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Distance Magic Labeling of Corona Product of Graphs

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Abstract

Let $G=(V,E)$ is a graph with order n, and $f: V(G) \rightarrow \{1,2,\ldots,n\}$ is a bijection. For any vertex $v \in V$, $\sum_{u\in N(v)} f(u)$ is called the weight of vertex v, denoted by $w(v)$, where $N(v)$ is the set of neighbors of vertex v. If the labeling f satisfies that there exists a constant k such that $w(v)=k$, for every vertex v in the graph G, then f is called a distance magic labeling for the graph G. If a graph G has a distance magic labeling, then G is called a distance magic graph. In this paper, we present a novel result that has not been extensively explored in previous research, on the distance magic labeling for the corona product between several families of graphs, such as complete graph, cycle graph, path graph, and star graph.
Keywords: distance magic labeling; corona product; complete graph; cycle graph; path graph; star graph.

Abstrak

Misalkan $G=(V,E)$ *adalah graf berorde* n, *dan* $f: V(G) \rightarrow \{1,2,\ldots,n\}$ *merupakan suatu bijeksi. Untuk sebarang titik* $v \in V$, $\sum_{u\in N(v)} f(u)$ *merupakan bobot dari titik* v *dan dinotasikan dengan* $w(v)$, *dengan* $N(v)$ *merupakan himpunan tetangga dari titik* v. *Jika pelabelan* f *memenuhi terdapat suatu konstanta* k *sehingga* $w(v)=k$, *untuk setiap titik* v *yang terdapat pada graf* G, *maka* f *disebut sebagai pelabelan ajaib jarak bagi graf* G. *Jika suatu graf* G *memiliki pelabelan ajaib jarak, maka* G *disebut sebagai graf ajaib jarak. Paper ini memberikan hasil yang belum pernah dibahas sebelumnya, yaitu pelabelan ajaib jarak untuk operasi korona antara beberapa keluarga graf, seperti graf lengkap, graf siklus, graf lintasan, dan graf bintang.*
Kata Kunci: *pelabelan ajaib jarak; operasi korona; graf lengkap; graf siklus; graf lintasan; graf bintang.*

2020MSC:

1. INTRODUCTION

The concept of magic labeling based on distance arises from an observation regarding the construction of magic squares. A magic square of size n is an $n \times n$ array with each entry being a permutation of the integers $\{1,2,3,\ldots,n^2\}$, such that the sum of entries in each row, each column, and each diagonal of the magic square is the same. Motivated by this observation, Vilfred introduced the concept of sigma labeling in his doctoral thesis in 1994 [1]. This concept was separately introduced by Miller, Rodgers, and Simanjuntak in 2003 [2]. A magic labeling based on the distance of a graph G is a bijection $f: V(G) \rightarrow \{1,2,3,\ldots,|V(G)|\}$ that satisfies the existence of a constant k, such that for every vertex v in $V(G)$, the weight of vertex v, $\omega(v)=\sum_{y\in N(v)} f(v)=k$, where $N(v)$ denotes the set of vertices adjacent to v.

In the same reference, Miller et al. provided several examples of multipartite graph operations with distance magic labelings and some other graphs such as path P_1 and P_3, cycle graph C_4, and wheel graph W_4. With the variety of types and kinds of graphs, many researchers have provided distance

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magic labelings for certain types of graphs or have provided proof that a graph does not have a distance magic label. Some researchers have provided conditions under which a regular graph is a distance magic graph [3, 4, 5, 6], including circulant graphs [7, 8]. Additionally, there are several results of distance magic labelings for operations between two or more graphs, such as lexicographic [9, 10], direct product [11], cartesian product [12, 13], and join graph operations [12, 14].

One operation between two or more graphs has not been identified regarding its distance magic properties, namely the corona operation, which will be discussed in this research.

2. METHODS

The method used in this study is the literature review method, which involves researching to obtain data, information, and objects used in the discussion. The steps include reviewing the definitions of magic labeling of distance and corona graph multiplication operations and then providing necessary conditions for the magic distance property of the corona operation if one of the graphs is a complete graph, cycle graph, path graph, or star graph. Finally, the research results are used to prove the magic distance property for both graphs, where one of them is a complete cycle, path, or star.

