} \quad (5)$$

In [6-9], Tilukay et al. also came to the same conclusion for fans, wheels, triangular books, friendship graphs, complete graphs, complete bipartite graphs, and corona product of paths. As also concluded by Ramdani et al. in [10-12] and Marzuki et al. that improved their previous results by working on m-copy of paths and m-copy of cycles in [13].

Research on total irregularity strength of class of trees given by Indriati et al. in [14] for determined the ts of a double star graph $C(m,n)$, with $m+n$ pendant vertices, where $m,n \geq 2$. They [14] proved that

$$ts(S_{m,n}) = \left\lceil\frac{m+n+1}{2}\right\rceil. \quad (6)$$

They also provided the exact value of ts for a class of caterpillars with the center path of order three $C(n, 0, n)$, $n \geq 2$. Later in [15], Indriati et al. determined the ts of two classes of caterpillars, that is

$C(p,0,0,q)$ and $C(p,0,0,0,p)$. Other results were given by Rosyida et al. in [16-17] for several class of caterpillars. In their research [16-17], they improved the results of ts for some caterpillars with longer center paths. For further results, one can refer to Galian's survey in [1]. In this paper, we deal with a class of trees. We investigate and determine the total irregularity strength of a comb product of stars, where the contact vertex is the center vertex of the star.

2. DEFINITIONS

Definition 1. Let G and H be two graphs. Let o be a vertex of H. A comb product of G and H with a contact vertex o is a graph $G \rhd_o H = (V,E)$ with the vertex set $V = V(G) \times V(H)$ and the edge set $E = \{(x,y)(x',y') | (x,y),(x',y') \in V, (x,y) \sim (x',y')\}$, where $(x,y) \sim (x',y')$ if either $x \sim x'$ and $y = y' = o$, or $x = x'$ and $y \sim y'$. G and H are called a backbone and a finger, respectively.

In other word, the comb product between G and H, in the vertex o is a graph obtaining by taking one copy of G and $|V(G)|$ copies of H and grafting the i-th copy of H on the contact vertex o to the i-th vertex of G. For example, a graph $S_m \rhd_x S_n$ in Figure 1 is a comb product of star graphs S_m and S_n with the contact vertex x is the center vertex of S_n.

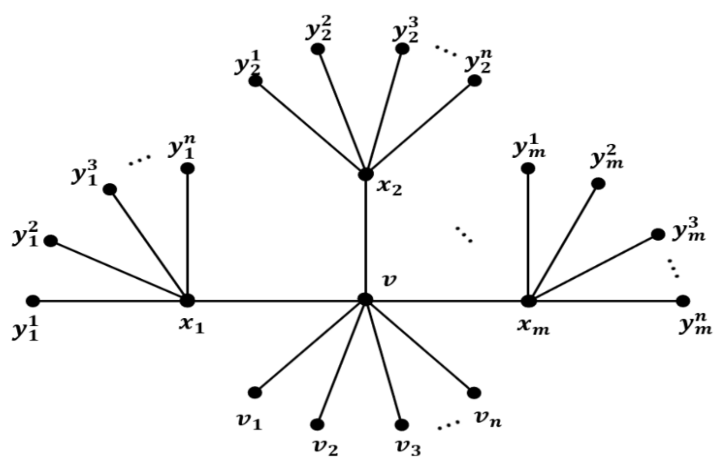

Figure 1. Graph $S_m \rhd_x S_n$

Definition 2. For nonnegative integers m and n, a double star graph $S_{m,n} \cong C(m,n)$ is a caterpillar with the center path of order two, where each of both center vertices adjacent to m and n pendant vertices, respectively.

Definition 3. For $m,p,n \geq 0$, a triple star graph $S_{m,n,p} \cong C(m,n,p)$ is a caterpillar with the center path of order two three, where each of its center vertices adjacent to m, n, and p pendant vertices, respectively.

3. RESULTS

Our first result provides the exact value of the total irregularity strength of a comb product of stars, where the contact vertex is the center vertex.

