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Optimizing Modem Placement in UPI Building FPMIPA using the Illumination Model

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Abstract

Since a reliable internet signal has become an essential and significant need nowadays, the existence of modems as transmitters of internet signals is also crucial; modems transmit the internet signal (without cable), which is then captured by devices. This research aims to construct a mathematical model to determine the minimum number of modems and their placement so that the entire building FPMIPA-A of UPI has a good internet signal. In this research, we assume that the modem can pass through at most two walls, and the area studied is limited to the first floor of FPMIPA-A. The model is based on the illumination problems theorems, one of which states that every monotone 6-gon can be covered by a single 2-modem point placed at one of its two leftmost (or rightmost) vertices. By the theorem, we view the layout of the rooms in the building as a combination of polygons. The results show that 12 modems are required to cover all areas on the first floor of FPMIPA-A to get a good signal.
Keywords: illumination problem; modem; polygonal regions; optimal modem placement; building FPMIPA A.

Abstrak

Saat ini, kebutuhan akan sinyal internet yang andal menjadi kebutuhan penting dan utama. Modem sebagai pemancar sinyal internet mengirimkan sinyal internet (tanpa kabel) dan kemudian ditangkap oleh perangkat. Penelitian ini bertujuan untuk membangun model matematika yang menentukan jumlah minimum modem dan penempatannya agar seluruh gedung FPMIPA-A UPI mempunyai sinyal internet yang baik. Pada penelitian ini diasumsikan modem dapat menembus paling banyak dua dinding dan area yang diteliti dibatasi pada lantai 1 gedung FPMIPA-A UPI. Model matematika untuk masalah penempatan modem ini didasarkan pada teorema masalah iluminasi, yang salah satunya menyatakan bahwa setiap 6-gon monoton dapat ditutupi oleh satu titik 2-modem yang ditempatkan di salah satu dari dua simpul paling kiri (atau paling kanan). Berdasarkan teorema tersebut, tata ruang pada lantai 1 gedung FPMIPA-A UPI dipandang sebagai kombinasi poligon. Hasil penelitian menunjukkan bahwa dibutuhkan 12 modem untuk mencakup seluruh area di lantai 1 FPMIPA-A guna mendapatkan sinyal yang baik.
Kata Kunci: *masalah iluminasi; modem, daerah poligon; penempatan modem secara optimal; gedung FPMIPA A.*

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1. INTRODUCTION

One critical aspect of using technology is internet connectivity. Modems placed in all faculty buildings of UPI, including building FPMIPA-A, are crucial to ensure the quality of the internet signal. However, modem placement often has challenges, such as how to fulfil internet needs with a minimal number of modems. UPI constantly improves the quality of its public facilities, providing a good internet signal in all parts of the building by placing modems optimally; internet needs are fulfilled with a minimal number of modems.

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Aicholzer et al. [1] mention that to connect a laptop to a modem, at least two factors must be considered: the distance between the laptop and the modem and the number of walls separating the laptop from the modem. Insufficient illumination can affect modem performance and internet signal quality, harming the user's network. This problem is known as the modern illumination problem, an extension of the Art Gallery Problem. In this context, illumination means ensuring that every point in the gallery can be reached by at least one modem signal. The term modem, in the context of this study, refers to a wireless modem. We study the k-modem illumination problem, which is a solution to place several modems with signal coverage penetrating a maximum of k sides in a polygon P with n points so that the signal can reach the entire P area. The main goal is to ensure that all areas in the building are illuminated or receive a signal from the k-modem.

