Comparison of AUV Position Estimation Using Kalman Filter, Ensemble Kalman Filter and Fuzzy Kalman Filter Algorithm in the Specified Trajectories

Ngatini[1]*, Zunif Ermayanti[2], Erna Apriliani[2], and Hendro Nurhadi[3]
[1]Department of Informatics, Universitas Internasional Semen Indonesia, Indonesia
[2]Department of Mathematics, Institut Teknologi Sepuluh Nopember, Indonesia
[3]Department of Mechanical Engineering, Institut Teknologi Sepuluh Nopember, Indonesia
Email: ngatini@uisi.ac.id

Abstract

This research compares AUV position estimations using Kalman Filter (KF), Ensemble Kalman Filter (EnKF), and Fuzzy Kalman Filter (FKF) algorithm for some specified trajectories. Assessment is performed on AUV Segorogeni ITS developed by the Institute Technology of Sepuluh Nopember (ITS), Indonesia. The specified trajectories are actual trajectories, i.e., diving, straight, and turning paths. The comparisons for each trajectory are made according to the simulation results and the RMSE (Root Mean Square Error) values. The best estimation is given by different methods depending on the trajectories. Fuzzy Kalman Filter gives the best result on the diving trajectory (Y-position and angle) and the straight trajectory. Ensemble Kalman Filter (EnKF) provides the best result on the X-position in the diving trajectory. At the same time, Kalman Filter gives the best result on a straight trajectory.
Keywords: AUV; Kalman Filter (KF); Ensemble Kalman Filter (EnKF); Fuzzy Kalman Filter (FKF); AUV Segorogeni ITS.

Abstrak

Penelitian ini menjelaskan tentang perbandingan estimasi untuk posisi AUV *antara algoritma* Kalman Filter (KF)*, *Ensemble Kalman Filter (EnKF) *dan* Fuzzy Kalman Filter (FKF) *untuk trayektori tertentu. Estimasi dilakukan terhadap* AUV *Segorogeni ITS yang dibuat oleh ITS* (*Institut Teknologi Sepuluh Nopember*)*, Indonesia. Trayektori yang diberikan adalah menyelam, lurus dan lintasan membelok yang merupakan lintasan real. Peneliti melakukan perbandingan untuk setiap lintasan berdasarkan hasil simulasi dan* Root Mean Square Error (RMSE)*. Pada kasus ini estimasi terbaik diberikan oleh metode yang berbeda.* Fuzzy Kalman Filter *memberikan hasil terbaik untuk lintasan berbelok pada posisi-Y dan pada garis lurus.* Ensemble Kalman Filter *memberikan estimasi terbaik untuk posisi-X pada lintasan menyelam. Sedangkan* Kalman Filter *memberikan hasil terbaik untuk lintasan lurus.*
Kata kunci: AUV; Kalman Filter (KF); Ensemble Kalman Filter (EnKF); Fuzzy Kalman Filter (FKF); AUV Segorogeni ITS.

1. INTRODUCTION

There are many methods for estimating mathematical models, both linear and nonlinear. Several ways use data assimilation that combines it with measurement data [1]. We can build an estimation based on linear or nonlinear mathematical models using data assimilation. One of the data assimilation methods for a linear model is Kalman Filter [2]. Various mathematical models need some modification of the Kalman Filter (KF) to solve more accurately. Different variations of Kalman Filter (KF) algorithms are Fuzzy Kalman Filter (FKF),

Unscented Kalman Filter (UKF), Ensemble Kalman Filter (EnKF), and other modifications. EnKF is an algorithm to estimate nonlinear models [3], while FKF is an algorithm to estimate linear models with a Fuzzy state variable [4]. This paper investigates the comparison of AUV (Autonomous Underwater Vehicle) position estimation between KF, FKF, and EnKF.

AUV (Autonomous Underwater Vehicle) is one of the Unmanned Underwater Vehicle (UUV). UUV has better performance to drive than humans [5]. AUV, a type of UUV, is an autonomous underwater vehicle that is moved and controlled by the computer, with a propulsion system in the water and has three dimensions of maneuver [6]. Recently, AUV has been used for many underwater tasks like underwater biology, geology, and others, and it has a reasonable cost for underwater technology [7]. In this research, AUV Segorogeni ITS is used for this work (Fig. 1).

