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# The Optimal Bonus-Malus System: Case of The Democratic Republic of Congo

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## Abstract

Automobile insurance is required in most African nations, and it is the most significant branch in the Democratic Republic of the Congo (DRC); if the automobile branches are poorly managed, this could even result in the insurance company's insolvency. A priori pricing does not improve the danger parameter (the variance), which measures the difference between the estimated model and the observed reality; since the pricing characteristics do not take into account the driver's experience, the portfolio remains heterogeneous. To ensure the insurer's solvency, a more refined post-season pricing model is necessary, one that accounts for driver behavior. Our research introduces an innovative approach to a posteriori pricing in the DRC, using the Bonus-Malus System. In this model, policyholders are divided into classes based on the frequency of claims to preserve the insurer's solvency. The Bonus-Malus System will serve as the basis for the automobile portfolio's a posteriori pricing: the driver who has not declared a claim receives a reduction in his premium in the year $t_{n+1}$ (Bonus), and the wrong driver who has declared more than one claim will see his premium increased to the year $t_{n+1}$ (Malus). Inspired by models from Belgium (the class model) and France (the multiplicative model), we develop a Bonus-Malus model applicable to the DRC. The results found that the class system outperforms the other model for the DRC due to its clarity and fairness. We also emphasize the need for SONAS to centralize data to effectively implement this system and optimize motor vehicle claim management.
**Keywords:** bonus-malus; insurance policy; risk; frequency of claims; prior pricing; ex post facto pricing.

## Abstrak

*Asuransi mobil diwajibkan di sebagian besar negara Afrika, dan merupakan cabang yang paling signifikan; di Republik Demokratik Kongo (DRC),. Jika cabang mobil dikelola dengan buruk, hal ini bahkan dapat mengakibatkan kebangkrutan perusahaan asuransi. Penetapan harga secara apriori tidak memperbaiki parameter bahaya (varians) yang mengukur perbedaan antara model yang diestimasi dengan kenyataan yang diamati, karena karakteristik penetapan harga tidak memperhitungkan pengalaman pengemudi, maka portofolio tetap heterogen. Untuk memastikan solvabilitas perusahaan asuransi, diperlukan model penetapan harga pasca musim yang lebih terperinci, yang memperhitungkan perilaku pengemudi. Penelitian kami memperkenalkan pendekatan inovatif untuk penetapan harga a posteriori di DRC, menggunakan Sistem Bonus-Malus. Dalam model ini, pemegang polis dibagi menjadi beberapa kelas berdasarkan frekuensi klaim, untuk menjaga solvabilitas perusahaan asuransi. Sistem Bonus-Malus akan menjadi dasar penetapan harga a posteriori portofolio kendaraan bermotor: pengemudi yang belum pernah mengajukan klaim akan menerima pengurangan premi pada tahun tn+1 (Bonus) dan pengemudi nakal yang mengajukan lebih dari satu klaim akan mengalami kenaikan premi pada tahun tn+1 (Malus). Terinspirasi oleh model dari Belgia (model kelas) dan Prancis (model multiplikatif), kami mengembangkan model Bonus-Malus yang dapat diterapkan di DRC. Hasil penelitian menemukan bahwa sistem kelas lebih unggul dibandingkan model lain untuk DRC karena kejelasan dan keadilan. Kami juga menekankan perlunya SONAS untuk mengkonsolidasi data guna mengimplementasikan sistem ini secara efektif dan mengoptimalkan manajemen klaim kendaraan bermotor.*
**Kata Kunci:** *bonus-malus; polis asuransi; risiko; frekuensi klaim; penetapan harga sebelumnya; penetapan harga* ex post facto.

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## 1. INTRODUCTION

According to [1] a French actuary, "insurance is an operation by which one party, the insured, is promised, in return for a remuneration, the premium, for himself and for a third party, in the event of the realization of a risk, a benefit by another party, the insurer, who, taking charge of a set of risks, composes them by the laws of probability and statistics". When purchasing motor insurance, the policyholder is safeguarded against various types of material damage to the covered vehicle (also known as property insurance) and bodily harm to the driver. Liability insurance [2] is car insurance that may also cover material damage or bodily injury caused by the insured vehicle to third parties, depending on the type of policy taken out. In the Democratic Republic of Congo (DRC), as in most countries, car insurance is compulsory for any vehicle that travels on a public road.

