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Uncoupled Two Agents Modeling Via Bilinear Optimal Control

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Abstract

In this paper, uncoupled two agents modeling is proposed using an optimal bilinear control approach. The model is proposed using assumptions: an absence of the multi agent leader, each agent cannot control the others, each agent never collides with the others, and each agent has the same properties. The special functional cost consisting of a repellent cost is considered. The Pontryagin Maximum Principle is used to determine the optimal path for each agent. After control and optimal path for each agent are obtained some of the simulation results are exposed in this paper.
Keywords: uncoupled agent; modeling; bilinear system.

Abstrak

Dalam penelitian ini, pemodelan dua agen yang tidak berpasangan disajikan dengan pendekatan kontrol optimal bilinear. Model yang diusulkan dalam paper ini ditulis dengan asumsi: tidak adanya pemimpin dalam sistem multi agen, setiap agen tidak dapat mengendalikan atau mempengaruhi agen yang lain, setiap agen tiak boleh bertabrakan satu sama lain, dan para agen mempunyai sifat-sifat yang identik. Fungsional biaya khusus yang membuat para agen tidak bertabrakan dipertimbangkan dalam penulisan paper ini. Prinsip maksimum Pontryagin digunakan dalam penentuan lintasan optimal dari para agen. Beberapa hasil simulasi disajikan dalam paper ini.
Kata Kunci: *agen tak berpasangan; pemodelan; sistem bilinear.*

1. INTRODUCTION

A cluster phenomenon is a natural phenomenon happening in groups of animals. The other phenomena like schooling, flocking, and herding are similar. Only the place where the phenomenon occurs is different. Some animals like bees, geese, fish, zooplankton, birds, wolves, and other organisms perform together moving. The clustering phenomenon is also happening in a group of animals. The clustering phenomenon has happened on UAVs, robots, and airplane. The phenomenon can be viewed as a multi agent system in more general modeling. This paper exposes uncoupled agents which move together and use bilinear optimal control. The author's motivation to present bilinear optimal control in this paper is to imitate what is in nature. It will be efficient and optimal when applied in an artificial system, as exemplified by nature.

Some research papers that expose a topic close to this paper can be listed as follows. [1] wrote linear coupled model, different from this paper which uses a bilinear model. Next, in [2], web design is used for design control for a coupled system. This paper uses the Pontryagin Maximum Principle to design the control. After that, [3] temporal logic time is applied to control a coupled system, and this paper uses continuous time. Moreover, in [4] PID control used for coupled system control design, this paper uses an optimal control approach. Also paper [5] utilized a coupled system in food, this paper utilizes an optimal path design. As well as [6], stability in a coupled system is considered, different from this paper, we do not consider stability. Next, [7] described the application of a coupled

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system in signal processing, this paper exposes the optimal path of an uncoupled system. Besides this, [8] wrote a coupled system in Hammerstein integral equations. Besides [9] revealing a coupled constraint in a multi agent system, this paper does not expose a coupled constraint, but the uncoupled system has initial and final conditions. With [10] exposed a coupled demand, the demand can be viewed as a constraint, but this paper has an uncoupled system. The system considered in this paper is a coupled system. Along with [11] optimized for a network of the coupled systems, this paper optimizes the path of the uncoupled system. Furthermore, [12] exposed a discrete model for the coupled system, this paper considers a bilinear continuous system. Besides [13] decentralized discrete-time for a coupled system, so different from this paper that considers continuous time. Too [14] reported globally coupled constraint, this paper exposes the bilinear uncoupled system. Furthermore, [15] used on line optimization for a coupled inequality constraint; however, in this paper, the uncoupled is the system, not the constraint.

