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Optimal Reinsurance for the Solvency of Automobile Portfolio: Application to Sub-Saharan Africa

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Abstract

This paper examines actuarial strategies to maintain the solvency of automobile insurance portfolios in sub-Saharan Africa, where motor insurance is mandatory and a significant revenue source, representing approximately 60% of total premiums in the CIMA (the Inter-African Conference on Insurance Markets) region. Poorly managed auto insurance portfolios risk pushing insurers toward insolvency, necessitating proactive financial measures. The study evaluates a priori and a posteriori pricing methods, concluding that neither approach alone sufficiently mitigates solvency risks due to the portfolio's heterogeneity and the potential for premium default. Our proposed solution, surplus share reinsurance, is a proportional and individualized strategy in which the insurer sets a retention limit per policy, only retaining premiums below this threshold. Additionally, establishing a reserve fund is essential to cover any potential shortfalls. The probability of ruin, assessed through a random walk analysis of risk reserves, is vital for evaluating the portfolio's financial stability and guiding risk management decisions.
Keywords: Priori pricing; Full surplus reinsurance; Thirst for insurance bonuses; Bonus-malus system; Line-by-line provisioning.

Abstrak

Artikel ini mengkaji strategi aktuaria untuk mempertahankan solvabilitas portofolio asuransi mobil di Afrika sub-Sahara, dimana asuransi kendaraan bermotor bersifat wajib dan merupakan sumber pendapatan yang signifikan, yang mewakili sekitar 60% dari total premi di kawasan CIMA (Konferensi Antar-Afrika tentang Pasar Asuransi). Portofolio asuransi kendaraan bermotor yang dikelola dengan buruk berisiko mendorong perusahaan asuransi menuju kebangkrutan, sehingga memerlukan langkah-langkah keuangan yang proaktif. Studi ini mengevaluasi metode penetapan harga apriori dan a posteriori, dan menyimpulkan bahwa tidak satu pun pendekatan yang cukup memitigasi risiko solvabilitas karena heterogenitas portofolio dan potensi gagal bayar premi. Solusi yang kami usulkan, reasuransi surplus share, adalah strategi proporsional dan individual di mana perusahaan asuransi menetapkan batas retensi per polis, hanya menahan premi di bawah ambang batas ini. Selain itu, pembentukan dana cadangan sangat penting untuk menutupi potensi kekurangan. Peluang kebangkrutan, yang dinilai melalui analisis random walk dari cadangan risiko, sangat penting untuk mengevaluasi stabilitas keuangan portofolio dan memandu keputusan manajemen risiko.
Kata Kunci: *Penetapan harga apriori; Reasuransi surplus penuh; Kehausan akan bonus asuransi; Sistem bonus-malus; Provisi lini per lini.*

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1. INTRODUCTION

Regarding car insurance, the insured is shielded from material damage to the covered vehicle (also known as property insurance) and bodily harm to the driver. Car insurance providers want every insured to pay a fair price commensurate with the risk they are taking. The challenge that emerges is figuring out specific standards by which policyholders can be distinguished.

Liability insurance is a subset of car insurance that covers material or physical harm to third parties caused by an insured vehicle, depending on the type of coverage taken out. Motor responsibility is mandatory in most sub-Saharan African nations and is the most significant branch. For instance, in Algeria, mandatory motor insurance accounts for nearly 57% of the market for property and casualty insurance [1], in the Democratic Republic of Congo, it covers more than 80% of the turnover of the Société Nationale d'Assurance (Automotive, Fire, Life, Maritime, ARD, etc.) [2]; and in the CIMA zone (the Inter-African Conference on Insurance Markets, which consists of 13 French-speaking sub-Saharan African nations), health and motor insurance accounts for 60% of the turnover of all 164 insurance companies. The insurance company can even become insolvent due to a poorly managed auto line.

The decision of the Inter-African Conference of Insurance Markets is another notable example of how insurance companies are forced to adapt to new regulations that are more demanding regarding risk quantification. Probabilistic methods, which are based on modeling the annual frequency of losses and their ultimate individual severity, have replaced the empirical methods that were previously used [3]. We describe a priori pricing in section 2, wherein the insurer attempts to project a new policyholder's future loss experience based on predetermined parameters agreed upon at the subscription time. Due to the ongoing variety of vehicle portfolios, a statistical examination of claims made, such as in Kinshasa in 2016, has demonstrated that the Bonus Malus system can be used in the Democratic Republic of the Congo. A finite horizon is reasonable since we cannot remain in the system indefinitely, or more accurately, we expect the insured to exit the system at a specific age (for example, after 40 years of driving) due to their inability to operate a vehicle [4].