Motor insurance largely dominates the insurance market in the DRC. The insurance company can even become insolvent due to a poorly managed car portfolio. Car insurance providers want every insured to pay a fair price commensurate with the risk they are taking. The challenge that emerges is figuring out specific standards by which policyholders can be distinguished. In part 2, we introduce a priori pricing, in which the insurer attempts to project a new policyholder's future loss experience based on predetermined standards chosen at the subscription time. By making a statistical analysis of the claims reported in Kinshasa in 2016, we note a persistent heterogeneity of the motor portfolio and show the need to apply a posteriori pricing, which consists of charging a premium taking into account the insured's history (their number of reported claims) [3]. This personalization of the premium according to the number of reported claims is often called the Bonus-malus system.

The government imposes the bonus-malus system in some countries, in which case all insurers are required to use the same system (number of classes, transition regulations, etc.). Every insurer creates its own system in other nations, and the market is free. Building upon theoretical foundations in section 3, we present two novel variations of the bonus-malus system, drawing from global implementations: the multiplicative Bonus- malus system (French type) and the Bonus Malus system with classes (Belgian type) [4]. The multiplicative Bonus Malus system (French type): The Reduction coefficient increase is obtained as follows: For each year without an at-fault accident, you benefit from a 5% reduction on your previous year's coefficient; for each at-fault accident, the insured is subject to a 25% surcharge. The Bonus Malus system with classes (Belgian type): The set of policies in the portfolio can be partitioned into a finite number of $s$ classes $c_i, i = 1, \dots, s$, so that the annual premium depends only on the class. In section 5, we conclude this paper by showing that the Bonus Malus Class System is more appropriate for the Democratic Republic of Congo, as it is fair and balanced.

## 2. METHOD

### 2.1. Car Pricing System

Pricing is the fair distribution of the total burden on the community. Its purpose is to estimate the risk of each insurance policy to distribute the total burden of the community fairly. That is, calculating a risk premium for each policyholder according to several observed factors. The actuary, using reliable statistical data, is able to determine the appropriate premium rates that will allow the constitution of reserves and technical provisions to safeguard the company's solvency [5].

Since each policyholder must contribute according to the risk they pose to the community, the actuary must subdivide the insurance portfolio into homogeneous classes when it is heterogeneous [6]. The problem is to determine some criteria for differentiating between policyholders. These criteria are called tariff characteristics.

### 2.1.1. A Priori Pricing

Based on particular parameters decided upon at the time of subscription, the insurer attempts to forecast a new policyholder's future loss experience from the point of enrollment. Policyholders A priori variables, often classification variables, are the observable qualities [7] [8]. Therefore, a priori pricing depends on the insured's (driver profile) and the vehicle attributes. According to [9], these traits are exogenous, a priori, or categorization variables. Since it is impractical and statistically challenging to consider every feature, each business chooses the ones that it deems most important [9]. For example, only four factors are considered for a priori pricing at the Société National d'Assurances (SONAS) in the Democratic Republic of the Congo (DRC) (see table 1): vehicle power (commercial, taxi, rental, or Super) and age of vehicle.

**Table 1**. A Priori pricing applied to SONAS/DRC

| Class | Vehicle category | Annual premium ($) by age group | |
|---|---|---|---|
| | | **≥ 6 years** | **< 5 years** |
| 1 | 1 to 5 Horsepowers | 173 | 163 |
| 2 | 6 to 9 Horsepowers | 217 | 201 |
| 3 | 10 to 13 Horsepowers | 285 | 262 |
| 4 | 14 to 17 Horsepowers | 375 | 343 |
| 5 | ≥ 18 | 508 | 466 |

### 2.1.2. Limit of use of a priori pricing; a posteriori pricing

We collected data on claims in different Sona branches in Kinshasa (Gombe, Limété, Ngaba, N'djili, Masina, Kasavubu, UPN). These data are shown in the claims distribution table 2, using a sample of 6475 cars in 2021 and selected using a straightforward random sampling technique.

**Table 2**. Distribution of claims by frequency

| Number of claims | Workforce |
|---|---|
| 0 | 5797 |
| 1 | 564 |
| 2 | 96 |
| 3 | 12 |
| 4 | 5 |
| 5 | 1 |

To show the limit of using a priori pricing, we will experiment with two model cases and then choose the one that does not deviate from the real model: hence, the model with a homogeneous portfolio (Poisson model) and model with a heterogeneous portfolio (Negative Binomial model) [10].

