

A Bibliometric Analysis of Mathematical Problem-Solving: A State of The Art

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Abstract

This study aims to explore research on problem-solving in various countries using complete and systematic bibliometrics, mainly Publish or Perish (PoP) and Vos viewer, to analyze data systematically. This analysis reveals the development of research related to problem-solving to provide new ideas and raise research gaps related to this topic. The data used in 2016-2022 is 2959 articles using keywords, namely problem solving and mathematics. The method used in this study used bibliometric analysis with interconnected keyword maps. Through this bibliometric, researchers can find research trends by the chosen topic, explain the updates that can be presented, and determine research gaps. So, this research is the first step to making it easier for researchers to report the results of previous studies and offer further research.

Keywords: Bibliometric; Mathematics; Problem solving; Vos viewer.

Abstrak

Tujuan penelitian ini untuk mengeksplorasi penelitian tentang pemecahan masalah diberbagai negara dengan menggunakan analisis data bibliometric vos viewer dan publish or perish secara lengkap dan sistematis. Analisis ini mengungkapkan perkembangan penelitian terkait pemecahan masalah untuk memberika ide baru serta memunculkan kesenjangan penelitian terkait topik ini. data yang digunakan pada tahun 2016-2022 sebanyak 2959 artikel. Metode yang digunakan dalam penelitian ini menggunakan analisis bibliometric dengan peta kata kunci yang saling berhubungan. Melalui bibliometric ini peneliti dapat menemukan trend penelitian yang sesuai dengan topik yang dipilih, menjelaskan kebeharuan yang dapat disajikan serta menentukan gap penelitian. Sehingga penelitian ini sebagai langkah awal untuk memudahkan peneliti melaporkan hasil penelitian terdahulu untuk dapat menawarkan penelitian yang berbeda dari penelitian yang sudah ada.

Kata Kunci: Bibliometrik; Matematika; Penyelesaian masalah; Vos viewer.

2020MSC: 97D50, 97U60.

1. INTRODUCTION

Problem-solving is an activity that everyone engages in regularly. For students, solving problems is an essential part of learning, particularly in mathematics, where it is a primary educational goal (Garcia, [1]). Developing problem-solving skills can enhance other abilities, such as critical thinking, representation skills, and reflective thinking (see [2], [3], [4]). Therefore, according to the National Council of Teachers of Mathematics (NCTM, [5]), problem-solving is a crucial skill that all students must develop. The 2016 Ministry of Education and Culture regulations on primary and secondary education content standards state that in mathematics learning, students should be able to think both concretely and abstractly to solve mathematical problems.

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A problem-solving situation involves a challenge that students must address using their knowledge. According to Goldin ([6]), a problem consists of three components: (1) the initial condition (given state), (2) the goal or end condition (goal state), and (3) obstacles that hinder the transition from the initial to the goal condition. To describe a problem, one must: (1) define a state space, (2) establish one or more initial states, (3) set one or more goals, and (4) establish a set of rules. Jonassen ([7]) defines problems based on (1) the problem domain, (2) the type of problem, (3) the problem-solving process, and (4) the solution. In mathematics, the problem domain includes concepts, rules, proofs, posing, and solving problems. A problem arises from a gap between the initial condition and the desired goal, creating a "problem situation." Therefore, research on problem-solving must clearly define the issue to be explored, helping students understand the difficulties they encounter.

Problem-solving can be approached in various ways, including routine and non-routine problems. Yee ([8]) distinguishes between closed problems, which have specific solutions and methods, and open-ended problems, which require multiple solutions. Jonassen ([7]) also discusses well-structured and ill-structured problems. Despite these various perspectives, Polya's fundamental principles of problem-solving are widely used. Polya ([9]) identifies four basic steps: (1) Understanding the problem, where students restate the problem information; (2) Devising a plan, where students develop a strategy to solve the problem; (3) Carrying out the plan, where students execute their strategy; and (4) Looking back, where students review their work. These steps form the basis for much problem-solving research.