Ngatini et al. [4] had estimated the trajectory of AUV Segorogeni ITS using Ensemble Kalman Filter (EnKF) and Fuzzy Kalman Filter (FKF). The AUV position was estimated using the model of AUV motion, a nonlinear mathematical model. The estimation result showed that EnKF has a better estimation for the AUV position than FKF. In this research, the authors develop a comparison between Kalman Filter (KF), Fuzzy Kalman Filter (FKF), and Ensemble Kalman Filter (EnKF) to estimate the AUV trajectories. Each error estimation and computation time are compared and simulated using GNU Octave-4.2.2 (GUI). Section 2 describes the dynamical model of AUV motion that consists of the nonlinear mathematical model and specification of the AUV Segorogeni ITS. In section 3, the authors explain the estimation using the Kalman Filter algorithms, including the state space development, linearization, and the steps of the Kalman Filter. In this Section authors also introduce previous results for the AUV position estimation using EnKF and FKF in terms of description and implementation of both algorithms in the AUV position estimation. The estimation comparisons between KF, EnKF, and FKF are given in Section 4. The comparison of simulation and error estimation is also explained in this section. Before applying the estimation algorithm, we develop three trajectories of the AUV position as the actual trajectories. We summarized our research in Section 5.
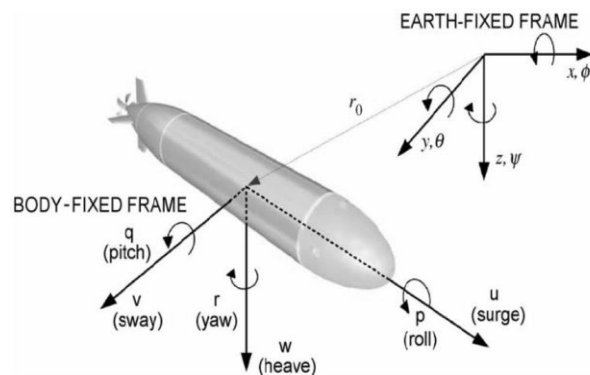

Figure 1. AUV Segorogeni ITS.　　　　　　**Figure 2.** AUV coordinates [8, p.14].

2.　METHODS

In this study, the estimation of AUV position was carried out using three methods, namely Kalman Filter, Ensemble Kalman Filter, and Fuzzy Kalman Filter. The model of the motion equation

of the AUV is nonlinear, so it needs to be linearized before being estimated using the Kalman Filter algorithm and Fuzzy Kalman Filter. However, the nonlinear model was not changed for estimation using the Ensemble Kalman Filter algorithm. The first stage is the formation of state states from the model, which is then continued for the stages of each algorithm. This stage can be seen in the following sub-chapter.

2.1 Mathematical model of AUV motion

AUV is an autonomous underwater vehicle that is moved and controlled by the computer, with a propulsion system in the water; and has three dimensions of maneuver [6]. The motions of AUV are in 6 degrees of freedom (DOF). They are the position and translational motion and the orientation and rotational motion [9]. The AUV coordinates can be seen in Fig. 2, while the notation used for AUV is described in Table 1.

Table 1. Notations for AUV [8].

DOF	Motion	Forces and Moments	Linear and Angular Velocities	Position and Euler Angles
1	Surge (x-direction)	X	u	x
2	Sway (y-direction)	Y	v	y
3	Heave (z-direction)	Z	w	z
4	Roll (rotation about-x)	K	p	φ
5	Pitch (rotation about-y)	M	q	θ
6	Yaw (rotation about-z)	N	r	ψ

According to Yang (2007), the nonlinear models of AUV motion are expressed as follows [8]:

- Translation along x-direction:
$$m[\dot{u} - vr + wq - x_G(q^2 + r^2) + y_G(pq - \dot{r}) + z_G(pq + \dot{q})] = X_{res} + X_{u|u|}u|u| + X_{\dot{u}}\dot{u} + X_{wq}wq + X_{qq}qq + X_{vr}vr + X_{rr}rr + X_{prop}, \tag{1}$$