**Model 1: Poisson model (Homogeneous portfolio)**

In the first approximation, we assume that all policyholders are equal before the risk and that each policyholder has an equal chance of experiencing an accident. Since the occurrence of claims in this instance is random. Additionally, assuming the following hypotheses [11]:

a) We have, on the one hand, calculated the average number of claims and the variance, which gave respectively $\bar{x} = 0.126$ and $s^2 = 0.163$; for a Poisson distribution, the variance should be equal to the mean;
b) On the other hand, we can fit these observations by a fish distribution with parameter $\lambda = \bar{x} = 0.126$ where $\lambda$ is the mathematical expectation of the number of claims in the distribution (or claim frequency).

**Table 3**. Adjustment of the data by the fish distribution

| Number of claims | Numbers Observed $n_k$ | Theoretical numbers $np_k$ |
|---|---|---|
| 0 | 5797 | 5708.5 |
| 1 | 564 | 719.2 |
| 2 | 96 | 45.31 |
| 3 | 12 | 1.90 |
| 4 | 5 | 0.059 |
| 5 | 1 | 0 |

Where

$$p_k = e^{-\lambda} \frac{\lambda^k}{k!}. \tag{1}$$

By grouping neighboring classes so that the new classes have a theoretical size of more than 5, we have table 4. Using the Pearson goodness of fit test, we obtain $\chi^2_{\text{calculate}} = 201{,}17.$ At the 0.05 threshold, using the tables of $\chi^2$ tables at 2 degrees of freedom, we find 5.591; the fit is weak, and the homogeneity hypothesis, according to which all policyholders are equal before the risk, is rejected. Consequently, model 1 (homogeneous portfolio) must be rejected.

**Table 4**. Aggregation of adjusted data

| Number of claims | Numbers Observed $n_k$ | Theoretical numbers $np_k$ |
|---|---|---|
| 0 | 5797 | 5708.5 |
| 1 | 564 | 719.2 |
| ≥2 | 114 | 47.26 |

**Model 2: Negative Binomial Model (Heterogeneous Portfolio)**

Here, we're assuming that policyholders differ in risk, meaning each has a unique distribution of claims. Therefore, a driver's attributes can be fully summed up by the value of his claims frequency λ. The portfolio is therefore made up of the good and bad drivers; by once again positing the hypothesis a) and b) of the previous model, the distribution of the number of claims for each insured is a Poisson distribution with parameter λ, which changes from policy to policy and whose repartition function (structure-function) is U(λ), represents the distribution of the number of claims made by each insured. The gamma (Γ) distribution describes the distribution of the random variable λ [12].

If the gamma ($\Gamma$) distribution describes the distribution of the random variable $\lambda$ with frequency function

$$dU(\lambda) = \frac{\tau^a\, e^{-\lambda\tau}\, .\, \lambda^{a-1}}{\Gamma(a)} \text{ where } a, \tau > 0. \tag{2}$$

Consequently, the distribution of the number of claims in the portfolio is a Negative Binomial [13]. We'll get its probability distribution from

$$p_k = \binom{k+a-1}{a-1}\left(\frac{\tau}{1+\tau}\right)^a\left(\frac{1}{1+\tau}\right)^k,\ k = 0,1,2,\dots, \tag{3}$$

of mean $m = \frac{a}{\tau}$, and variance $\sigma^2 = \frac{a}{\tau^2}(1+\tau)$. Additionally, let's try to do a statistical analysis of this heterogeneous portfolio: Let us estimate the parameters a and $\tau$ of the model from the average number of claims $\bar{x}$ and the variance $\sigma^2$. From $\bar{x} = m = \frac{a}{\tau}$ and $\sigma^2 = \frac{a}{\tau^2}(1+\tau)$, we have: $a = \frac{\bar{x}^2}{s^2-\bar{x}} = 0.4290$ and $\tau = \frac{\bar{x}}{s^2-\bar{x}} = 3.4054$, hence table 5, adjusting the data.