Despite extensive research on problem-solving, gaps remain in the literature. Goldin's and Jonassen's perspectives highlight the need for clearly defined problems and research updates. One way to trace previous research and map relationships between studies is through bibliometric mapping, which visualizes research trends and developments. Vos Viewer is a tool that uses bibliometric network mapping to group publications and analyze trends (Eck, [10]). It visualizes the network of interrelated articles or authors, helping researchers find new ideas in problem-solving research, especially in mathematics education.

This study aims to display the steps of bibliometric data analysis on mathematical problem-solving using Vos Viewer. It is expected to provide a reference for identifying new research ideas in problem-solving, particularly in mathematics education.

2. METHODS

The review is systematically established on the methodology to clarify and simplify the search process. This is done in three phases: Planning, Implementation, and Reporting. The planning stage prepares the topic or problem being investigated, determining specific keywords to make searching easier, determining year limits in capturing articles, and preparing tools to capture and map data. In the implementation process, the collected articles are then extracted, and the identity of the netted article is to match the specified keywords. Finally, a report on the results obtained in the search process is presented, considering their relevance and impact.

2.1. Planning

This discussion focuses on studies of problem-solving, particularly in mathematics education for elementary, middle, and high school students. Research from various countries can contribute to the

development of these studies. Although problem-solving has been extensively researched, its definition and objectives can vary among researchers.

The search process begins with using the tool Publish or Perish (PoP) to collect article data from 2,959 articles using the keywords "problem-solving and mathematics," sourced from Google Scholar and Scopus. The data is limited to the years 2016-2022 and saved in Bibtex or Ris formats for further use. Searches on Google Scholar and Scopus are capped at 200 articles per year. These articles are then imported into Mendeley Desktop or Mendeley Reference Manager, as illustrated in Figure 1. It is essential to complete the required metadata for all articles in Mendeley, as shown in the right column, to ensure data completeness for analysis in the Vos Viewer tool.

2.2. Implementation

This section gathers all published articles that include the author keywords "problem-solving" and "mathematics problem-solving," ensuring that the search keywords match the article's author keywords. We filter the results to retain only articles, excluding other types of publications. Additionally, the collected articles must have updated details in Mendeley, as illustrated in Figure 1.

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Figure 1. File view Publish or perish at Mendeley

3. RESULTS

This section presents the results of mapping using Vosviewer, which analyzes the development of journal publications focused on solving mathematical problems in the Scopus and Google Scholar databases from 2016 to 2022. Figure 2 illustrates the relationship between problem-solving and other themes based on the authors' keywords. Figure 3 visualizes the trends in problem-solving research over the years, with darker colors representing earlier years (2016) and lighter colors representing more recent years (2022). The overlapping colors suggest that this research area is likely to continue evolving, offering further opportunities for exploration. Figure 4 depicts the study's density: lighter colors indicate more common research themes, while darker colors or faded keywords suggest topics that remain under discussion and are promising for ongoing research.

The visualization display in Figure 2 is grouped into various interconnected clusters, each represented by different colors. Cluster 3, which includes the keyword "problem-solving," shows its relationship with other themes, such as word problem-solving, as illustrated in Figure 5. In Figure 5, the keyword "problem-solving" is associated with several other terms and is prominently marked with a blue circle, indicating its central role in this research area. Figure 5 highlights the density of the study themes, where lighter colors indicate well-researched topics, and darker or faded keywords suggest less explored areas, presenting potential ideas for future research.