3. RESULTS AND DISCUSSIONS

This discussion begins with some basic definitions of graphs and some crucial families of graphs. Then, we will introduce the concept of distance magic labeling, which is the main focus of this research. Next, we will explain the corona operation, which is the focus of this study. Some lemmas used to observe specific properties will also be introduced. All of these will form a solid foundation for understanding the concept of distance magic labeling in the context of the corona operation.

Graph G is an ordered pair $G = (V, E)$, where V is the set of vertices in G, and E is the set of edges in G, and $E \subseteq V^2$ [15]. The cardinality of the set of vertices in G is defined as the order of graph G, and the cardinality of the set of edges in graph G is defined as the size of graph G. A pair of vertices is called adjacent if there is an edge connecting between the pair of vertices. The set of all vertices adjacent to a vertex v in graph G is denoted by $N(v)$. The cardinality of $N(v)$ is called the degree of vertex v, and is denoted by $d(v)$.

A Complete Graph with n vertices, denoted by K_n, is a graph where every pair of vertices is adjacent. A Cycle Graph of order n, denoted by C_n, is a graph with $V(C_n) = \{v_1, v_2, \dots, v_n\}$, and $E(C_n) = \{v_i v_{i+1}: 1 \leq i \leq n-1\} \cup v_n v_1$. A Path Graph of order n, denoted by P_n, is a graph with $V(P_n) = \{v_1, v_2, \dots, v_n\}$ and $E(P_n) = \{v_i v_{i+1} : 1 \leq i \leq n-1\}$. A Star Graph with n vertices, denoted by S_n, is a graph with $V(S_n) = \{v_1, v_2, \dots, v_n\}$ and the condition that 1 vertex v_1 is adjacent to the other $n-1$ vertices, and vertices $v_2, v_3, \dots, v_n$ are adjacent only to v_1.

Definition 1 [2]. The magic distance labeling of a graph $G = (V, E)$ with order n is a bijection $f: V \to \{1, 2, \dots, n\}$ such that there exists a positive integer k satisfying $\sum_{y \in N(x)} f(y) = k$ for every $x \in V$. The constant k is referred to as the magic constant of the labeling f. The sum $\sum_{y \in N(x)} f(y)$ is called the weight of vertex x and is denoted as $w(x)$. If a graph G has a distance magic labeling, then G is called a distance magic graph.

Lemma 1 [2]. If the graph G has two vertices, u and v, such that $|N_G(u) \cap N_G(v)| = d(u) - 1 = d(v) - 1$, then G is not a distance magic graph.

Lemma 2 [2]. If the graph G is a distance magic graph with order n and magic constant r, then $rn = \sum_{x \in V} d(x) f(x)$.

Definition 2 [16]. Let G and H be graphs with orders m and n, respectively. The Corona Product $G \odot H$ is a graph obtained from G and H by taking one copy of graph G and m copies of graph H and connecting each vertex in the i-th copy of graph H with the i-th vertex of graph G.

Let in the graph $G \odot H$, $V(G) = \{v_1, v_2, \dots, v_m\}$ and $V(H_i) = \{v_{i1}, v_{i2}, \dots, v_{in}\}$ for $1 \leq i \leq m$. We start with some simple observations for various families of graphs.

Observation 1.

The graph $G \odot K_n$ is not a distance magic graph. Every vertex in $(K_n)_i$ has a degree of n, and $\left|N(v_{ij}) \cap N(v_{ik})\right| = n - 1$, for every $1 \leq j, k \leq n$. Based on Lemma 1, the graph $G \odot K_n$ is not a distance magic graph. For $n = 1$, consider two distinct vertices in $(K_1)_i$.
Furthermore, the graph $G \odot H$ is not a distance magic graph if H contains a complete graph component.

Observation 2.