**Table 5.** Negative binomial fitted data

| Claims | Numbers Observed | Theoretical numbers |
|---|---|---|
| 0 | 5797 | 5797.715 |
| 1 | 564 | 564.60 |
| 2 | 96 | 91.57 |
| 3 | 12 | 16.8 |
| 4 | 5 | 3.27 |
| 5 | 1 | 0.6 |

Where $k$ is the number of accidents, $n_k$ is the observed number, $np_k$ is the theoretical number. By grouping neighboring classes so that the new classes have a theoretical size of more than 5, we have table 6. Using the Pearson goodness of fit test, we obtain $\chi^2{}_{calculate} = 0.56$. At the 0.05 threshold, using the $\chi^2$ tables at 2 degrees of freedom, we find 3.84; adjustment is reasonable. The second model illustrates the variety of portfolios that can be used to implement a Bonus Malus system in the Democratic Republic of the Congo. As a result, it is the one that comes closest to reality.

**Table 6**. Grouped adjusted data

| Number of claims | Numbers Observed $n_k$ | Theoretical numbers $np_k$ |
|---|---|---|
| 0 | 5797 | 5797.715 |
| 1 | 564 | 564.60 |
| 2 | 96 | 91.57 |
| 3 | 18 | 20.67 |

In this model, each policyholder will be characterized within the portfolio by the frequency of their claims and the number of accidents they cause per year. This system has the property of being equitable (it makes each person pay at any time a premium proportional to his own estimated frequency $\lambda_{t+1}$ at time $t+1$) and balanced (the average of the estimated claim frequencies is constantly equal to $\frac{a}{\tau}$ the overall average). In other words, the amount collected by the company is stationary, and financial equilibrium is achieved yearly [14].

## 3. RESULTS

### 3.1. Construction of the Bonus-Malus System Model

The transition rules built into the SBM indicate the likelihood of an insured individual switching from one class to another [2] [14]. The transition rules that enable the insured to transfer from one class to another are as follows, assuming that the insured has reported $k$ accidents:

$$t_{ij}^{(k)} = \begin{cases} 1 & if \quad T_k(i) = j \\ 0 & if \quad T_k(i) \neq j \end{cases}. \tag{4}$$

Let $N_t$ be the annual number of claims brought about by an insured, and let λ be the average yearly frequency of claims in the portfolio. Think about an insurance provider that employs a bonus-malus structure. Each policyholder resides in a class in the bonus-malus scale, which has $(s + 1)$ classes (numbered from 0 to $s$). The maximum bonus is given by degree 0, and the relative bonus rises with the level to its maximum in $s$ [15].

To predict the insured's level next year, one only has to know the current level and the total number of claims the insured has made throughout the year. Thus, understanding how the insured got to the level that they currently hold is not required [16] [17].

### 3.2. Bonus-Malus system applicable in the Democratic Republic of Congo

In the Democratic Republic of Congo, as in most countries worldwide, 'third party liability' motor insurance to compensate victims of accidents caused by the insured vehicle is compulsory. Congolese motorists see their premiums increase almost every year and sometimes double [18]. This situation has always caused discontent among some drivers, who consider themselves 'rightly or wrongly' good drivers and who even refuse to pay these extra premiums.

It has been shown that the bonus-malus system can be used in the Democratic Republic of Congo. Thus, by introducing this post-ante pricing system, this problem can be solved. By comparing and criticizing the 2 types of Bonus Malus System applied worldwide [19], we propose in this section a Bonus Malus System applicable in the Democratic Republic of Congo.

**Case 1: Application to the class system**

The class system we propose has 23 classes; level 22 corresponds to double the insurance premium (Table 7). The premium level is expressed in dollars ($). For example, an insured in class 9 will pay a premium of $268, and an insured in class 14 will pay a premium of $343.

In the Democratic Republic of Congo, a priori pricing is based on vehicle horsepower; thus, users whose cars have between 1 and 9 horsepower enter the system at class 9, while other users enter class 14. The transition rules for moving from one class to the other are as follows:

1) Each year in which there is no claim, we go down a class.
2) Each year, if there are one or more claims:
    a) An increase of four classes for the first accident was declared.
    b) Five additional classes are added for subsequent accidents.

c) This system constrains the insured to not exceed classes 0 and 22, regardless of the number of accidents he causes.