- Translation along y-direction:
$$m[\dot{v} - wp + ur - y_G(r^2 + p^2) + z_G(qr - \dot{p}) + x_G(pq + \dot{r})] = Y_{res} + Y_{v|v|}v|v| + Y_{r|r|}r|r| + Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{ur}ur + Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r, \tag{2}$$

- Translation along z-direction:
$$m[\dot{w} - uq + vp - z_G(p^2 + q^2) + x_G(rp - \dot{q}) + y_G(rq + \dot{p})] = Z_{res} + Z_{w|w|}w|w| + Z_{q|q|}q|q| + Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uq}uq + Z_{vp}vp + Z_{rp}rp + Z_{uw}uw + Z_{uu\delta_s}u^2\delta_s, \tag{3}$$

- Rotation along x-direction:
$$I_x\dot{p} + (I_z - I_y)qr + m[y_G(\dot{w} - uq + vp) - z_G(\dot{v} - wp + ur)] = K_{res} + K_{p|p|}p|p| + K_{\dot{p}}\dot{p} + K_{prop}, \tag{4}$$

- Rotation along y-direction:
$$I_y\dot{q} + (I_x - I_z)rp + m[z_G(\dot{u} - vr + wq) - x_G(\dot{w} - uq + vp)] = M_{res} + M_{w|w|}w|w| + M_{q|q|}q|q| + M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uq}uq + M_{vp}vp + M_{rp}rp + M_{uw}uw + M_{uu\delta_s}u^2\delta_s, \tag{5}$$

- Rotation along z-direction:

$$I_z \dot{r} + (I_y - I_x)pq + m[x_G(\dot{v} - wp + ur) - y_G(\dot{u} - vr + wq)] = N_{res} + N_{v|v|}v|v| + N_{r|r|}r|r| +$$
$$N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{ur}ur + N_{wp}wp + N_{pq}pq + N_{uv}uv + N_{uu\delta_r}u^2\delta_r, \tag{6}$$

where

m	: total mass of AUV,
x_G, y_G, z_G	: the gravity of AUV's center in body-fixed coordinates,
$\dot{u}, \dot{v}, \dot{w}, \dot{p}, \dot{q}, \dot{r}$	: the acceleration,
I_x, I_y, I_z	: the inertia moment of the x,y, z-axes, respectively,
X_{prop}	: the thrust force of the propeller,
K_{prop}	: the additional moment of a propeller,
$X_{u|u|}$	: the axial drag coefficient,
$K_{p|p|}$	: the rolling drag coefficient,

$X_{res}, Y_{res}, Z_{res}, K_{res}, M_{res}, N_{res}$	: the hydrostatic force,
$X_{\dot{u}}\dot{u} + X_{wq}wq + X_{qq}qq + X_{vr}vr + X_{rr}rr$	: the added mass force in surge motion,
$Y_{\dot{v}}\dot{v} + Y_{\dot{r}}\dot{r} + Y_{ur}ur + Y_{wp}wp + Y_{pq}pq$	: the added mass force and body lift force in sway motion,
$Z_{\dot{w}}\dot{w} + Z_{\dot{q}}\dot{q} + Z_{uq}uq + Z_{vp}vp + Z_{rp}rp$	: the added mass force and body lift force in heave motion,
$K_{\dot{p}}\dot{p}$	: the added mass force in roll motion,
$M_{\dot{w}}\dot{w} + M_{\dot{q}}\dot{q} + M_{uq}uq + M_{vp}vp + M_{rp}rp + M_{uw}uw$	: the added mass force, lift force and body lift force in pitch motion,
$N_{\dot{v}}\dot{v} + N_{\dot{r}}\dot{r} + N_{ur}ur + N_{wp}wp + N_{pq}pq + N_{uv}uv$	: the added mass force, lift force and body lift force in yaw motion,
$Y_{uv}uv, Z_{uw}uw$	: the lift force (fin lift and body lift)
$Y_{uu\delta_r}u^2\delta_r, Z_{uu\delta_s}u^2\delta_s, M_{uu\delta_s}u^2\delta_s, N_{uu\delta_r}u^2\delta_r$	: the lift force (fin lift),
$Y_{v|v|}, Z_{w|w|}, M_{w|w|}, N_{v|v|}, Y_{r|r|}, Z_{q|q|}, M_{q|q|}, N_{r|r|}$	: the crossflow drag coefficients,
$X_{wq}, X_{qq}, X_{vr}, X_{rr}, Y_{ur}, Y_{wp}, Y_{pq}, Z_{uq}, Z_{vp}, Z_{rp}$	: the rolling added mass coefficient,
$M_{uq}, M_{vp}, M_{rp}, M_{uw}, N_{ur}, N_{wp}, N_{pq}, N_{uv}$	: the rolling added mass coefficient.