The former study, such as Chvátal (see [2]), formulated a principle that is now known as Chvátal's Art Gallery Theorem (1973), commonly called the Watchman Theorem: for any natural number n, $\lfloor n/3 \rfloor$ guards are needed to watch n-wall polygons (for more detail see [1], [2], [3], [4], [5], [6], [7]). This theorem has been extended and developed in depth by other mathematicians and computer scientists who study partition algorithms to obtain various possible variations, such as exploring multiple aspects of the illumination problem and providing theoretical constraints and practical algorithms for placing modems in various polygonal arrangements. For example, Monroy [8] studied variations of the illumination problem for families of lines, families of line segments, orthogonal polygons, and sets of vertical or horizontal disjoint segments or sets of lines; Duque [9] determined an upper bound on the k-modem illumination problem; and Shamaee [10] determined the minimum number of modems to cover a monotone polygon. Ballinger et al. in [11] developed a constraint on the k-modem illumination problem, specifically for $k = 2$; that is, all simple polygons require at least $\lfloor n/6 \rfloor$2-modem to illuminate, where the modems are placed on the interior boundaries of the polygon. Cannon et al. [12] improved research on this limitation, stating that all simple polygons with n points require as many as $\lfloor n/5 \rfloor$ 2-modem to illuminate the polygon's interior. Mahdavi [13] examined a new variant of covering in an orthogonal art gallery problem where each guard is a sliding k-transmitter.

Although research on modem illumination problems has been widely studied, implementing these theoretical findings in real-world scenarios still needs to be improved. The plan of a building consisting of connected rooms is usually depicted as a combination of several divided quadrangles. Therefore, it is necessary to build a model and make modifications to apply the theorems in Cannon et al. [12]. This research aims to construct mathematical models and implement theorems related to geometry and polygons to solve the modem illumination problem in building FPMIPA-A. It is assumed that the value $k = 2$, representing the modem, has good signal strength to penetrate a maximum of two walls. This assumption is used to ensure smooth internet access from the modem signal so that laptops or other devices in the building can receive the modem signal optimally and that adequate signal availability can be guaranteed to support the functionality of the internet network in building FPMIPA-A of UPI.

2. METHODS

Research methods used in this study consist of several phases as follows.

1) **Literature review on the theorems and lemmas**

In the context of implementing the k-modem illumination problem, the area on a building floor is a graph consisting of several polygons. The edges of the graph are the walls of the building while the vertices are the intersection points of the two edges of the polygon. Suppose that one of the polygons is called polygon P. In a polygon P with n vertices, suppose that there are points $p_i \in P$ for all $i \in \mathbb{N}$. In this article, we say interchangeably between vertices and points, as well as between edges and walls. We recall that the k-modem illumination problem is a solution to place the number of modems needed to reach all points p_i in the polygon P from the modem $q \in \mathbb{R}_2$ which cuts a maximum of k sides if a straight-line segment is constructed (see [1], [13]). A point $q \in \mathbb{R}_2$ is called a k-modem point, and a point $p \in P$ is said to be a 2-modem illuminated from $q \in \mathbb{R}_2$, if the line-segment $\overline{qp}$ intersects at most 2 edges in P.

We implement practically the k-modem illumination problem in the building FPMIPA-A to install and arrange modems, particularly 2-modems. Furthermore, our discussion is limited to the number and position of the 2-modem needed to illuminate the monotonous polygon horizontally in the building FPMIPA- A; that is, the 2-modem signal range applies only to the same floor.

The set of points on P illuminated by the 2-modem of $q \in \mathbb{R}_2$ is called the 2-modem illuminated region, denoted by $2VR(q)$. For $S \subseteq P$, define $2VR(S) := \cup_{q \in S} 2VR(q)$. Any set $C \subseteq P$ is called a valid 2-modem point in P if $2VR(C) = P$ (see [12]). As an illustration, see Figure 1 below. Suppose that there is a polygon P. In Figure 1 (a), suppose that a 2-modem is placed at the point $q_1 \in \mathbb{R}_2$. For $i = 1, \dots, 7$, let p_i be the points representing the region P. We observe that $2VR(q_1) = \{p_i | i = 1,2,3,4,6,7\}$, because the points $p_1, p_2, p_3, p_4, p_6, p_7 \in P$ are illuminated by the 2-modem point q_1 due to the straight-line segment q_1 to $p_1, p_2, p_3, p_4, p_6, p_7$ intersects at most 2 sides. However, for the same reason, $p_5 \notin 2VR(q_1)$. A point illuminates the area is colored red as in Figure 1(a). In Figure 1 (b), suppose that a 2-modem is placed at the point q_3. The entire polygon area P is $2VR(q_3)$. In other words, point q_3 is a valid 2-modem point because q_3 illuminates P.