**Table 7**. The Class system/DRC

| Premium payable | Class | Premium payable | Class |
|---|---|---|---|
| 508 | 22 | 282 | 10 |
| 482 | 21 | 268 | 9 |
| 459 | 20 | 256 | 8 |
| 437 | 19 | 243 | 7 |
| 416 | 18 | 232 | 6 |
| 397 | 17 | 221 | 5 |
| 378 | 16 | 210 | 4 |
| 360 | 15 | 200 | 3 |
| 343 | 14 | 190 | 2 |
| 326 | 13 | 181 | 1 |
| 311 | 12 | 172 | 0 |
| 296 | 11 | | |

Table 8 sets out the transition rules and the premium scale:

**Table 8.** Premiums based on the number of claims reported

| Premium payable | Class | $T_k\ (k \geq 5)$ | $T_4$ | $T_3$ | $T_2$ | $T_1$ | $T_0$ |
|---|---|---|---|---|---|---|---|
| 508 | 22 | 22 | 22 | 22 | 22 | 22 | 21 |
| 482 | 21 | 22 | 22 | 22 | 22 | 22 | 20 |
| 459 | 20 | 22 | 22 | 22 | 22 | 22 | 19 |
| 437 | 19 | 22 | 22 | 22 | 22 | 22 | 18 |
| 416 | 18 | 22 | 22 | 22 | 22 | 22 | 17 |
| 397 | 17 | 22 | 22 | 22 | 22 | 21 | 16 |
| 378 | 16 | 22 | 22 | 22 | 22 | 20 | 15 |
| 360 | 15 | 22 | 22 | 22 | 22 | 19 | 14 |
| 343 | 14 | 22 | 22 | 22 | 22 | 18 | 13 |
| 326 | 13 | 22 | 22 | 22 | 22 | 17 | 12 |
| 311 | 12 | 22 | 22 | 22 | 21 | 16 | 11 |
| 296 | 11 | 22 | 22 | 22 | 20 | 15 | 10 |
| 282 | 10 | 22 | 22 | 22 | 19 | 14 | 9 |
| 268 | 9 | 22 | 22 | 22 | 18 | 13 | 8 |
| 256 | 8 | 22 | 22 | 22 | 17 | 12 | 7 |
| 243 | 7 | 22 | 22 | 21 | 16 | 11 | 6 |
| 232 | 6 | 22 | 22 | 20 | 15 | 10 | 5 |
| 221 | 5 | 22 | 22 | 19 | 14 | 9 | 4 |
| 210 | 4 | 22 | 22 | 18 | 13 | 8 | 3 |
| 200 | 3 | 22 | 22 | 17 | 12 | 7 | 2 |
| 191 | 2 | 22 | 22 | 16 | 11 | 6 | 1 |
| 182 | 1 | 22 | 21 | 15 | 10 | 5 | 0 |
| 173 | 0 | 22 | 20 | 14 | 9 | 4 | 0 |

$T_k$(i)=j means that any insured in class i will be transferred to class j following k reported claims.

Dollars ($) represent the premium level. If an insured person falls under class 11, they must pay $296. $T_1$(11)=15, meaning that any insured in class 11 will be moved to class 15 following the filing of a claim.

Similarly, if an insured person is in class 7 and does not make a claim, they will be placed in class 6; otherwise, after a claim has been made, they will be placed in class 11, i.e., $T_1(7) = 11$; if he declares 3 claims he will be transferred to the "malus" class 22; $T_3(7) = 22$

**Case 2: the Multiplicative System**

In the multiplier system, the premium to be paid by the insured is determined by multiplying the basic (reference) premium by a multiplier (reduction factor) [20] and [21]. According to the French reduction coefficient increase system (1) the premium is reduced by 5% for each claim-free year, so the basic premium is multiplied by 0.95, and (2) for each claim reported during the year, the premium is increased by 25%, i.e., the basic premium is multiplied by 1.25; the same applies to each additional claim. Applying this system, we have table 9 as a result.