2. METHODS

The modeling method used in this paper uses a bilinear system approach. The multi agent model as bilinear control generally is described as a function of t as follows

$$\begin{aligned} \dot{x}_1 &= u_1 A_1 x_1 + v_1 B_1 x_1, \\ \dot{x}_2 &= u_2 A_2 x_2 + v_2 B_2 x_2, \\ &\vdots \\ \dot{x}_m &= u_m A_m x_m + v_m B_m x_m. \end{aligned} \tag{1}$$

Consider (1), generally $x_i \in \mathbf{R}^n, n = 1, 2, \ldots, k$, if n=1, we obtain uncoupled multi agents in R. Here k is a finite integer number. Still from (1), u_i and v_i are the controls for the *i*-th agent, A_i and B_i are scalar in R. Through the system of equation (1), the initial and boundary value conditions can be listed as follows

$$\begin{aligned} x_1(0) &= S_1, \quad x_1(T) = Q_1, \\ x_2(0) &= S_2, \quad x_2(T) = Q_2, \\ &\vdots \\ x_m(0) &= S_m, \quad x_m(T) = Q_m. \end{aligned} \tag{2}$$

The trajectory is optimized with respect to a cost functional defined as follows

$$J = -\frac{1}{2}\int_0^T g(x_1,...,x_m) + h(u_1,v_1,u_2,v_2...,u_m,v_m)dt. \tag{3}$$

In this modeling, $\dot{x}_i$ is the dynamic equation of the *i*-th agent, S_i is the initial position of the *i*-th swarm agent, and Q_i is the final position of the *i*-th agent. The solutions of the system of differential equations are $x_i(t), i = 1, 2, \ldots, m$ and $x_i(t)$ describe equations of trajectory or path of the *i*-th agent.

We want to minimize cost functional (3), and we use the Pontryagin Maximum Principle and then maximize the negative of cost functional. The cost functional (3) is also a model to describe collective behavior for two agents. The agent must move from the initial to a determined position without collision. In cost functional (3), generally consists of a function g and h. The function g denotes a function that makes two agents do not collide one each other, and the function h describes the control of each agent.

The model's assumptions, which are translated into a model, can be detailed as follows. First, an absence of the multi agent leader can be translated into the independent equation of $x_i(t)$ means that $x_i(t)$ is not influenced by $x_j(t)$. The model (1-3) does not exist a special equation for the leader. Second, each agent cannot control the others. The agent depends on the other agents in collective duty, like migrating from a place to the other warmer place, catching prey for more effective hunting. The collective responsibility is to search for food or forage for food, avoid predators, or another special duty. Third, each agent never collides with one of the others and can be translated in function g in cost functional (3). The last one is each agent has the same properties, translated in among the equations of $x_i(t)$ it is similar.

The next step is to combine (1) and (3) in Hamiltonian System and solve the Hamiltonian system for each control for each agent. Moreover, each agent obtains the optimal path equation, and the optimal trajectory can be plotted in simulation results.

3. RESULTS AND DISCUSSIONS

In this section, the main result will be exposed. The main result of this paper is the solution to this paper's main problem. What is the main problem of this paper? This paper's main problem is the control of two uncoupled agents modeled through bilinear optimal control. The two agents, move from the starting point to the specified endpoint without colliding with each other. The main result consists of two parts, the first part in R and the second part in R^2. Therefore, this section is divided into two subsections.

3.1 Uncoupled Two Agents in R

Follows (1), uncoupled two agents in R model as bilinear control is described as follows

$$\dot{x}_1 = u_1 A_1 x_1 + v_1 B_1 x_1,$$

$$\dot{x}_2 = u_1 A_2 x_2 + v_2 B_2 x_2.$$

For simulation, the values of A_1 and A_2 are 10. Still, also for simulation, the values of B_1 and B_2 are 5. The initial and boundary conditions for an uncoupled agent in R given as follows:

$$x_1(0) = 3, \quad x_1(1) = 4,$$
$$x_2(0) = 4, \quad x_2(1) = 5.$$

The cost functional is defined by

$$J = -\frac{1}{2}\int_0^1 (\delta u_1^{\;2} + \delta u_2^{\;2} + \delta v_1^{\;2} + \delta v_2^{\;2} + \frac{\gamma}{|x_1 - x_2|^2})dt.$$