Theorem 1. Let m, n be positive integers and $m \leq n$. Let S_n be a star with the center vertex x. Let $S_m \rhd_x S_n$ be a comb product of S_m and S_n with the contact vertex x and $mn+n$ pendant vertices. Then

$$ts(S_m \rhd_x S_n) = \left\lceil \frac{mn+n+1}{2} \right\rceil.$$

Proof.
Let $S_m \rhd_x S_n$, where $m, n \geq 1$ and $m \leq n$, be a comb product of a star S_m and S_n at the center vertex x of S_n. The proof is divided into the following two cases.

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Case 1. For $m = 1$ and $n \geq 1$.
For $m = n = 1$, we obtained that $S_1 \rhd_x S_1$ is isomorph to path graph P_4, then by (5), we have $ts(S_1 \rhd_x S_1) = 2$. Next, for $m = 1$ and $n \geq 2$, we obtained that $S_1 \rhd_x S_n$ is isomorph to double star graph $S_{n,n}$, then by (6), we have $ts(S_1 \rhd_x S_n) = \left\lceil \frac{2n+1}{2} \right\rceil$.

Case 2. For $m, n \geq 2$.
Consider that $S_m \rhd_x S_n$ form a tree with $mn+m+n$ edges, $mn+n$ pendant vertices, maximum degree $m+n$, and contains no vertex of degree 2. Then, by (1) and (3), we have, $tes(S_m \rhd_x S_n) = \max\left\{\left\lceil \frac{mn+n+1}{2} \right\rceil, \left\lceil \frac{\Delta+1}{2} \right\rceil\right\}$ and $tvs(S_m \rhd_x S_n) = \left\lceil \frac{mn+n+1}{2} \right\rceil$. Hence, by (4), we have $ts(S_m \rhd_x S_n) \geq \left\lceil \frac{mn+n+1}{2} \right\rceil$. Next, let $V(S_m \rhd_x S_n) = \{v\} \cup \{x_i, v_j, y_i^j \mid 1 \leq i \leq m, 1 \leq j \leq n\}$ and $E(S_m \rhd_x S_n) = \{vx_i, vv_j, x_i y_i^j \mid 1 \leq i \leq m, 1 \leq j \leq n\}$, as depicted in Figure 1. Let $t = \left\lceil \frac{mn+n+1}{2} \right\rceil$ and $r = \left\lceil \frac{t}{n} \right\rceil$.

To establish the reverse inequality, we construct a totally irregular total labeling $\lambda : V \cup E \rightarrow \{1, 2, \cdots, t\}$ defined below.
For the vertex set:

$$\lambda(v) = r(n+1) - t + 1;$$

$$\lambda(x_i) = \begin{cases} 1, & \text{for } 1 \leq i \leq r; \\ m+1, & \text{for } r+1 \leq i \leq m; \end{cases}$$

$$\lambda(v_j) = mn - t + j + 1, \text{ for } 1 \leq j \leq n;$$

$$\lambda(y_i^j) = \begin{cases} 1, & \text{for } 1 \le i \le r-1, 1 \le j \le n; \\ 1, & \text{for } i = r, 1 \le j \le n(1-r)+t; \\ n(i-1)-t+j+1, & \text{for } i = r, n(1-r)+t+1 \le j \le n; \\ n(i-1)-t+j+1, & \text{for } r+1 \le i \le m, 1 \le j \le n. \end{cases}$$

For the edge set:

$$\lambda(vx_i) = \begin{cases} t-r+i, & \text{for } 1 \le i \le r; \\ t-r-m+i, & \text{for } r+1 \le i \le m; \end{cases}$$

$\lambda(vv_j) = t$, for $1 \le j \le n$;

$$\lambda(x_i y_i^j) = \begin{cases} ni-n+j, & \text{for } i = 1, 1 \le j \le n-1; \\ t, & \text{for } i = 1, j = n; \\ ni-n+j, & \text{for } 2 \le i \le r-1, 1 \le j \le n; \\ ni-n+j, & \text{for } i = r, 1 \le j \le n(1-r)+t-1; \\ n, & \text{for } i = r, j = n(1-r)+t; \\ t, & \text{for } i = r, n(1-r)+t+1 \le j \le n; \\ t, & \text{for } r+1 \le i \le m, 1 \le j \le n. \end{cases}$$

From the construction of an irregular total labeling λ above, the minimum label is 1 and the maximum label is t.