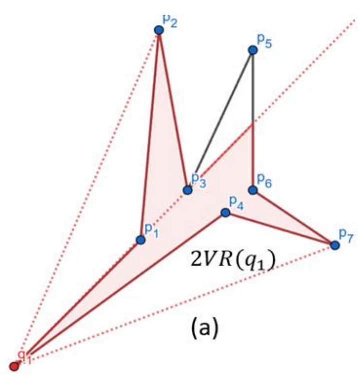

Figure 1. Illustration.

The following are the lemmas and theorems for the 2-modem illumination problem based on [12]. The proof which is given here following [12] but in a more detailed version.

Lemma 1. For each 5-gon can be illuminated by a 2-modem point that is placed anywhere (either on the sides or in the interior of the polygon).

Proof.

It will be shown that every 5-gon is 2-convex. Note that, a pentagon or 5-gon is a polygon with 5 sides and 5 points with a total angle of 540° or a combination of 3 triangle angles, namely 3×180° (see [14]). To prove every 5-gon is 2-convex, the 5-gons are divided into 2 groups. The first group consists of 5-gons where all angles have measures less than 180°. Meanwhile, the second group consists of 5-gons with several angle measures of more than 180°.

Figure 2 shows the shape of the first group of 5-gons. Figure 2 (a) is a 5-gon shape with angles of the same measure, that is 108°. The 5-gon shape in Figure 2 (b) has 2 pairs of equal angle measures and 1 different angle. Figure 2 (c) shows a 5-gon shape where all the angle sizes are different and are less than 180°. Meanwhile, Figure 2 (d) is a 5-gon shape with 1 pair of equal angle measures and 3 different angles.

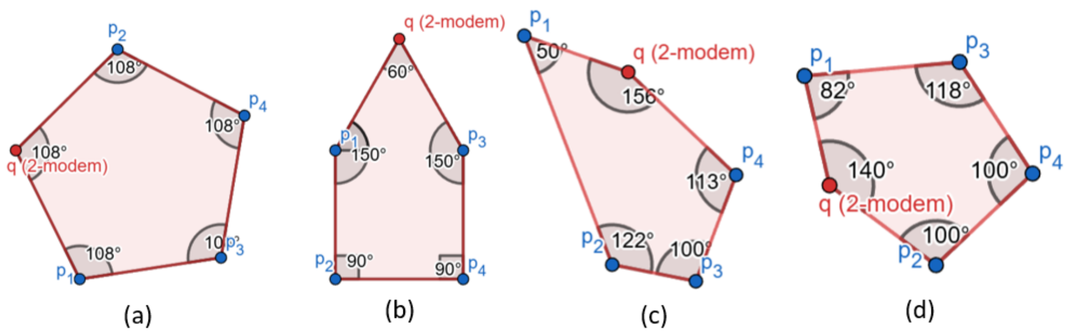

Figure 2. Group 1 of 5-gons; the size of each angle < 180°.

Figure 2 shows that 2 points in a 5-gon with an angle of less than 180° can be connected by a straight-line segment that passes through a maximum of 2 sides in each 5-gon. Thus, every 5-gon with an angle <180° is 2-convex.

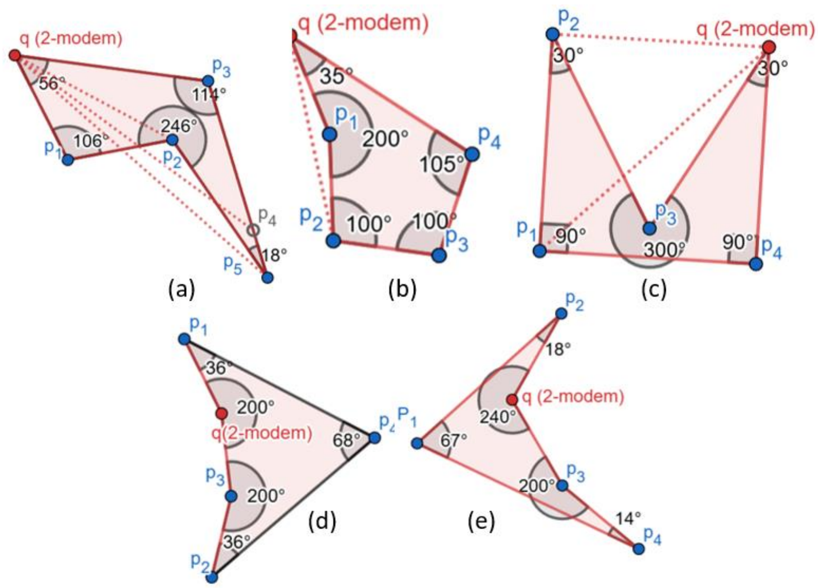

Figure 3. Group 2 of 5-gons; the size of some angles >180.