In this paper δ is a control constant, and for simulation, it is given a value of 1. Next, γ is a repellent constant which is currently also given a value of 1 for simulation. The Hamiltonian function, which combines the bilinear model and the functional cost is

$$H = p_1\left(u_1A_1x_1 + v_1B_1x_1\right) + p_2\left(u_2A_2x_2 + v_2B_2x_2\right) - \frac{1}{2}p_0\delta u_1^{\;2} - \frac{1}{2}p_0\delta u_2^{\;2} - \frac{1}{2}p_0\delta v_1^{\;2} - \frac{1}{2}p_0\delta v_2^{\;2} - \frac{1}{2}\frac{p_0\gamma}{|x_1 - x_2|^2}.$$

In the Hamiltonian function above, p_0 =-1, next as a consequence of optimal control, we get the additional co-state variables p_1, and p_2. The Hamiltonian System is derived as follows

$$\frac{\partial H}{\partial p_1} = \dot{x}_1 = u_1A_1x_1 + v_1B_1x_1,$$

$$\frac{\partial H}{\partial p_2} = \dot{x}_2 = u_1A_2x_2 + v_2B_2x_2,$$

$$\frac{\partial H}{\partial x_1} = -\dot{p}_1 = p_1u_1A_1 + p_1v_1B_1 - \frac{\gamma p_0}{|x_1 - x_2|^3},$$

$$\frac{\partial H}{\partial x_2} = -\dot{p}_2 = p_2u_2A_2 + p_2v_2B_2 - \frac{\gamma p_0}{|x_1 - x_2|^3}.$$

Based on the Pontryagin Maximum Principle, the necessary condition such that the Hamiltonian System optimal is

$$\frac{\partial H}{\partial u_1} = 0 = p_1A_1x_1 - p_0\delta u_1,$$

$$\frac{\partial H}{\partial u_2} = 0 = p_2A_2x_2 - p_0\delta u_2,$$

$$\frac{\partial H}{\partial v_1} = 0 = p_1B_1x_1 - p_0\delta v_1,$$

$$\frac{\partial H}{\partial v_2} = 0 = p_2B_2x_2 - p_0\delta v_2.$$

From the condition of the Hamiltonian system optimal, each agent's control is obtained. For i = 1 and 2 the control for each agent can be written as follows.

$$u_i = \frac{p_iA_i}{p_0\delta},$$

$$v_i = \frac{p_iB_i}{p_0\delta}.$$

After the control for each agent is obtained and substituted in the Hamiltonian System, then the Hamiltonian system can be solved. The movement equation of agents 1 and 2 are found, and plot the result given in Figure 1. From figure 1, it can be seen that the initial position of agent 1 is $x_1(0)=3$ and the final position of agent 1 is $x_1(1)=4$. Also, the initial position of agent 2 is $x_2(0)=4$ and the final position of agent 1 is $x_2(1)=5$. The simulation result in Figure 1 shows that the simulation is successful.

3.2 Uncoupled Two Agents in R^2

The uncoupled two agents model as bilinear control in R^2 is described as follows.

$$\dot{y}_1 = u_1 A_1 y_1 + v_1 B_1 y_1,$$

$$\dot{y}_2 = u_2 A_2 y_2 + v_2 B_2 y_2.$$

with y_i in R^2. In this paper $y_1 = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $y_2 = \begin{bmatrix} x_3 \\ x_4 \end{bmatrix}$.

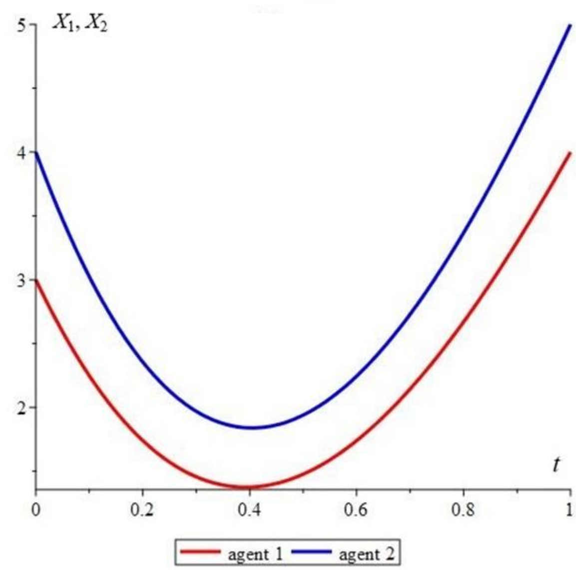

Figure 1. The optimal path of a coupled agents in R.