To demonstrate that λ is a totally irregular total labeling, we must evaluate the vertex-weight set and edge-weight set to ensure that no two vertices have the same weight and no two edges share the same weight. By calculating all vertex weights and edge weights, we arrive at the following results.

For the vertex-weight set:

$w(v) = \frac{m}{2}(2t-m+1) + r(n+1) + t(n-1) + 1$;

$$w(x_i) = \begin{cases} t-r-\frac{n}{2}(n-1)+(n^2+1)i+1, & \text{for } 1 \le i \le r-1; \\ \frac{1}{2}(n-nr+t)(nr-n+t+1)+(nr-t+1)t+1, & \text{for } i = r; \\ t(n+1)-r+i+1, & \text{for } r+1 \le i \le m; \end{cases}$$

$w(v_j) = mn+j+1$, for $1 \le j \le n$;

$w(y_i^j) = ni-n+j+1$, for $1 \le i \le m, 1 \le j \le n$.

Next, we verify the vertex-weight set and obtain that:

i) $\{w(y_i^j | 1 \le i \le m, 1 \le j \le n\} = \{2, 3, \cdots, mn+1\}$;

ii) $\{w(v_j | 1 \le j \le n\} = \{mn+2, mn+3, \cdots, mn+n+1\}$;

iii) $w(v_n) < w(x_1)$;

iv) $\{w(x_i) | 2 \le i \le r-1\}$ form a consecutive sequence of different n^2+1; and $\{w(x_i) | r+1 \le i \le m\}$ form a consecutive sequence of different 1;

v) And $w(x_i) < w(x_j) < w(v)$ where $1 \le i < j$.

Hence, the vertex weights of $S_m \rhd_x S_n$ are pairwise distinct.

For the edge-weight set:
$w(vx_i) = nr + i + 2,$ for $1 \leq i \leq m$;
$w(vv_j) = r(n+1) + mn - t + j + 2$, for $1 \leq j \leq n$;

$$w(x_i y_i^j) = \begin{cases} ni - n + j + 2, & \text{for } i = 1, 1 \leq j \leq n-1; \\ t + 2, & \text{for } i = 1, j = n; \\ ni - n + j + 2, & \text{for } 2 \leq i \leq r-1, 1 \leq j \leq n; \\ ni - n + j + 2, & \text{for } i = r, 1 \leq j \leq n(1-r) + t - 1; \\ n + 2, & \text{for } i = r, j = n(1-r) + t; \\ ni - n + j + 2, & \text{for } i = r, n(1-r) + t + 1 \leq j \leq n; \\ m + ni - n + j + 2, & \text{for } r + 1 \leq i \leq m, 1 \leq j \leq n. \end{cases}$$

By verifying the edge weights, we obtain that
i) $\{w(x_i y_i^j) | 1 \leq i \leq m, 1 \leq j \leq n\} \cup \{w(vx_i | 1 \leq i \leq m\} = \{3, 4, \cdots, mn + m + 2\}$;
ii) $w(x_i y_i^j) < w(vv_j)$;
iii) and $w(vx_i) < w(vv_j)$.
This implies that the edge weights of $S_m \rhd_x S_n$ are pairwise distinct. Hence, we have proven that λ is a totally irregular total labeling and that $ts(S_m \rhd_x S_n) \leq \left\lceil \frac{mn+n+1}{2} \right\rceil$, $m, n \geq 2$.

Therefore, based on Case 1 and Case 2, it can be concluded that for $m \leq n$, $S_m \rhd_x S_n$ is a totally irregular total graph with $ts(S_m \rhd_x S_n) = \left\lceil \frac{mn+n+1}{2} \right\rceil$. ■

Furthermore, it can be seen that $S_2 \rhd_x S_n$ is isomorphic to a triple star graph $S_{n,n,n}$. This leads to Lemma 2.

Lemma 2. For a positive integer n, let $S_{n,n,n}$, with $n \geq 2$, be a triple star graph with $3n$ pendant vertices. Then

$$ts(S_{n,n,n}) = \begin{cases} 3, & \text{for } n = 1; \\ \left\lceil \frac{3n+1}{2} \right\rceil, & \text{otherwise.} \end{cases}$$

Proof.
Let $S_{n,n,n}$, with $n \geq 1$, be a triple star graph. The proof is divided into the following two cases.