A similar way to show that 2 points in a 5-gon with some angles size more than 180°, the 2 points can be connected by a straight-line segment that passes through a maximum of 2 sides in each 5-gon as shown in Figure 3. Thus, every 5-gon with an angle >180° is 2-convex, and so every 5-gon is 2-convex.

Lemma 2. Let P be a 6-gon and let $e = \{v, w\}$ be a side of P. A 2-modem point on v illuminates P or a 2-modem point on w illuminates P.

Lemma 3. Each monotone 6-gon P can be illuminated by a 2-modem point placed at one of its two leftmost (or rightmost) vertices: if the two vertices are v_1 and v_2, a 2-modem point either in v_1 or in v_2 , illuminates P.

Theorem 4. A simple polygon with n points requires $\lfloor n/5 \rfloor$ 2-modem to illuminate the interior of the polygon.

Proof.

If we partition P_n into several 5-gons, by **Lemma 1**, a 2-modem can be placed anywhere in the interior of the 5-gon such that the 5-gon is illuminated by a 2-modem. Choose an arbitrary point in the 5-gon area as the 2-modem point. The combination of the partitions of several 5-gons will again form P_n, if the number of n is a multiple of 5 then the 2-modems needed are $n/5$ 2-modems. If $n \geq 5$ and n is not a multiple of 5, then there are 5-gon partitions that intersect each other and have the same 2-modem points for different partitions. Therefore, a simple polygon with n points requires as many as $\lfloor n\ /5 \rfloor$ 2-modems to illuminate the interior of the polygon. The area-colored blue is the area illuminated by a 2-modem because the 2-modem point and any point in the 5-gon can be connected by a straight-line segment that passes through a maximum of 2 sides in the 5-gon.

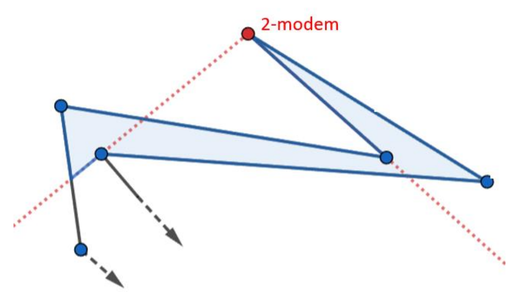

Figure 4. Theorem 4: illustration.

2) **Model construction**

We determine the placement point and number of k-modems by applying the lemmas and theorems in [12]. The stages of completing the model are as follows.

i. Identify n as the points and k as the number of walls penetrated most by a straight line from the k-modem point to the points in the building *P*. Fix the value $k = 2$.
ii. Compute the number of 2-modems needed by applying **Theorem 4**.

iii. Determine a valid 2-modem point such that each floor of the building can be illuminated by a minimum of modems by dividing the graph into several regions that can form 6-gons and then applying **Lemma 2** and **Lemma 3**. If there is a side in the interior of the 6-gon, then modifications need to be made to form a 6-gon with holes, by making the only side in the interior of the area into a 4-sided polygon.

iv. Analyze each direction of the building separately. To carry out this analysis, it is necessary to name the buildings according to their directions. We label the West building with W, the North building with N, the East building with E, and the South building with S. For example: label W-1 refers to the West building of the first floor of building FPMIPA-A.

3) Model validation

Check the modem placement results again by subtracting a 2-modem from the analysis results and placing them randomly to ensure the solution obtained is optimal.

4) Visual representation

Create a visual representation of the modem placement in the building using *GeoGebra* and *PowerPoint* to show the modem position and signal coverage area.