The graph $G \odot C_n$ is not a distance magic graph if $n \neq 4$. Consider in the graph $(C_n)_i$, for $1 \leq i \leq m$, $N(v_{i1}) = \{v_i, v_{i2}, v_{in}\}$, $N(v_{in}) = \{v_i, v_{i1}, v_{i,n-1}\}$, and $N(v_{ij}) = \{v_i, v_{i,j-1}, v_{i,j+1}\}$. For $n \neq 4$, every vertex in $(C_n)_i$ has a degree of 3, but $|N(v_{i1}) \cap N(v_{i3})| = 2$. Whereas for $n = 4$, it satisfies $|N(v_{i1}) \cap N(v_{i3})| = 3$. Based on Lemma 1, the graph $G \odot C_n$ is not a distance magic graph if $n \neq 4$. Therefore, it suffices to examine the distance magic property for $n = 4$.

For $n = 4$, observe that $N(v_{i1}) = N(v_{i3}) = \{v_i, v_{i2}, v_{i4}\}$, and $N(v_{i2}) = N(v_{i4}) = \{v_i, v_{i1}, v_{i3}\}$, for $1 \leq i \leq m$. Thus, $w(v_{i1}) = w(v_{i3}) = f(v_i) + f(v_{i2}) + f(v_{i4})$ and $w(v_{i2}) = w(v_{i4}) = f(v_i) + f(v_{i1}) + f(v_{i3})$. Suppose the graph $G \odot C_n$ is a distance magic graph with a magic constant r. Then,

$$w(v_{i1}) + w(v_{i2}) = f(v_i) + f(v_{i2}) + f(v_{i4}) + f(v_i) + f(v_{i1}) + f(v_{i3}) = 2f(v_i) + \textstyle\sum_{j=1}^{4} f(v_{ij}) = 2r.$$

If summed for $1 \leq i \leq m$, it holds true that

$$2\sum_{i=1}^{m} f(v_i) + \sum_{i=1}^{m}\sum_{j=1}^{4} f(v_{ij}) = 2mr. \tag{1}$$

The corona product for graph $G \odot C_4$ can be shown in Figure 1(a).

Observation 3.

The graph $G \odot P_n$ is not a distance magic graph if $n \neq 3$. Consider in the graph $(P_n)_i$, for $1 \leq i \leq m$, $N(v_{i1}) = \{v_i, v_{i2}\}$, $N(v_{in}) = \{v_i, v_{i,n-1}\}$, and $N(v_{ij}) = \{v_i, v_{i,j-1}, v_{i,j+1}\}$. Note that the vertices v_{i1} and v_{in} have a degree of 2, but for $n \neq 3$, it can be seen that $|N(v_{i1}) \cap N(v_{in})| = 1$, whereas for $n = 3$, $|N(v_{i1}) \cap N(v_{in})| = 2$. Based on Lemma 1, the graph $G \odot P_n$ is not a distance magic graph if $n \neq 3$. So, it suffices to examine the distance magic property for $n = 3$.

For $n = 3$, observe that $N(v_{i1}) = N(v_{i3}) = \{v_i, v_{i2}\}$, and $N(v_{i2}) = \{v_i, v_{i1}, v_{i3}\}$, for $1 \le i \le m$. Thus, $w(v_{i1}) = w(v_{i3}) = f(v_i) + f(v_{i2})$ and $w(v_{i2}) = f(v_i) + f(v_{i1}) + f(v_{i3})$. Suppose the graph $G \odot P_n$ is a distance magic graph with a magic constant r. Then,