Case 1. For $n = 1$.
Let $V(S_{1,1,1}) = \{x_1, x_2, x_3, y_1, y_2, y_3\}$ and $E(S_{1,1,1}) = \{x_1x_2, x_2x_3, x_1y_1, x_2y_2, x_3y_3\}$. By (1) and (4), we have $ts(S_{1,1,1}) \geq 3$. Next, we construct an irregular total labeling $\lambda : V \cup E \to \{1, 2, 3\}$ defined by $\lambda(y_1) = \lambda(y_3) = \lambda(x_1y_1) = 1$, $\lambda(y_2) = \lambda(x_2y_2) = \lambda(x_3y_3) = \lambda(x_1x_2) = 2$, $\lambda(x_1) = \lambda(x_2) = \lambda(x_3) = \lambda(x_2x_3) = 3$. It is easy to check that the vertex-weights are $w(x_1) = 6, w(x_2) = 10, w(x_3) = 8, w(y_1) = 2, w(y_2) = 4$, and $w(y_3) = 3$; and the edge-weights are $w(x_1x_2) = 8, w(x_2x_3) = 9, w(x_1y_1) = 5, w(x_2y_2) = 7,$ and $w(x_3y_3) = 6$. Since the maximum label is 3, the vertex-weights and the edge-weights are each pairwise distinct. Therefore $ts(S_{1,1,1}) \leq 3$.

Case 2. For $n \geq 2$.
It follows from Theorem 1.
Therefore, by Case 1 and Case 2, it can be concluded that for a positive integer n, $S_{n,n,n}$ is a totally irregular total graph, with $ts(S_{1,1,1}) = 3$, and $ts(S_{n,n,n}) = \left\lceil \frac{3n+1}{2} \right\rceil$, where $n \geq 2$. ■

4. DISCUSSION

The results of this study show that a comb product of stars with center vertex as contact vertex and triple star graphs are totally irregular total graphs with the total irregularity strengths equal to the lower bound, implying that the lower bound is sharp. This lower bound corresponds to the total vertex irregularity strength. We also investigate the results provided by [14-15] which discuss several classes of caterpillars which have vertices of degree 2 as well as those in [16-17] that explore several classes of caterpillars which have no vertex of degree 2, but with many vertices of even degree. In both cases, their total irregularity strengths are equal to the total vertex irregularity strength. This is due to the fact that $tvs(T)$ has a wider range of values, depending on the degree of their center vertices. However, constructing a concise definition of a totally irregular labeling corresponding to $ts(T)$, as presented in our results, is challenging. The labeling is set such that the edge weights follow an arithmetic sequence with a common different of 1, while $ts(T) = tvs(T)$.

Furthermore, by considering results of various labeling of a caterpillar graph, we found that a caterpillar is graceful, harmonious, super edge-magic, and antimagic [1]. Following the work of [3], we conclude that a caterpillar is also total edge irregular graph. Hence, determining whether it is a totally irregular total graph is important. Our result contributes significantly to this area of research. Additionally, it is known a comb product of stars with center vertex as contact vertex is a subtree of a firecracker (a tree where if every pendant vertex is removed, obtained a caterpillar). Therefore, determining that a comb product of stars is a totally irregular total graph is crucial for building patterns for larger structure.

5. CONCLUSIONS

In this paper, we determine the total irregularity strength of the comb product S_m and S_n, where the contact vertex is the center vertex x of S_n, resulting in $mn + n$ pendant vertices, with $m \leq n$. Specifically, we establish that $ts(S_m \rhd_x S_n) = \left\lceil \frac{mn+n+1}{2} \right\rceil$. Additionally, we provide the exact total irregularity strength of the triple star graph $S_{n,n,n}$, showing that $ts(S_{1,1,1}) = 3$ and $ts(S_{n,n,n}) = \left\lceil \frac{3n+1}{2} \right\rceil$, $n \geq 2$.