**Table 9.** The multiplicative system / DRC

| Premium payable | Number of claims | | | | | |
|---|---|---|---|---|---|---|
| | 5 | 4 | 3 | 2 | 1 | 0 |
| 508 | 1547 | 1238 | 991 | 793 | 635 | 482 |
| 488 | 1487 | 1190 | 952 | 762 | 610 | 463 |
| 473 | 1441 | 1153 | 923 | 739 | 591 | 449 |
| 458 | 1395 | 1116 | 893 | 715 | 572 | 435 |
| 443 | 1351 | 1081 | 865 | 692 | 553 | 420 |
| 428 | 1303 | 1043 | 835 | 668 | 535 | 406 |
| 413 | 1258 | 1007 | 806 | 645 | 516 | 392 |
| 398 | 1212 | 970 | 776 | 621 | 497 | 378 |
| 383 | 1166 | 933 | 747 | 598 | 478 | 363 |
| 368 | 1121 | 897 | 718 | 575 | 460 | 349 |
| 353 | 1075 | 860 | 688 | 551 | 441 | 335 |
| 338 | 1035 | 825 | 660 | 528 | 422 | 321 |
| 323 | 983 | 787 | 630 | 504 | 403 | 306 |
| 308 | 938 | 751 | 601 | 481 | 385 | 292 |
| 293 | 891 | 713 | 571 | 457 | 366 | 278 |
| 278 | 846 | 677 | 542 | 434 | 347 | 264 |
| 263 | 800 | 640 | 512 | 410 | 328 | 249 |
| 248 | 753 | 603 | 483 | 387 | 310 | 235 |
| 233 | 710 | 568 | 455 | 364 | 291 | 221 |
| 218 | 663 | 531 | 425 | 340 | 272 | 207 |
| 203 | 618 | 495 | 396 | 317 | 253 | 192 |
| 188 | 571 | 457 | 366 | 293 | 235 | 178 |
| 173 | 526 | 421 | 337 | 270 | 216 | 164 |

The premium is always expressed in $ (dollars). If an insured whose basic premium is $308 does not make a claim, he/she will pay $292 the following year, a reduction of 5%; however, if he/she does make a claim, the premium will be $385, an increase of 25%.

## 4. COMPARISON AND DISCUSSION

After applying a priori pricing to these two systems, we find that for the class system, for example, an insured who pays $343 at the beginning of the system, if he does not declare a claim, will go to class 13 and pay $326 the following year, whereas if he declares a claim, he will go to class 18 and pay

\$416; for the multiplicative system, he will pay \$343*1.25=\$438 in the case of an increase, and \$343*0.95=\$326 in the case of a decrease. Also, for the class system, if the insured declares two claims during the year, an insured with a base premium of \$343 will pay \$508 the following year, whereas in the multiplier system, he will pay \$343$\times$ **1.5625 = 536 \$.** Let us summarize this in table 10.

**Table 10**. Comparison of two bonus-malus models

| | Disaster / Basic premium (\$) | 0 | 1 | 2 |
|---|---|---|---|---|
| Class system | 343 | 326 | 360 | 508 |
| Multiplicative system | 343 | 326 | 438 | 536 |

We note that the malus varies less rapidly in the class system than in the multiplicative system; the multiplicative system is more severe in this case than the class system. In fact, in the multiplicative system, everything depends on the multiplication or increase coefficient set by the insurer. In contrast, in the class system, the number of classes being well defined, the decision rule is very precise for the insured's evolution in the system [13]. In addition, the class system has the merit of being balanced and fair, qualities that the multiplicative system lacks. So, the class system is best suited to the DRC.

## 5. CONCLUSION

In this paper, we have made a general presentation of the Bonus-Malus System, with the application of a Bonus-Malus System (BMS) in the Democratic Republic of Congo, based on the a priori pricing of the Société Nationale d'Assurance (SONAS). The work of [21] has shown that SBM can be introduced in the DRC. For our part, the statistical analysis of accidents that occurred in the DRC in 2020 has enabled us to prove that the a priori pricing system practiced by SONAS does not improve the danger parameter (variance), which measures the difference between the estimated model and the observed reality, as the characteristics of the tariff do not take into account the driver's experience (number of claims over time), and the portfolio, therefore, remains heterogeneous. This is why there is a need for a posteriori pricing that considers the driver's behavior.

In this work, we drew inspiration from two SBM models, SBM with classes (Belgian) and Multiplicative SBM (French), to propose an SBM applicable in the DRC. Comparing the two models, we proposed the "class system," which is best used in the DRC because of the well-defined number of classes and the precise decision rule for the evolution of the insured in the system. In addition, the class system has the merit of being balanced and fair. The development of a Bonus Malus system requires serious field studies. This is facilitated if the company's statistics on claims are well archived and updated. SONAS has great difficulty archiving and consolidating data from various agencies. It should create a body within the general services to centralize all the data into a database. The SBM proposed in this paper should be given special attention by the SONAS authorities as it can be applied to optimize the management of motor vehicle claims.

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