Figure 2. Network visualization of research themes in mathematical problem solving (2016-2022)



Figure 3. Temporal evolution and network of research themes in mathematical problem solving (2016-2022)



Figure 4. Density visualization of research themes in mathematical problem solving



Figure 5. Visualization in cluster 3

Figure 6 visualizes cluster 5 with the keywords used by problem-solving strategies only related to assessment keywords. In contrast to Figure 7a in cluster 7, there are three keywords used for problem-solving; the first-word problem-solving is related to engineering and mathematics, computer science education, blended learning, curriculum, educational robotics, computational thinking, data mining, collaboration, working memory, problem-posing, mathematics, Chinese, childhood. The two collaborative problem-solving approaches are related to STEM education, learning analytics, and problem-posing. Figure 7b in cluster 7 is computational problem-solving related to STEM education and computational thinking.



Figure 6. Visualization in cluster 5



Figure 7. Visualization in cluster 7

Figure 8, the keyword arithmetic problem solving, relates to game-based learning, reading, and embodied cognition. Research with this keyword can be in the form of the development of a game-based arithmetic learning methods, integration of literacy and mathematics in arithmetic learning, development of integrated curriculum, and several similar studies.

Figure 9, with the keywords mathematical problem solving, is related to embodied cognition, anxiety, metacognition, assessment, and comparative judgment. Research with these keywords can be, the role of metacognition in solving mathematical problems, the Development of innovative assessments for solving mathematical problems, Comparative studies on problem-solving assessment methods, and several similar studies.



Figure 8. Visualization in cluster 11

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Figure 9. Visualization in cluster 12

Furthermore, in Figure 10, the keywords mathematics problem solving are related to digital games, learning disabilities, at-risk, intervention programs, anxiety, and classroom practice. Examples in mathematics education research can be Using digital games to reduce students' mathematics anxiety, analyzing the impact of educational digital games on students' motivation and mathematical problem-solving performance, developing classroom practice models that combine digital games and intervention strategies to support at-risk students in mathematics learning, and several similar studies.



Figure 10. Visualization in cluster 14

In Figures 11(a) and (b), cluster 18, there are only two keywords that are not related to each other, namely problem-solving ability and problem-solving skills. Although these two keywords are used at the same time, they have different objectives, such as research examples, Meta-analysis of recent studies that distinguish between problem-solving abilities and skills in mathematics education, or Effectiveness of interventions targeted to improve general problem-solving abilities vs. specific mathematical problem-solving skills.







Figure 12. Visualization in cluster 19

Figure 12 Cluster 19, with the creative keyword "problem-solving," is related to exceptional talent, identification of gifted students, and STEM and spatial ability. Examples in mathematics education research can be the relationship between mathematical problem-solving creativity and exceptional talent in spatial abilities in secondary school students, Correlation analysis between STEM abilities and mathematical problem-solving creativity in identified gifted students, and several similar studies.



Figure 13. Visualization in cluster 20

In Figure 13, cluster 20 has two main themes, namely problem-solving and mathematics. These two themes are interrelated, and researchers use many as research themes. So that it causes a lot of themes or keywords that are researched a lot such as general curriculum access, autism, dyslexia, children, bilingualism, access to the general curriculum, attention, arithmetic, diagram, cognitive development, conceptual understanding, working memory, mathematical achievement, learning, teacher education, reasoning, cognitive reflection test, autism spectrum disorder, assessment, instruction, mathematics, mathematical equivalence, direct instruction, critical thinking, prospective teacher, academic achievement, challenging task, elementary mathematics, decimals, student attitude, elementary, elementary mathematics, meta-analysis, schema instruction, mathematics learning, mathematics education, engineering, feedback, affect, erroneous examples, problem posing, elementary school, cognitive load, calculus, augmented reality, blended learning, spatial ability, computational thinking, advance computing environment, active learning, creativity,and intelligent tutoring system.

The cluster above describes the networks that are interrelated with problem-solving mathematics. This visualization of the network is essential as a first step to understanding how previous research with similar themes is. With this analysis, we can find various related keywords that have been widely researched and research that is still small to see updates in research based on this bibliometric analysis technique.