The right-hand side of equations (1)-(6) describes the total force and moment from combining the hydrostatic force (subscript res), lift force, added mass force, body lift, and fin lift [8]. These equations explain the total force and moment in Table 1. Equations (1) – (6) are nonlinear dynamic equations of the AUV motion. These equations can be written by [9]:

$$[M_{RB} + M_A]\dot{v} + [C_{RB}(v) + C_A(v)]v + D(v)v + g(\eta) = \tau_E + \tau, \tag{7}$$

where

$\eta = [x, y, z, \varphi, \theta, \psi]^T$	: the linear and the angular position vector in the earth-fixed coordinates (EFF),
$v = [u, v, w, p, q, r]^T$	: the linear and the angular velocity in the body-fixed coordinates (BFF),
$\tau = [X, Y, Z, K, M, N]^T$	: the forces and the moments acting on the vehicle in the body-fixed frame,
M_A	: added mass matrix,
M_{RB}	: AUV rigid body mass and inertia matrix,
C_{RB}	: rigid body Coriolis and centripetal matrix,
C_A	: added mass induced Coriolis-centripetal matrix,

$D(v)$	: damping matrix,
$g(\eta)$	: vector of gravitational forces and moments,
τ_E	: environmental forces and moments,
τ	: propulsion forces and moments.

The AUV estimation is performed by calculating the position of AUV from the velocity estimation based on the dynamical model. We use the below equations to transform the linear and angular velocity into the position and orientation of AUV [10].

$$\dot{x} = u\cos(\psi) - v\sin(\psi), \tag{8}$$
$$\dot{y} = u\sin(\psi) + v\cos(\psi), \tag{9}$$
$$\dot{z} = w, \tag{10}$$
$$\dot{\psi} = r, \tag{11}$$

Where $\dot{x}, \dot{y}, \dot{z}$ are the linear velocities in the EFF coordinate system, $\dot{\psi}$ is the angular velocity in the EFF coordinate system, and u, v, w, r are the velocities in the BFF coordinate system.

2.2 AUV Segorogeni ITS

The AUV Segorogeni ITS is an AUV developed by the Institute Technology of Sepuluh Nopember (ITS) (see Fig. 1). That AUV has a propeller on the tail, making the thrust force and additional moments [11]. AUV Segorogoeni ITS has several advantages: the unmanned vehicle is capable of monitoring underwater conditions, has a hydrodynamic profile, the navigation system is equipped with a compass and GPS, and provided a motion sensor IMU. The AUV Segorogeni ITS can observe underwater conditions with camera vision and wireless cable [12]. The details of AUV Segorogeni ITS are described in Table 2.

Table 2. The specification of AUV Segorogeni ITS [12].

Specification	Size
Weight	15 Kg
Overall Length	980 mm
Beam	188 mm
Controller	Ardupilot Mega 2.0
Communication	Wireless Xbee 2.4 GHz
Camera	TTL Camera
Battery	Li-Po 11.8 v
Propulsion	12 V motor DC
Propeller	3 blades OD; 40 mm
Speed	1,94 knots (1 m/s)

3. AUV POSITION ESTIMATION

3.1 AUV Position Estimation using Kalman Filter (KF)

Kalman Filter is an algorithm to estimate the linear dynamical model constructed by R.E. Kalman [1]. For convenient reference, the algorithm is summarized in Table 3. The dynamical model of AUV motion is a nonlinear mathematical model, so firstly, we need to do linearization [14] for Eq. (1) – (6). We build a state space from those equations as below.