In some countries, the government imposes the bonus-malus system, wherein all insurers must use the same system (number of classes, transition regulations, etc.). Every insurer creates its own system in other nations, and the market is free. We build a Bonus system based on the two types of Bonus Malus systems that are used in the world: the multiplicative Bonus Malus system (French type) and the Bonus Malus system with classes (Belgian type), after providing some theoretical underpinnings for the construction of a Bonus Malus system in section 3 [2].

According to Lemaire's research, policyholders familiar with dynamic programming and its applications can save up to 36% of the total amount paid to the insurance company due to their craving for bonuses. This encourages them to manage small claims payments themselves. These savings could amount to nearly 38% in the Democratic Republic of Congo [4]. In this case, the insurer will discover that policyholders only pay large claims, making them vulnerable to financial disaster. As part of a third strategy, we examine at the end of this article the possibility that the insurance company may turn to reinsurance to protect its solvency.

Actuaries sometimes need help implementing their pricing methodology due to the limited statistical information available and the nature of the peak exposures that reinsurance companies accept. As a result, they have significant challenges when deciding which probability distributions to use and how to parameterize them for risk modeling [1]. For insurance companies in sub-Saharan Africa, excess share (XP) reinsurance is the best option because it reduces the risks assumed by the ceding company; this is because the insurer knows the maximum amount it will have to pay in the event of a claim, and premiums and claims are distributed according to a predetermined ratio. Only policies above a specific coverage threshold, known as full retention or line, will trigger the reinsurer's intervention [5]. Other strategies, such as using financial products, can keep the probability of an insurer's ruin below a certain threshold. Given that few publicly-traded insurance companies exist in sub-Saharan Africa, this article will not address this case. This article aims to find the mechanisms that can enable a motor insurance company (or the motor industry) to maintain its solvency, which is its financial capacity to meet its commitments to its policyholders (reimbursement in the event of a claim) and third parties (commission agents, taxes, etc.).

2. METHODS

Constructing the optimum bonus-malus system for sub-Saharan African countries

Hypothesis

A bonus-malus system is used by an insurance company when:

1) There is a finite number of classes c_i $(i = 1, \dots, s)$ that can be created from the collection of policies in a portfolio so that the annual premium depends solely on the class,
2) The number of claims reported during the period and the class from the previous period uniquely determine the class at any given time,
3) There are two final classes: One in which all policies with a large number of accidents are located and another in which all policies have a sufficient number of loss-free years.

The following three elements influence such a system:

i. The number of classes (represented by s),
ii. The premium scale b_i $(i = 1, \dots, s)$ such that the premium is paid by policyholders in class i, b_i and $\forall i, i = 1, \dots, s$ on a : $b_i \leq b_{i+1}$,
iii. The transition rules are the rules guiding the change from one class to another when the number of claims is known.

These transition rules can be presented as transformations T_k transformations such as $T_k(i) = j$, which means that the font is transferred from class c_i to class c_j if k claims have been reported. These transformations can also be presented in matrix form $\left(t_{ij}^{(k)}\right) \in \mathrm{IR}^{sxt}$, where s is number of classes, and t is highest number of claims an insured person can make combined classes. These transition rules can be expressed as transformations T_k, such as $T_k(i) = j$, which indicates that if k claims have been made, the police is moved from the class c_i to the c_j class. Additionally, these transformations can be shown as matrix $\left(t_{ij}^{(k)}\right)$.

The system's transition mechanisms determine the probability of an insured individual switching from one class to another in the SBM [3]. The transition rules that permit the insured to be transferred from one class to another, assuming that k accidents have been reported by the insured, are as follows:

$$t_{ij}^{(k)} = \begin{cases} 1 & \text{if } T_k(i) = j, \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

Let N_t be the annual number of claims brought about by an insured, and let λ be the average yearly frequency of claims in the portfolio. Think about an insurance that employs a bonus-malus system structure. Each policyholder is assigned to a class in the bonus-malus scale, which has $(s+1)$ classes (numbered from 0 to s).