Problem-solving is one of the discussions that has been studied for many years. However, its existence is still very much needed, considering that problem-solving is not only related to science at the education level but can also be applied in everyday life. For that research, we must choose the

primary source as a solid foundation to start a study. Based on the visualization results, some relationships between problem-solving and other keywords can be seen. Although various problem-solving deals with different topics, after further investigation, problem-solving itself is divided into domains, types of problems, processes, and solutions described by Jonassen [7] and Goldin [6]. Referring to the two expert opinions, the discussion focuses more on solving problems that have been researched by previous research. The searches can group the number of studies that are often and rarely studied. So, the results of this grouping offer gaps and novelty in problem-solving research.

Goldin [6] states there are three components of the type of problem, namely (1) initial condition (given state), (2) goal state, and (3) obstacle, namely anything that gets in the way between the initial condition and the goal condition. Thus, in describing the Goldin [6] problem, it must be (1) define a state space, (2) establish one or more initial states, (3) set one or more goals (goal states), and (4) establish a set of rules. Research that has been netted shows that many researchers consider the process of problem-solving or solving. Even though a problem that occurs in students begins with an initial state, in this condition, students may experience issues that will affect the final stage or goal state. This set of conditions is interrelated and requires special attention to overcome students' difficulties, so the table below compiles the research results related to problem-solving mathematics in school students.

			Problem-Solving											
No	Reference			Domain			Type of Task		k	Process Problem Solving	Solution			
		Concept	Rule	Proof	Posing	Solving	(Given State)	(State Place)	(Goal State)					
1	Abdullah et al. 2014									~	Students' attitudes to- wards mathe- matical word problem solving among elementary school students			
2	Abdullah et al. 2019					~				\checkmark				
3	Alibali et al. 2018					\checkmark					The new solution			
4	Amir, et al. 2018									\checkmark				
5	Bahar et al. 2015					V					Mathematical problem- solving performance in closed and open-ended problems.			
6	Barana et al. 2022					✓			\checkmark	~	Comprehensi on of the problem-atic situati-on, identification of the solving			

Table 1. SOTA (State of The Art) table of articles on mathematics problem-solving

		Problem-Solving									
No	Reference			Domain			Ту	pe of Tas	k	Process Problem Solving	Solution
		Concept	Rule	Proof	Posing	Solving	(Given State)	(State Place)	(Goal State)		
7	Bass et al		×								strategy, development of the solving process, argumentation of the chosen strategy, and appropriate
'	2017		·								
8	Bataluna et al. 2021									✓	The imple- mentation of a problem- solving approach is evaluated through problem- solving
9	Bazzini et al. 2014									~	Narratively- based problem- solving activity
10	Bossé et al. 2021					~		~		~	Cognitive processes used by students in mathematical problem- solving
11	Bruce et al. 2017					~					
12	Bubno et al 2019					\checkmark					
13	Caviola et al. 2018					~					Solve complex arithmetic problems.
14	Desli et al. 2020					V					Provide solutions from four different areas of mathematics (problem- solving tasks).
15	Doorman et al. 2019					\checkmark				\checkmark	Open-ended problem
16	Dung 2017	~									Assessing Vietnamese students' problem- solving skills