Consider the equation

$$\dot{y}_1 = u_1 A_1 y_1 + v_1 B_1 y_1,$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = u_1 A_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + v_1 B_1 \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} u_1 A_1 x_1 + v_1 B_1 x_1 \\ u_1 A_1 x_2 + v_1 B_1 x_2 \end{bmatrix}.$$

Similarly, from equation

$$\dot{y}_2 = u_2 A_2 y_2 + v_2 B_2 y_2,$$

we infer

$$\begin{bmatrix} \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} u_2 A_2 x_3 + v_2 B_2 x_3 \\ u_2 A_2 x_4 + v_2 B_2 x_4 \end{bmatrix}.$$

Similar to the uncoupled two agents' model in R, for simulation, the values of A_1 and A_2 are 10. Still, also for simulation, the values of B_1 and B_2 are 5. Next, the initial and boundary conditions of two agents are given as follows

$$x_1(0) = 8, x_2(0) = 8, x_1(4) = 4, x_2(4) = 20,$$
$$x_3(0) = 9, x_4(0) = 9, x_3(4) = 5, x_4(4) = 24.$$

The functional cost is defined by

$$J = -\frac{1}{2}\int_0^4 (\delta u_1{}^2 + \delta u_2{}^2 + \delta v_1{}^2 + \delta v_2{}^2 + \frac{\gamma}{|x_1 - x_3|^2 + |x_2 - x_4|^2})dt.$$

The constant δ is a control constant and similar to simulation in R, for simulation in R^2 it is given a value of 1. Next, γ is a repellent constant which is currently also given a value of 1 for simulation. The Hamiltonian function is

$$\begin{aligned} H = {} & p_1(u_1 A_1 x_1 + v_1 B_1 x_1) + p_2(u_2 A_2 x_2 + v_2 B_2 x_2) + p_3(u_3 A_3 x_3 + v_3 B_3 x_3) \\ & + p_4(u_4 A_4 x_4 + v_4 B_4 x_4) - \frac{1}{2} p_0 \delta u_1{}^2 - \frac{1}{2} p_0 \delta u_2{}^2 - \frac{1}{2} p_0 \delta v_1{}^2 - \frac{1}{2} p_0 \delta v_2{}^2 \\ & - \frac{1}{2} \frac{p_0 \gamma}{|x_1 - x_3|^2 + |x_2 - x_4|^2}. \end{aligned}$$

Consider the Hamiltonian function above, $p_0 = -1$ and appears co-state variables of p_i for $i = 1,2,3,4$. The Hamiltonian System is

$$\frac{\partial H}{\partial p_1} = \dot{x}_1 = u_1 A_1 x_1 + v_1 B_1 x_1,$$
$$\frac{\partial H}{\partial p_2} = \dot{x}_2 = u_1 A_2 x_2 + v_2 B_2 x_2,$$
$$\frac{\partial H}{\partial p_3} = \dot{x}_3 = u_2 A_3 x_3 + v_2 B_3 x_3,$$

$$\frac{\partial H}{\partial p_4} = \dot{x}_4 = u_2 A_4 x_4 + v_2 B_4 x_4,$$

$$\frac{\partial H}{\partial x_1} = -\dot{p}_1 = p_1 u_1 A_1 + p_1 v_1 B_1 + \frac{\gamma p_0 (x_3 - x_1)}{\left\| |x_1 - x_3|^2 + |x_2 - x_4|^2 \right\|^2},$$

$$\frac{\partial H}{\partial x_2} = -\dot{p}_2 = p_2 u_2 A_2 + p_2 v_1 B_1 + \frac{\gamma p_0 (x_4 - x_2)}{\left\| |x_1 - x_3|^2 + |x_2 - x_4|^2 \right\|^2},$$