Degree 0 gives t

The maximum bonus is given by degree 0, and the relative premium rises with level to its maximum in s. Note: L_t is the class the insured belongs to between t and $t+1$. The trajectory of the insured is represented by the discrete-time stochastic process $\{L_t, t \in \mathbb{N}\}$. According to the method, the number of claims filed during a prior period and the insured's degree decide the degree of an insurance period [6].

In the event where each claim results in a move up one ω degree and the insured moves unconditionally down one class each year on the scale, the class L_{t+1} in which the insured will be positioned at time $t+1$ is determined by:

$$L_{t+1} = \max\{\min\{L_t + \omega N_{t+1} - 1, s\}, 0\}. \tag{2}$$

Generally, $L_{t+1} = \Psi(L_t;\ N_{t+1})$, where Ψ in its two arguments is an increasing function. Depending on how good the risk is

$$\mathbb{P}[L_{t+1} = l_{t+1} | L_t = l_t, \dots, L_0 = l_0, \theta] = \mathbb{P}[L_{t+1} = l_{t+1} | L_t = l_t, \theta]. \tag{3}$$

Provided that the trajectory $l_0, \dots, l_t$ is achievable:

$$\mathbb{P}[L_t = l_t, \dots, L_0 = l_0] > 0. \tag{4}$$

The fact that the insured person's current status in the scale summarizes all the knowledge necessary to understand his future progression is expressed in relation (4). This indicates that knowing the levels occupied at times 1, 2,... improves the forecast of future developments. This characteristic makes it possible to use Markov processes to model the evolution of an insured individual [7]. This is because a Markov chain is a stochastic process, meaning that its future evolution solely depends on its current state and not on its past or how it arrived at its current state. The process is memoryless, meaning that the various states of the chain correspond to the multiple rungs of the bonus-malus hierarchy.

To predict the insured's level in the next year, one only has to know the current level and the total number of claims the insured has made throughout the year. Therefore, it is optional to be aware of how the insured got to the position that they currently hold [8]. This system's limitation is that the insured cannot exceed classes 0 and 22 regardless of the accidents they cause.

Thirst for the insurance company bonus on a finite horizon

Assuming that the insured cannot remain in the system indefinitely, that is, that the insured will eventually exit the system because they will no longer be eligible to drive at a specific age, say after 40 years of driving, is far more realistic. As a result, we presume that the maximum duration of the insured is N periods [2]. Given that the risk was insured in the $(n-1)^{th}$ period let W_n be the probability that it will be insured in the nth period. $W_1 = 1$ (because the insured entered the system in period 0, his probability of being in the system in period 1 is 100%) and $W_{N+1} = 0$ (because it is assumed that the insured leaves the system after N periods) are apparent. A logical policyholder would try to reduce the discounted expectation of future payments.

The following procedure operates across a finite horizon in hindsight analysis. Determine each period's best course of action, $\bar{x}_n$, and the associated discounted expectation, $\bar{v}^n$. R. Bellman's optimality theorem [9] ensures the existence of the optimal strategy. In retrospective analysis over a finite horizon, we calculate the optimal policy $\bar{x}_n$ and the corresponding discounted expectation v_i^n for each period. R. Bellman's optimality theorem guarantees the existence of the optimal policy:
Given a policy $\bar{x}_n = (x_1(n), \ldots, x_s(n))$; solving the system.

$$x_i(n) = W_{n+1} \cdot \beta^{1-t} \cdot \sum_{k=0}^{L} \bar{p}_k^i[\lambda(1-t); n] \cdot \left[v_{T_{k+m+1(i)}}^{n+1} - \bar{v}_{T_{k+m(i)}}^{n+1} \right]. \tag{5}$$

Given a vector $\bar{v}^n = (v_1^n, v_2^n, \ldots, v_s^n)$, determine an improved policy by solving the system

$$v_i^n = E[x_i(n)] + W_{n+1} \cdot \beta \cdot \sum_{k=0}^{L} \bar{p}_k^i[\lambda, n] \cdot v_{T_{k\,(i)}}^{n+1}. \tag{6}$$

At the start of the nth period in class ci, v_i^n represents the insured person's discounted anticipation of all future payments. $x_i(n)$ denotes the estimated cost (premium plus personally indemnified claims) for the n^{th} period. Retention cap for an insured individual in class C for his nth term $x_i(n)$.
$t \in [0.1]$ is the instant in which a claim happens.