			Problem-Solving									
No	Reference	Domain T		pe of Tas	k	Process Problem Solving	Solution					
		Concept	Rule	Proof	Posing	Solving	(Given State)	(State Place)	(Goal State)	0		
											in dealing with	
											errors using	
											statistical tools.	
17	Fyfe et al.					\checkmark					Minimal	
	2017										corrective	
											feedback in a	
											problem-	
											solving task.	
18	Garcia et al. 2019					\checkmark				\checkmark		
19	Green et al.					\checkmark					To solve more	
	2017										complex	
											mathematical	
											problems.	
20	Haataja et al. 2019									\checkmark		
21	Harisman et					\checkmark					Background	
	al. 2020										behavior	
											students in	
											problem-	
											solving	
22	Hasan et al.									\checkmark	Problem-	
	2017										solving skills,	
											who are aware	
											of their think-	
											ing processes	
											and have self-	
											learning skills	
23	Hooglandet					\checkmark					Solve	
	al 2018										problem-	
	al. 2010										solving tasks	
											the posed	
											problems	
24	Hornburg					\checkmark					Solving	
	et al. 2017										mathematical	
											equivalence	
											problems	
25	Hwang et	\checkmark				✓					Five domains	
	al. 2019										when teaching	
											iractions in	
											problem	
											solving	
26	Irving et al					\checkmark					Solve	
20	2017										nonroutine	
	2017										algebraic tasks	
27	Iamaludin					\checkmark				\checkmark	Narrativized	
<u> </u>	et al. 2017										problem to be	
											solved	

			Problem-Solving									
No	Reference	Domain Type of Task					Process Problem Solving	Solution				
		Concept	Rule	Proof	Posing	Solving	(Given State)	(State Place)	(Goal State)			
28	Julie et al. 2018									✓	Understand- ing of its role during group mathematical problem- solving.	
29	Kenedi et al. 2019					\checkmark						
30	Lee et al. 2017			~							Polya	
31	Leo et al. 2019					~			\checkmark		Problem- solving skills and abilities	
32	Luria et al. 2017	~				\checkmark				~	Open-ended problems	
33	Masson et al. 2017					~					Solving arithmetic problems	
34	Mogari et al. 2017					~					Regular and non-routine problem- solving	
35	Mohtarom et al. 2017					V				\checkmark	Thinking process in solving mathematical problems	
36	Morsanyi et al. 2019						\checkmark		\checkmark			
37	Nur et al. 2020	~				~					One's thinking level to solve problems	
38	Ozcan et al. 20									\checkmark	•	
39	Parwati et al. 2018					~					Open and close the problem	
40	Passolunghi et al. 2019					~					Arithmetical problem- solving.	
41	Pelczer et al. 2014 Peltier et al.					✓				✓	Students' responses to multiple choice problems students' problem- solving behavior. To solve story	
	2017										problems	

		Problem-Solving									
No	Reference	Domain			Ту	pe of Tas	k	Process Problem Solving	Solution		
		Concept	Rule	Proof	Posing	Solving	(Given State)	(State Place)	(Goal State)		
43	Peltier et al. 2018					~					Solving word problem mathematics
44	Peltier et al. 2018					\checkmark				\checkmark	
45	Phonapichat et al. 2014.									\checkmark	The difficult- ies in mathe- matical problem solving for students' learning process
46	Psycharis et al. 2017									\checkmark	
47	Rahmi et al 2019					~					Solving mathematical problem
48	Root et al. 2018					\checkmark					
49	Root et al. 2019					\checkmark					
50	Samo et al. 2017	\checkmark									
51	Santia et al. 2019					\checkmark					Ill-structured problem
52	Savard et al. 2017					~					Solving word problem
53	Shen et al. 2018					\checkmark				\checkmark	Solve aljabar problem
54	Shin et al. 2017					 ✓ 					Word problem solving with fractions.
55	Sidney et al. 2019					~					Solved fraction division problems
56	Suarsana et al. 2019									√	Use online problem- posing to investigate non-routine problem solving
57	Suponta- wanit et al. 2021					~					Improve word problem- solving skills
58	Suprotun et al. 2019					~					Problem- solving skills and abilities

						Prot	olem-Solv	ing			
No	Reference			Domain			Ту	pe of Tas	k	Process Problem Solving	Solution
		Concept	Rule	Proof	Posing	Solving	(Given State)	(State Place)	(Goal State)		
59	Sutama et al. 2021					√					Students' metacognition cognitive styles in mathematics problem- solving
60	Swason et al. 2018									V	Math problem- solving processes and accuracy task
61	Tachie et al. 2019					~					Strategies in mathematics problem- solving
62	Viitala 2015									~	In problem- solving, the main focus is on the cognitive problem- solving process written
63	Wu et al. 2017					~					Arithmetic word problem- solving
64	Yeo et al. 2017						\checkmark		\checkmark	\checkmark	Open-ended tasks problem solving
65	Young et al. 2018					×				~	Stimulated- recall interviews allow researchers to ask questions that may reveal additional information about students' thinking and problem- solving processes.
66	Yuanita et al. 2018	~				~					Solve problems in the learning context.