3. RESULTS

In this section, we discuss construction of a mathematical model and implementation of theorems related to geometry and polygons to solve the modem illumination problem in building FPMIPA-A. In this study, there are several ***standing assumptions*** as follows.

i. The modem signal can penetrate a maximum of 2 uniform walls, i.e., the value of $k = 2$.
ii. The main limitation of the modem illumination problem is influenced by the number of walls that restrict the position of the modem to the laptop or other devices, while distance is not.
iii. All types of modems used are considered to have similar signal strength.
iv. The modem signal is unable to penetrate the floor below or above it; the modem signal strength only applies to one floor (horizontal).

To solve the 2-modem illumination problem on the first floor of building FPMIPA-A, we need to present the first-floor plan of building FPMIPA-A in a graph model as shown in Figure 5. The first-floor plan of building FPMIPA-A consists of four parts, namely the West (W-1), East (E-1), North (N-1), and South (S-1). We determine the number of modems and their locations separately in each part.

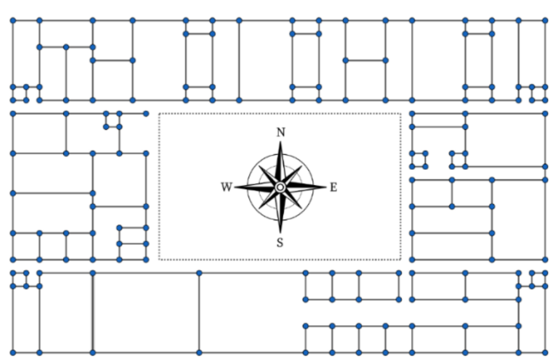

Figure 5. Graph model of the first floor of building FPMIPA-A.

First, we describe the problem solution for the West (W-1). Figure 6 is a sketch of the plan of the West (W-1).

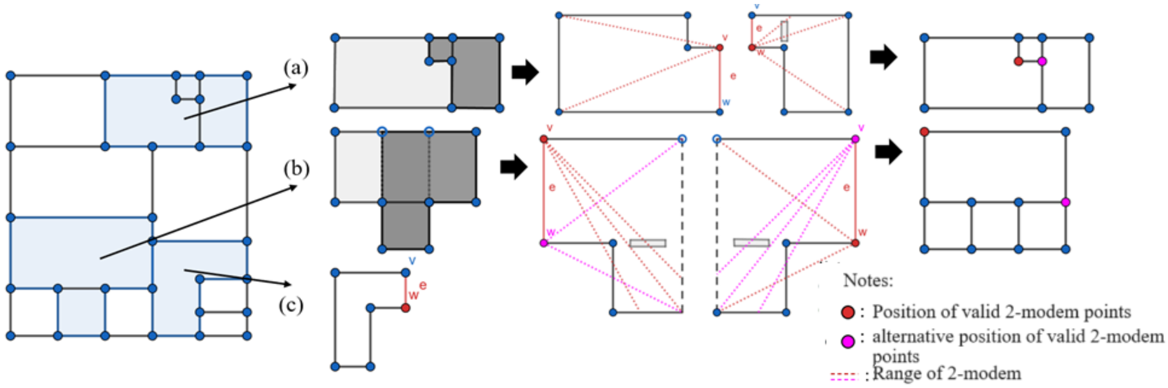

Figure 6. The solution of the 2-modem illumination problem for the graph model W-1.

The stages of solving the 2-modem illumination for the graph model W-1 are as follows. First, partition the graph model W-1 into 6-gons. We observe that W-1 can be divided into 3 6-gon regions, namely regions (a), (b), and (c). The region (a) can be simplified again into smaller 2 6-gons so that **Lemma 3** can be applied.

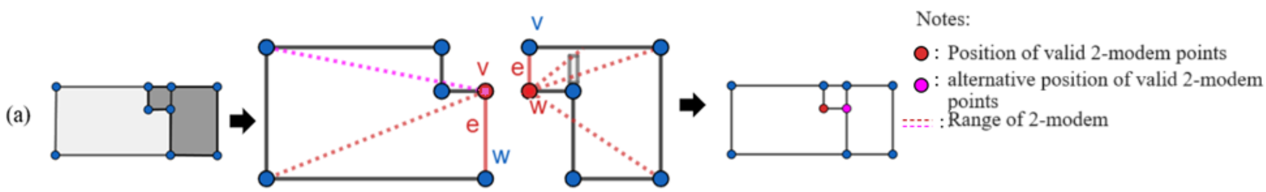

Figure 7. The graph model W-1 and its solution for the region (a).