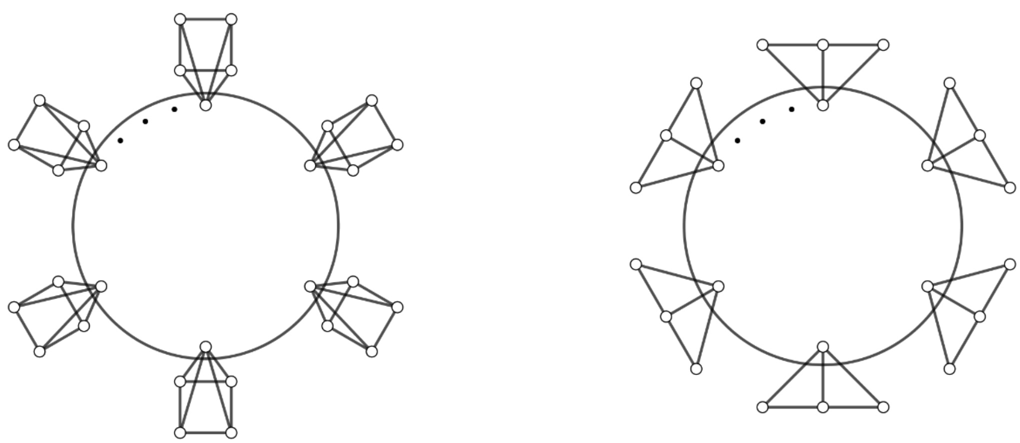
$$w(v_{i1}) + w(v_{i2}) = f(v_i) + f(v_{i2}) + f(v_i) + f(v_{i1}) + f(v_{i3}) = 2f(v_i) + \sum_{j=1}^{3} f(v_{ij}) = 2r.$$

If summed for $1 \le i \le m$, it holds true that

$$2\sum_{i=1}^{m} f(v_i) + \sum_{i=1}^{m}\sum_{j=1}^{3} f(v_{ij}) = 2mr. \tag{2}$$

The corona product for graph $G \odot P_3$ can be shown in Figure 1(b).

This case can be generalized by considering the graph $G \odot S_n$.



(a) (b)

Figure 1. (**a**) Graph $G \odot C_4$, and; (**b**) Graph $G \odot P_3$.

Observation 4.

Consider the graph $G \odot S_n$ with $n \ge 4$. Let in the graph $(S_n)_i$, for $1 \le i \le m$, $N(v_{i1}) = \{v_i, v_{i2}, v_{i3}, \dots, v_{in}\}$, and $N(v_{i2}) = N(v_{i3}) = \cdots = N(v_{in}) = \{v_i, v_{i1}\}$. Suppose the graph $G \odot S_n$ is a distance magic graph with a magic constant r. Then,

$$w(v_{i1}) + w(v_{i2}) = f(v_i) + f(v_{i1}) + f(v_i) + f(v_{i2}) + \cdots + f(v_{in}) = 2f(v_i) + \sum_{j=1}^{n} f(v_{ij}) = 2r.$$

If summed for $1 \le i \le m$, it holds true that

$$2\sum_{i=1}^{m} f(v_i) + \sum_{i=1}^{m}\sum_{j=1}^{n} f(v_{ij}) = 2mr. \tag{3}$$

Observation 5.

Consider the graph $K_m \odot H$. In the graph K_m, for every $1 \le i \le m$, $N(v_i) = \{v_1, v_2, \dots, v_{i-1}, v_{i+1}, \dots, v_m, v_{i1}, \dots, v_{in}\}$. Suppose the graph $K_m \odot H$ is a distance magic graph with a magic constant r. Then,

$$w(v_i) = \sum_{j=1, j\neq i}^{m} f(v_j) + \sum_{k=1}^{n} f(v_{ik}) = r.$$

If summed for $1 \le i \le m$, it holds true that

$$(m-1)\sum_{i=1}^{m} f(v_i) + \sum_{i=1}^{m}\sum_{j=1}^{n} f(v_{ij}) = mr. \tag{4}$$

We will use equation (4) to prove several theorems below for small values of m.

Theorem 1. If the graph $K_1 \odot H$ is a distance magic graph, then the labeling of the vertices of K_1 must satisfy

$$\frac{n}{2} + 1 \le f(v_1) \le n + 1.$$

Proof.

For $m = 1$, equation (4) will change to $\sum_{k=1}^{n} f(v_{1k}) = r$. The left-hand side of this equation represents the summation of all labels of vertices in the graph $K_1 \odot H$ excluding the label $f(v_1)$. Since $K_1 \odot H$ has order $n + 1$, the labeling on the graph $K_1 \odot H$ is $f: V_{K_1 \odot H} \to \{1,2, \dots, n+1\}$. Therefore,

$$\sum_{k=1}^{n} f(v_{1k}) = \big(1 + 2 + \dots + (n+1)\big) - f(v_1) = \frac{(n+1)(n+2)}{2} - f(v_1) = r.$$