						Prot	olem-Solv	ing			
No	Reference			Domain			Type of Task			Process Problem Solving	Solution
		Concept	Rule	Proof	Posing	Solving	(Given	(State	(Goal State)		
						,	State	Flace)	State)		
67	Yuanita et					\checkmark					Aim to solve
	al. 2018										arithmetic
											problems
68	Zakirova et					\checkmark	\checkmark			\checkmark	Solving
	al. 2019										problems with
											parameter
69	Zhou et al.										Solve the
	2017										task of
											arithmetical
											problem.

Table 1 presents a grouping derived from the Vosviewer visualization. Several articles center around the theme of "mathematical problem-solving." This grouping is categorized based on the contributions of key experts: (1) Polya for the problem domain, (2) Goldin for the type of task, and (3) Jonassen for the solution process. Each column reveals areas that have not been extensively explored by researchers, highlighting opportunities for new discussions to ensure that research on "mathematical problem-solving" continues to innovate and diversify.

This grouping is supported by primary expert sources, which serve as key references. The discussion section offers an overview of the gaps identified in Table 1, suggesting directions for future research. By focusing on these gaps, researchers can delve deeper into specific subjects and contribute to a broader understanding of mathematical problem-solving.

4. DISCUSSION

Table 2 provides examples of further research on problem-solving. Suggestions for further research can be combined with problematic research subjects. Table 2 shows gaps and novelty in a research topic.

Gaps	Research Topics or Research Design	Novelty
Problem domains that many researchers still need to explore.	Students' abilities or skills can explore problem situations from the type of problem students face. Some of these situations can later	In Figure 12 of cluster 20, there is an example of critical thinking. Further research could use thinking or similar
A type of problem in the problem	describe the difficulties/obstacles that students	abilities.
space (state place) that contains a series of steps/strategies that	have.	Types of problems in the problem space
researchers rarely notice	Some solutions of problem-solving such as open and close problems, runtime approaches,	Types of problems that are still little or not even used by other researchers.
Problem-solving can be considered the process of solving problems or just solving problems that adapt a theory.	and non-routine problems or elements of problem-solving. In this section, the researcher needs to explain the solutions offered to further explore the variety of research on problem-solving.	Offer solutions that can describe the problem clearly.

Table 2. Examples of research topics

Solutions to problem-solving can be viewed in several ways of solving.

By paying attention to the problem-solving process and offering solutions to problems such as routine and non-routine problems.

Problem-solving research with more specific methods such as qualitative that describes phenomena in a certain area / certain level of education.

Choosing the position of problemsolving as a process/applying a model from an expert theory By offering multiple solutions Qualitative methods of phenomenology troubleshooting process its domain to find evidence.

5. CONCLUSSION

Collecting data from over 2,959 articles related to problem-solving in mathematics, this research utilizes Vosviewer to visualize relationships based on the researchers' keywords. This method aids in identifying gaps and novelties within the specified research theme. The most frequently used keyword is "problem-solving," which appears in cluster 20 alongside the prominent theme of "mathematics." These two keywords are highly interconnected with other terms. Other clusters contain similar terms, showing relationships within each cluster.

However, further filtering by reading each article individually is necessary to identify more precise linkages. In mathematics education research, problem-solving studies can be applied to various cognitive and affective aspects, which can help describe and explore new findings in the field.

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