For the left 6-gon colored in grey in Figure 7, based on **Lemma 3**, we can directly choose the edge e on the rightmost or leftmost side of the 6-gon. Fix the rightmost side of the 6-gon and write it as e, then a vertex v colored in red can be a valid 2-modem point, or a vertex w colored in blue can be an alternative valid 2-modem point with the maximum number of walls penetrated by straight line segments from the 2-modem point to the vertices in graph W-1 is 2 edges.

Meanwhile, the right 6-gon region in dark grey needs to be modified so that it can form a hollow 6-gon by making the only side in the interior of the region a 4-sided polygon as in Figure 7. Furthermore, e can be directly selected on the right or left side of the 6-gon. In Figure 7, suppose we choose the left side of the 6-gon as e, then a vertex w in red can be a valid 2-modem point or a vertex v in blue can be an alternative valid 2-modem point with the number of walls penetrated by the most straight line segments from the 2-modem vertex to the points in the W-1 graph is 2 edges.

By **Theorem 4**, the region (a) with 9 vertices requires $\lfloor 9/5 \rfloor = 1$ 2-modem to illuminate the interior of the region (a). Therefore, the two 2-modem points in the two 6-gon regions are represented as red vertices (valid 2-modem points) or pink vertices (alternative valid 2-modem points).

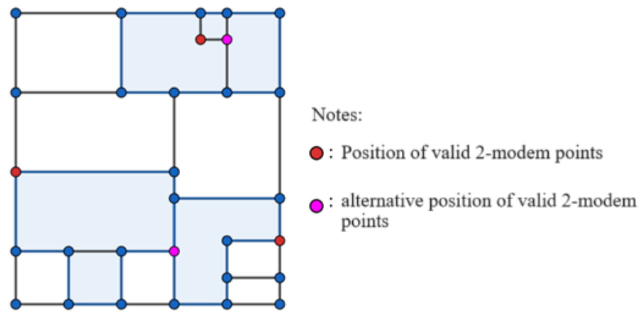

Figure 8. Valid 2-modem points for W-1.

By dividing the graph W-1 into 3 regions representing all points on the graph, we analyze that three 2-modems need to be placed in building W-1 to illuminate building W-1. The location of the three 2-modems can be seen in Figure 8, the valid 2-modem points are marked with red nodes. As an alternative position, the three 2-modems can also be placed in the nodes that are colored pink.

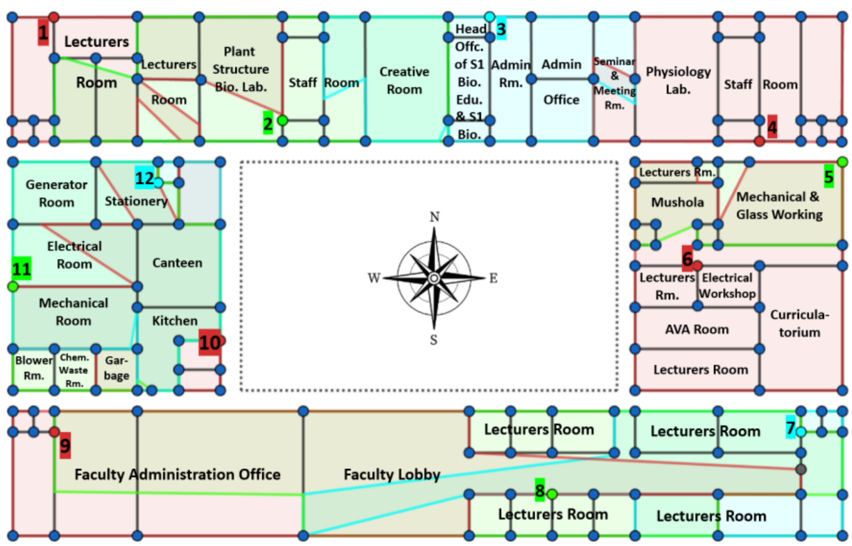

Figure 9. Valid 2-modem positions on the first floor of building FPMIPA-A.