Based on Lemma 2, if $K_1 \odot H$ is a distance magic graph, then it must satisfy

$$\sum_{x \in V(K_1 \odot H)} d(x) f(x) = r(n+1) = (n+1)\left[\frac{(n+1)(n+2)}{2} - f(v_1)\right].$$

Let δ be the minimum degree of the graph $K_1 \odot H$, and the maximum degree of the graph $K_1 \odot H$ is n. Then,

$$\delta \sum_{x \in V} f(x) \le \sum_{x \in V} d(x) f(x) \le n \sum_{x \in V} f(x).$$

The sum of all labels in the graph $K_1 \odot H$ is $\sum_{x \in V} f(x) = 1 + 2 + \dots + (n+1) = \frac{(n+1)(n+2)}{2}$, so

$$\delta \frac{(n+1)(n+2)}{2} \le (n+1)\left[\frac{(n+1)(n+2)}{2} - f(v_1)\right] \le n \frac{(n+1)(n+2)}{2}$$

$$\frac{\delta(n+2)}{2} \le \frac{(n+1)(n+2)}{2} - f(v_1) \le \frac{n(n+2)}{2}$$

$$\frac{(n+1)(n+2)}{2} - \frac{n(n+2)}{2} \le f(v_1) \le \frac{(n+1)(n+2)}{2} - \frac{\delta(n+2)}{2}$$

$$\frac{n}{2} + 1 \le f(v_1) \le \frac{(n+2)(n+1-\delta)}{2}.$$

Based on Observation 1, graph H cannot be a complete graph, meaning the maximum possible value of δ is $n-1$. Therefore,

$$\frac{(n+2)(n+1-\delta)}{2} \geq \frac{(n+2)(n+1-n+1)}{2} = n+2.$$

However, since the maximum label of the graph, $K_1 \odot H$ is $n+1$, then

$$\frac{n}{2}+1 \leq f(v_1) \leq n+1.$$

Theorem 2. If the graph $K_2 \odot H$ is a distance magic graph, then H has an odd order.

Proof.
For $m=2$, then equation (4) will change to:

$$\sum_{j=1}^{2} f(v_j) + \sum_{i=1}^{2}\sum_{k=1}^{n} f(v_{ik}) = 2r.$$

For the left-hand side, the equation above is the summation of each vertex label in the graph $K_2 \odot H$. Since $K_2 \odot H$ has an order of $2n+2$, the labeling on the graph $K_2 \odot H$ is $f: V_{K_2 \odot H} \to \{1,2,\ldots,2n+1,2n+2\}$. Therefore,

$$\sum_{j=1}^{2} f(v_j) + \sum_{i=1}^{2}\sum_{k=1}^{n} f(v_{ik}) = 1+2+\cdots+(2n+2) = \frac{(2n+2)(2n+3)}{2} = 2r.$$

Hence, if the graph $K_2 \odot H$ is a distance magic graph, then its magic constant is

$$r = \frac{(n+1)(2n+3)}{2}.$$

Since r is a positive integer, from the last equation, we conclude that n must be odd.

Observation 6.

Consider the graph $C_m \odot H$, for $m \geq 3$. In the graph C_m, $N(v_1) = \{v_2, v_m, v_{11}, \ldots, v_{1n}\}$, $N(v_m) = \{v_1, v_{m-1}, v_{m1}, \ldots, v_{mn}\}$, and $N(v_i) = \{v_{i-1}, v_{i+1}, v_{i1}, \ldots, v_{in}\}$, for every $2 \leq i \leq m-1$. Suppose the graph $C_m \odot H$ is a distance magic graph with a magic constant r. Then,

$$w(v_1) = f(v_2) + f(v_m) + \sum_{k=1}^{n} f(v_{1k}) = r, w(v_m) = f(v_1) + f(v_{m-1}) + \sum_{k=1}^{n} f(v_{mk}) = r.$$

and

$$w(v_i) = f(v_{i-1}) + f(v_{i+1}) + \sum_{k=1}^{n} f(v_{ik}) = r, \qquad 2 \leq i \leq m-1.$$

If we sum up the m equations above, then

$$2\sum_{i=1}^{m} f(v_i) + \sum_{i=1}^{m}\sum_{j=1}^{n} f(v_{ij}) = mr. \tag{5}$$

We will use the equation from the observation above to prove the following theorem.