The discount rate is $\beta < 1$

If an insured is in class i, the probability that he will report k claims in his nth period with a claims frequency of λ is $\bar{p}_k^i[\lambda, n]$. Then, it becomes evident that two factors determine the insured's retention limit, $x_i(n)$, for each period: λ is frequency of claims and β is discount rate. Which the insured typically needs help understanding. Therefore, examining how $x_i(n)$ varies about these parameters is interesting. Based on the Bonus Malus system of the Democratic Republic of Congo [2], the bonus scale and transition rules are shown in the Table 1.

Table 1. Premium scale and transition rules about the number of claims declared by the insured in the Democratic Republic of the Congo

Class	Premium level	T_0	T_1	T_2	T_3	T_4	$T_k\ (k \geq 5)$
22	508	21	22	22	22	22	22
21	482	20	22	22	22	22	22
20	459	19	22	22	22	22	22
19	437	18	22	22	22	22	22
18	416	17	22	22	22	22	22
17	397	16	21	22	22	22	22
16	378	15	20	22	22	22	22
15	360	14	19	22	22	22	22

Table 1. (Cont.)

Class	Premium level	T_0	T_1	T_2	T_3	T_4	$T_k\,(k \geq 5)$
14	343	13	18	22	22	22	22
13	326	12	17	22	22	22	22
12	311	11	16	21	22	22	22
11	296	10	15	20	22	22	22
10	282	9	14	19	22	22	22
9	268	8	13	18	22	22	22
8	256	7	12	17	22	22	22
7	243	6	11	16	21	22	22
6	232	5	10	15	20	22	22
5	221	4	9	14	19	22	22
4	210	3	8	13	18	22	22
3	200	2	7	12	17	22	22
2	191	1	6	11	16	21	22
1	182	0	5	10	15	20	22
0	173	0	4	9	14	19	22

In Table 1, $T_1(11) = 15$, i.e., after declaring a claim, any policyholder in class 11 will be transferred to class 15. We determine $\bar{x}_i(n)$ by adjusting variables such as λ. For all common values of λ, we calculated $\bar{x}$ for a constant interest rate of 4%. For typical values of λ, the highest retention limits are found in classes 16, 17, and 15. Consequently, policyholders must bear huge claims in this case [9], [10].

By using the best policy, policyholders can save significant money at the firm's expense. It $\bar{x}^*$ can be rather large. For example, Figure 1 illustrates the discounted expectation of all payments for a new sedentary policyholder (who joins the system in class 9) when all claims are reported ($V_9 0$) and when the best policy is applied (V_9^*). The hatching region, the difference between these two curves, indicates the company's loss. It can amount to up to 40% of the total amount vi0 paid by a policyholder familiar with dynamic programming and its applications [4]. To protect its solvency in this situation, the insurance firm will turn to reinsurance.

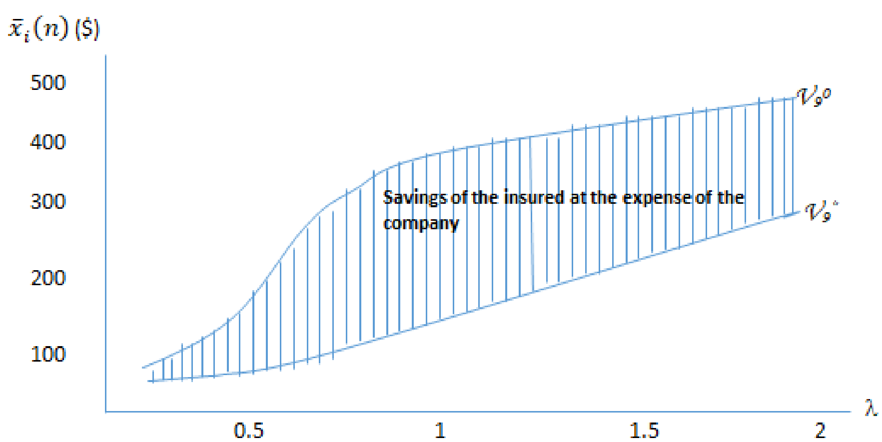

Figure 1. Payment expectancy as a function of λ for a newly insured

3. RESULTS

Since a posteriori pricing does not guarantee the solvency of an insurance company's motor business, it is necessary to use another method that will help the company ensure this solvency: reinsurance. The estimated global reinsurance market in 2015 was USD 230 million, with life accounting for 28% of the total and non-life for 72%.
Substantial insurance amounts exist for specific hazards. For instance, in aviation, the insured amounts may range from US$200 million to US$500 million for both people and cargo. A sub-Saharan insurance firm alone cannot take on such a risk without running the risk of bankruptcy in the event of a claim.