For future research on the comb product $S_m \rhd_x S_n$, one could consider using a pendant vertex of S_n as the contact vertex to verify that, in general, the comb product of stars is a totally irregular total graph. Moreover, our result in Lemma 2 can be combined with findings from [6] to extend the analysis to any triple star graph $S_{m,n,p}$ or caterpillar graph. There also remain many classes of trees for which the total irregularity strength has yet to be determined.

REFERENCES

[1] J. A. Galian, "A dynamic survey of graph labeling," *Electron. J. Combin.*, vol. 25, no. DS6, 2022.

[2] M. Bača, S. Jendrol', M. Miller, and J. Ryan, "On irregular total labelings," *Discrete Math.*, vol. 307, pp. 1378–1388, 2007.

[3] J. Ivanco and S. Jendrol, "Total edge irregularity strength of trees," *Discuss. Math. Graph Theory*, vol. 26, pp. 449–456, 2006.

[4] Nurdin, E. T. Baskoro, A. N. M. Salman, and N. N. Gaos, "On the total vertex irregularity strength of trees," *Discrete Math.*, vol. 310, pp. 3043–3048, 2010.

[5] C. C. Marzuki, A. N. M. Salman, and M. Miller, "On the total irregularity strengths of cycles and paths," *Far East J. Math. Sci.*, vol. 82, no. 1, pp. 1–21, 2013.

[6] M. I. Tilukay, A. N. M. Salman, and E. R. Persulessy, "On the total irregularity strength of fan, wheel, triangular book, and friendship graphs," *Procedia Comput. Sci.*, vol. 74, pp. 124–131, 2015.

[7] M. I. Tilukay, B. P. Tomasouw, F. Y. Rumlawang, and A. N. M. Salman, "The total irregularity strength of complete and complete bipartite graphs," *Far East J. Math. Sci.*, vol. 102, no. 2, pp. 317–327, 2017.

[8] M. I. Tilukay, P. D. M. Taihuttu, A. N. M. Salman, F. Y. Rumlawang, and Z. A. Leleury, "Complete bipartite graph is a totally irregular total graph," *Electron. J. Graph Theory Appl.*, vol. 9, no. 2, pp. 387–396, 2021.

[9] M. I. Tilukay, "On the total irregularity strength of the corona product of a path with a path," *Tensor: Pure Appl. Math. J.*, vol. 4, no. 1, pp. 21–26, 2023.

[10] R. Ramdani and A. N. M. Salman, "On the total irregularity strength of some cartesian product graphs," *AKCE Int. J. Graphs Comb.*, vol. 10, pp. 199–209, 2010.

[11] R. Ramdani, A. N. M. Salman, and H. Assiyatun, "On the total irregularity strength of regular graphs," *J. Math. Fund. Sci.*, vol. 47, no. 3, pp. 281–295, 2015.

[12] R. Ramdani, A. N. M. Salman, H. Assiyatun, A. Semaničová-Fenovčíková, and M. Bača, "Total irregularity strength of three families of graphs," *Math. Comput. Sci.*, vol. 9, pp. 229–237, 2015.

[13] C. C. Marzuki, F. N. Gianita, R. Fitri, Abdussakir, and F. Aryani, "On the total irregularity strength of m-copy cycles and m-copy paths," *Res. J. Appl. Sci.*, vol. 13, no. 10, pp. 582–586, 2018.

[14] D. Indriati, Widodo, I. E. Wijayanti, and K. A. Sugeng, "On the total irregularity strength of double-star and related graphs," *Procedia Comput. Sci.*, vol. 74, pp. 118–123, 2015.

[15] D. Indriati, Widodo, I. E. Wijayanti, K. A. Sugeng, and I. Rosyida, "Totally irregular total labeling of some caterpillar graphs," *Electron. J. Graph Theory Appl.*, vol. 8, no. 2, pp. 247–254, 2020.

[16] I. Rosyida, Widodo, and D. Indriati, "On total irregular strength of caterpillars with two leaves on each internal vertex," *IOP Conf. Ser.: J. Phys. Conf. Ser.*, vol. 1008, p. 012046, 2018.

[17] I. Rosyida, Mulyono, and D. Indriati, "On totally irregular total labeling of caterpillars having even number of internal vertices with degree three," *AIP Conf. Proc.*, vol. 2326, p. 020024, 2021.