The solution for the other building plans (North, South, and East) is done in a similar way. Based on the analysis results using the illumination problem approach, building W-1 requires three 2-modems, building E-1 requires two 2-modems, building N-1 requires four 2-modems, and building S-1 requires three 2-modems to illuminate the first floor of building FPMIPA-A. In Figure 10, the colored areas (red, green, and blue) indicate the areas inside the first floor of building FPMIPA-A that are illuminated by the 2-modems placed on the colored nodes (red, green, and blue). Thus, a total of twelve 2-modems are needed to illuminate the first floor of building FPMIPA-A. We conclude that twelve modems with signal strength that can penetrate a maximum of two walls are required so that all areas inside the first floor of building FPMIPA-A receive the modem signal, as shown in Figure 9.

Model Validation

Validation of the results is done by reducing a 2-modem and by randomly selecting valid 2-modem points on the graph model from the analysis results through the illumination problem approach. For example, according to the result of the analysis, building E-1 requires two 2-modems. To validate the result, a random position of a 2-modem is selected which is marked with a red point as in Figure 10.

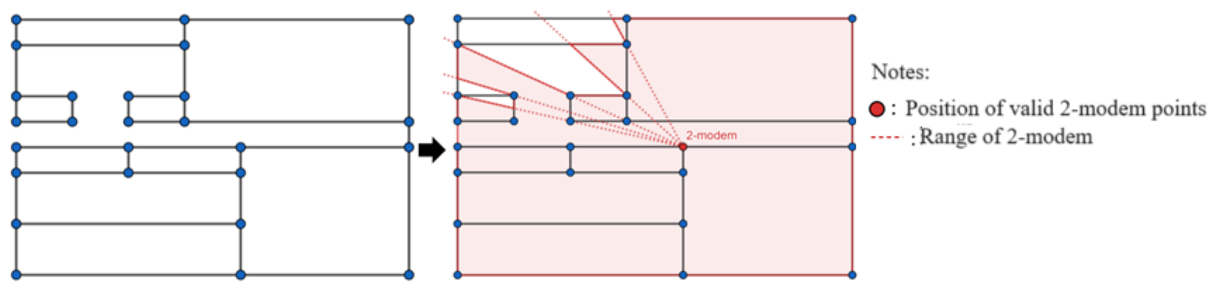

Figure 10. Result validation for graph model of the building E-1.

If a straight-line segment is constructed from a 2-modem point to each vertex of graph E-1 as shown in Figure 10, the results show an area that is not illuminated by a 2-modem. Observe that the colored area is the area illuminated by a 2-modem. This indicates that a 2-modem is not enough to illuminate graph E-1 and the analysis results using the illumination problem approach in this study have produced valid 2-modem points.

4. DISCUSSION

Illumination problems that generalize art gallery problems are applied to solve modem placement problems. In the result of Canon [12], a theorem shows that 1 modem with the power to penetrate two walls is enough to illuminate a hexagon. That result aligns with the study by Aicholzer [1], stating that every orthogonal polygon with at most (k +4) vertices can be illuminated by a k-modem placed anywhere, in the interior, or on the boundary of the polygon. Practically, we implement the theorems on a plan of a building that consists of rooms. However, we can not apply directly as the building plan is usually not in the form of a polygon, but a collection of some intersect rectangles. For that reason, we propose a solution to the problem by using a hexagon as a basis and some rectangles that form a hexagon are viewed as one unit that by the theorems can be illuminated by a single 2-modem.

5. CONCLUSIONS

In this article, a mathematical model has been constructed to solve the modem placement problem on the first floor of FPMIPA-A. The theorems contained in Cannon [12] are the basis for the formation and completion of the model. By using the illumination problem approach, the total minimum number of valid 2-modems on the first floor of building FPMIPA-A is twelve 2-modems; three 2-modems in the West, two 2-modems in the East, four 2-modems in the North, and three 2-modems in the South. The positions of the 2-modems are placed at the points shown in Figure 9. For further research development, an implementation study of the illumination problem can be carried out with a value of $k \geq 3$, and by considering variations such as wall thickness, varying modem signal strengths, and the ability of the modem signal to penetrate to the floor above or below.

ACKNOWLEDGEMENTS

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