Theorem 3. Graph $K_m \odot C_4$ is not a distance magic graph for $m \geq 2$.

Proof.
Suppose the graph $K_m \odot C_4$ is a distance magic graph with a magic constant r. The vertices in the graph $K_m \odot C_4$ must satisfy both of the equation (1) and (4):

$$(m-1)\sum_{i=1}^{m} f(v_i) + \sum_{i=1}^{m}\sum_{j=1}^{4} f(v_{ij}) = mr,$$

$$2\sum_{i=1}^{m} f(v_i) + \sum_{i=1}^{m}\sum_{j=1}^{4} f(v_{ij}) = 2mr.$$

By eliminating these two equations, we obtain:

$$mr = (3-m)\sum_{i=1}^{m} f(v_i).$$

Since m, r, and $\sum_{i=1}^{m} f(v_i)$ are positive integers, then m must be less than 3. If $m = 2$, then by substitution and simple elimination from the two equations above, we get $\sum_{i=1}^{m}\sum_{j=1}^{4} f(v_{ij}) = 0$, which is impossible.

For $m = 1$, the obtained graph is $K_1 \odot C_4$. It has been proven previously that the graph $K_1 \odot C_4$ is the wheel graph W_4, which is a distance magic graph. The distance magic labeling for $K_1 \odot C_4$ can be shown in Figure 2.

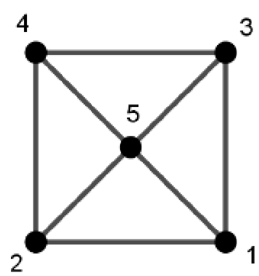


Figure 2. Distance magic labeling for W_4.

Theorem 4. The graph $K_m \odot S_n$ is not a distance magic graph.

Proof.
Suppose the graph $K_m \odot S_n$ is a distance magic graph with magic constant r. The vertices in the graph $K_m \odot S_n$ must satisfy both of the following equations:

$$(m-1)\sum_{i=1}^{m} f(v_i) + \sum_{i=1}^{m}\sum_{j=1}^{n} f(v_{ij}) = mr,$$

$$2\sum_{i=1}^{m} f(v_i) + \sum_{i=1}^{m}\sum_{j=1}^{n} f(v_{ij}) = 2mr.$$

By eliminating these two equations, we obtain:

$$mr = (3-m)\sum_{i=1}^{m} f(v_i).$$

Since $m, r,$ and $\sum_{i=1}^{m} f(v_i)$ are positive integers, m must be less than 3. If $m = 2$, then by substitution and simple elimination from the two equations above, we get $\sum_{i=1}^{m}\sum_{j=1}^{n} f(v_{ij}) = 0$, which is impossible. For $m = 1$, then we have $\sum_{j=1}^{n} f(v_{1j}) = r$ and $f(v_1) = \frac{r}{2}$. Since the magic constant is r, then for every $2 \leq j \leq n$, we have $w(v_{1j}) = f(v_1) + f(v_{11}) = r$. But because $f(v_1) = \frac{r}{2}$, then $f(v_{11}) = \frac{r}{2}$. This is a contradiction. This theorem concludes that $K_m \odot P_3$ is also not a distance magic graph.

The three theorems below are consequences of several previous observations. Assuming the distance magic property of those graphs and the results of elimination and substitution of equations lead to a contradiction, that is $mr = 0$.

Theorem 5. The graph $C_m \odot C_4$ is not a distance magic graph for $m \geq 3$.

Theorem 6. The graph $C_m \odot P_3$ is not a distance magic graph for $m \geq 3$.

Theorem 7. The graph $C_m \odot S_n$ is not a distance magic graph for $m \geq 3$.

4. CONCLUSIONS

Based on this discussion, it can be concluded that corona product graphs from certain families of graphs, such as complete graphs, cycles, paths, and stars, are not distance magic graphs. However, it has been proven that corona product graphs of small orders, such as wheel graphs, are indeed distance magic graphs.

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