$$\frac{\partial H}{\partial x_3} = -\dot{p}_3 = p_3 u_2 A_2 + p_3 v_2 B_2 + \frac{\gamma p_0 (x_1 - x_3)}{\left\| |x_1 - x_3|^2 + |x_2 - x_4|^2 \right\|^2},$$

$$\frac{\partial H}{\partial x_4} = -\dot{p}_4 = p_4 u_2 A_2 + p_4 v_2 B_2 + \frac{\gamma p_0 (x_2 - x_4)}{\left\| |x_1 - x_3|^2 + |x_2 - x_4|^2 \right\|^2}.$$

Based on the Pontryagin Maximum Principle, the necessary condition such that the system optimum is

$$\frac{\partial H}{\partial u_1} = 0 = p_1 A_1 x_1 + p_2 A_1 x_2 - p_0 \delta u_1,$$

$$\frac{\partial H}{\partial u_2} = 0 = p_2 A_2 x_3 + p_4 A_2 x_4 - p_0 \delta u_2,$$

$$\frac{\partial H}{\partial v_1} = 0 = p_1 B_1 x_1 + p_2 B_1 x_2 - p_0 \delta v_1,$$

$$\frac{\partial H}{\partial v_2} = 0 = p_3 B_2 x_3 + p_4 B_2 x_4 - p_0 \delta v_2.$$

The control of each agent can be written as follows:

$$u_1 = \frac{p_1 A_1 x_1 + p_2 A_1 x_2}{p_0 \delta}, \quad u_2 = \frac{p_2 A_2 x_3 + p_4 A_2 x_4}{p_0 \delta},$$

$$v_1 = \frac{p_1 B_1 x_1 + p_2 B_1 x_2}{p_0 \delta}, \quad v_2 = \frac{p_3 B_2 x_3 + p_4 B_2 x_4}{p_0 \delta}.$$

The obtained controls can be substituted to Hamiltonian System and its system solved, then the movement equation agent 1 and 2 are found, and the result is a plot in figure 2.

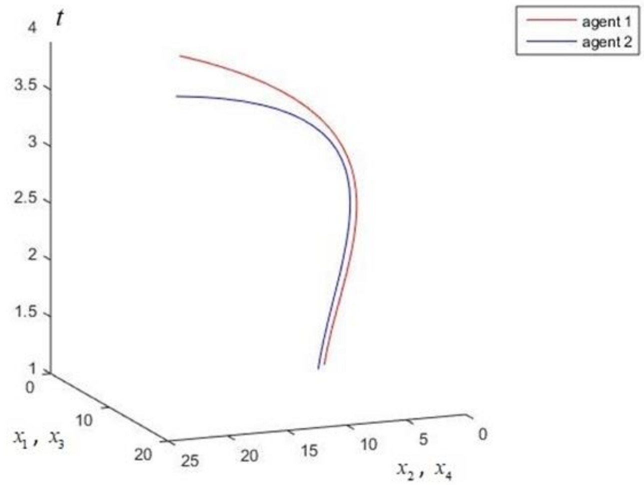

Figure 2. The optimal path of a coupled agent in R^2

From Figure 2, it can be seen that the simulation of a pair of agents in R^2 can run well. Each agent does not collide one to the other. Agents can move from the initial position to the final position in pairs without crashing.

4. CONCLUSIONS

The modeling of uncoupled agents with optimal control bilinear system has been successfully carried out. The bilinear dynamics model and the cost functional are combined in the Hamilton function. from the Hamilton function is derived to the Hamilton system. Control for agents was obtained using Pontryagin's Maximum Principle. After the control is obtained, the equations of motion of the agents are also obtained. Then it was done in two simulations. The simulation was successfully performed for the pair of moving agents in R and in R^2. In both simulations, the agents managed to move from the starting position to the end without hitting each other.

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CONFLICT OF INTEREST

The authors declare no conflict of interest. The funders had no role in the design of the study; in the collection, analyses, or interpretation of data; in the writing of the manuscript, or in the decision to publish the results.

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