Surplus share is the optimum reinsurance treaty for insurance firms operating in sub-Saharan Africa

The "surplus share" reinsurance is the best option for these insurance companies because it lowers the risks assumed by the ceding company; in fact, the insurer is aware of the maximum amount it must pay in the event of a claim, and premiums and claims are split according to a predetermined ratio. Only policies beyond a specific coverage threshold, known as the full retention or line, will trigger the reinsurer's intervention. As a result, it is a unique and proportionate contract [11]. We can define the retention rate or retention coefficient $a_j, (0 < a_j < 1)$ for each risk j in the portfolio where $j = 1,2, \dots, n$. The loss (and premium) ceded rate for policy i is ai. In this case, the cession rate is determined policy by policy.

Table 2 defines the 'Excess Full'(or 'surplus share') form. The surplus share reinsurance contract gives the shares of the risks retained and ceded by the ceding company, respectively.

Table 2. Form of surplus share reinsurance

	Original total risk	**Conserved risk**	**Ceded risk**
Claims	$S = \sum_{j=1}^{n} S_j$	$\sum_{j=1}^{n} a_j S_j$	$\sum_{j=1}^{n} (1 - a_j).S_j$
Premium	$P = \sum_{j=1}^{n} P_j$	$\sum_{j=1}^{n} a_j P_j$	$\sum_{j=1}^{n} (1 - a_j).P_j$

Where S is the total number of claims for this insurance portfolio must be paid annually, and P is the total premiums paid for this insurance portfolio each year. The initial portfolio is represented by the random variables X and P (prior to reinsurance). S_j is the annual risk for contract j or the yearly amount of risk j (where $j = 1,2, \dots, n$). P_j is the yearly premium the portfolio under consideration receives for risk j (where $j = 1,2, \dots, n$). In practice, for each policy, the reinsurer only assumes the portion of the risk above a certain capital level, called the full retention. The effective cession rate is actually

$$\theta = \frac{(min\{full\ subscription - insured\ capital\} - full\ subscription)_+}{min\{full\ subscription - insured\ capital\}}.$$

Note that the portfolio retained by the insurer is capped.

3. DISCUSSIONS

Surplus reinsurance, a type of proportional reinsurance, provides several specific benefits and considerations for insurers, especially when dealing with diverse portfolios. Key features of surplus reinsurance include:

a. This type of reinsurance is proportionate in that: a. reinsurance is determined for each risk j the reinsurer's share of the claim is known in advance risk-by-risk;
b. The total amount of the claim charged to the reinsurer depends on each of the Random Variables S_j; because the reinsurance is determined on a risk-by-risk basis (individual reinsurance),
c. This type of reinsurance is suitable in cases when the portfolio is heterogeneous, as is the case with an insurance company's vehicle portfolio;
d. Nevertheless, the insurer is still exposed to the risk of cumulative claims or a high volume of claims over time;
e. Surplus share maximizes retention since proportional reinsurance lowers the probability of ruin by raising the safety coefficient at a given level of reserves (or equity).

This method's sole drawback is that it necessitates defining and disseminating an exact rate schedule to reinsurers.

The line-by-line provisioning of a vehicle portfolio for third-party liability insurance

Technical provisions in non-life insurance include main provisions for claims payable, outstanding risks, unearned premiums, and equalization. The provisions for claims payable are the most significant component; on average, they account for 85% of non-life insurance companies' reserves [12]. The company should make as many provisions as possible to ensure solvency, but it wants to take as little as possible to achieve return and Profitability relative to shareholders. Accurately predicting future benefits is the hardest part. Therefore, getting a correct estimate of this is essential for the company.

One type of non-life insurance known as "long-term" coverage is third-party liability, which means that claims for losses that happened years ago still need to be paid by the insurer. This is because many claims are only reported a few years after they occur, and there is a waiting period between the incidence of the claim and the final court decision [13]. In this instance, the significance of allocating funds for claims at the time of inventory to cover future payments for claims about the current or prior fiscal years becomes evident.

The best technique for automobile insurance for sub-Saharan nations was created in [2] and involved calculating reserves for claims that have already been reported to the insurer on a claim-by-claim basis (sometimes referred to as a line-by-line reserving model) [14], [15], [16]. Each claim is evaluated separately for this purpose and is identified by its date of occurrence, settlement procedure, and status (closed or in settlement).

Even if using stochastic methods to decide the level of provisioning doesn't always lessen the cost, it reveals the risk associated with the level of provisioning that is chosen. The application of these methodologies highlights that provisioning may occasionally be made imprudent when utilizing the Chain Ladder method, which involves estimating by average. Quantiles are a better choice for determining the reserve if distribution can be linked to claims settlements. Because of this, the discussions surrounding the reform of insurance company solvency indicators revolve around the stochastic approach.

Solvency analysis

[17] covers the probability of ruin approach and the simulation-based solvency analysis criterion. Using a probability of ruin strategy aims to ascertain the lowest amount of registered capital needed to reduce the likelihood of ruin. From an economic point of view, an insurance company's assets are defined as a set of positive flows that represent the income generated by the assets of the investment portfolio and its liabilities as a set of negative flows that represent benefits that will be received in the future.

The asset/liability margin, i.e., the difference between the probable present value of the assets and the probable present value of the liabilities, must be covered by the initial funds for the insurance business to be viable and have the necessary resources to pay for future benefits.

Our goal is to determine the appropriate level of starting capital for the company to maintain solvency in $(1-x)\%$ of cases. This is the same as finding the starting capital level at which the probability of insolvency is less than a certain percentage, $x\%$.

Provided that:

Probability (Probable Present Value (Assets) - Probable Present Value (Liabilities) < Initial Popres Fund) $< x\%$. Solvency is defined as the amount of possible loss that will not be exceeded in $x\%$ of circumstances, which is comparable to the definition of VaR (Value at Risk) [18], [19]. The computer tool under VBA of Excel or Matlab can be used to simulate the trajectories of the Asset-Liability Margin of the Monte Carlo type.

4. CONCLUSIONS

This work examined the actuarial solutions required to protect a vehicle portfolio's solvency, focusing on sub-Saharan Africa. Motor responsibility is mandatory in most sub-Saharan African nations. It is the most significant branch: 60% of the total revenue of the 163 insurance businesses in the CIMA zone (the Inter-African Conference on Insurance Markets) comes from health and auto insurance. The insurance company may even become insolvent due to mismanagement of the car portfolio. The traditional empirical methods have been replaced by probabilistic methods based on modeling the annual frequency of claims and their ultimate individual severity. This is due to the internal needs of insurance companies to control the underwriting risks of their business better and to adapt to new, more stringent regulations regarding the quantification of risks (Solvency II for Europe, for example, with a Solvency Capital Requirement; another striking example is the decision of CIMA, which requires all insurance companies in French-speaking countries to gradually multiply their minimum share capital by 5, tripling it in three years, and quintupling it in five years...).

An insurer has four main tools, which it uses in combination to keep the probability of ruin below a certain level: charging the pure premium, building up a security reserve, using reinsurance, and using financial products.We have shown that even with the charging of the pure premium, the solvency of the vehicle portfolio cannot be guaranteed by a priori or a posteriori pricing (moral hazard, thirst for power, inadequate provisions, etc.). We've proposed additional options, including reinsurance and creating payment provisions.

We have selected two appropriate models because the car portfolio's pricing model is individual (each policyholder pays based on the risk or danger they bring to the community):

1) Surplus share reinsurance, which lowers the risks assumed by the ceding business, is appropriate when the portfolio is diverse, as is the case with an insurance company's automobile portfolio. It is evaluated on a risk-by-risk basis (individual reinsurance). Furthermore, this model's proportionality raises the safety factor, lowering the likelihood of disaster.
2) Implementing the provisioning model, sometimes called the line-by-line provisioning model, by calculating reserves on a claim-by-claim basis.
3) Ultimately, we decided to use a solvency analysis model based on the "ruin" probability method. The computer tool can be used to build the simulation of the trajectories of the Asset-Liability Margin of Monte Carlo type under Excel VBA or MATLAB.

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