$$\dot{u} - \frac{myg\dot{r}}{m-X_{\dot{u}}} = \frac{X_{res} + X_{u|u|}\,u|u| + X_{wq}wq + X_{qq}qq}{m-X_{\dot{u}}} + \frac{X_{vr}vr + X_{rr}rr + X_{pop} - m(-vr+wq)}{m-X_{\dot{u}}} +$$
$$\frac{-m(-x_G(q^2+r^2)+y_G(pq)+z_G(pr))}{m-X_{\dot{u}}}$$
(12)

$$\dot{v} - \frac{mzG\dot{p}}{m-Y_{\dot{u}}} + \frac{(mxG-Y_{\dot{r}})\dot{r}}{m-Y_{\dot{u}}} = \frac{Y_{res} + Y_{v|v|}\,v|v| + Y_{r|r|}r|r| + Y_{ur}ur}{m-Y_{\dot{v}}} + \frac{Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r}{m-Y_{\dot{v}}} -$$
$$\frac{m\left(-wp+ur-y_G(r^2+p^2)\right)}{m-Y_{\dot{v}}} + \frac{-m(-z_G(qr)+x_G(pq))}{m-X_{\dot{u}}}$$
(13)

$$\dot{w} - \frac{(Z_{\dot{q}}+mxG)\dot{p}}{m-Z_{\dot{w}}} + \frac{myG\dot{p}}{m-Z_{\dot{w}}} = \frac{Z_{res} + Z_{w|w|}\,w|w| + Z_{q|q|}q|q| + Z_{uq}uq}{m-Z_{\dot{w}}} + \frac{Z_{vp}vp + Z_{rp}rp + Z_{uw}uw + Z_{uu\delta_s}u^2\delta_s}{m-Z_{\dot{w}}} +$$
$$\frac{-m\left(-uq+vp-z_G(p^2+q^2)\right)}{m-Z_{\dot{w}}} + \frac{-m(-x_G(rp)+y_G(rq))}{m-Z_{\dot{w}}}$$
(14)

$$\dot{p} - \frac{myG\dot{w}}{I_x-K_{\dot{p}}} + \frac{mzG\dot{v}}{I_x-K_{\dot{p}}} = \frac{K_{res} + K_{p|p|}\,p|p| + K_{prop} - (I_z-I_y)qr}{I_x-K_{\dot{p}}} + \frac{-m(yG(-uq+vp)-zG(-wp+ur))}{I_x-K_{\dot{p}}}$$
(15)

$$\dot{q} + \frac{mzG\dot{u}}{I_y-M_{\dot{q}}} - \frac{(M_{\dot{w}}+xG)\dot{w}}{I_y-M_{\dot{q}}} = \frac{M_{res} + M_{w|w|}\,w|w| + M_{q|q|}q|q|}{I_y-M_{\dot{q}}} + \frac{M_{uq}uq + M_{vp}vp + M_{rp}rp}{I_y-M_{\dot{q}}} +$$
$$\frac{M_{uw}uw + M_{uu\delta_s}u^2\delta_s - (I_x-I_z)rp}{I_y-M_{\dot{q}}} - \frac{m(zG(-vr+wq))}{I_y-M_{\dot{q}}} + \frac{mxG(-uq+vp)}{I_y-M_{\dot{q}}}$$
(16)

$$\dot{r} + \frac{(mxG-N_{\dot{v}})\dot{v}}{I_z-N_{\dot{r}}} - \frac{myG\dot{u}}{I_z-N_{\dot{r}}} = \frac{N_{res} + N_{v|v|}\,v|v| + N_{r|r|}r|r|}{I_z-N_{\dot{r}}} + \frac{N_{ur}ur + N_{wp}wp + M_{pq}pq}{I_z-N_{\dot{r}}} +$$
$$\frac{N_{uv}uv + N_{uu\delta_r}u^2\delta_r - (I_y-I_x)pq}{I_z-N_{\dot{r}}} + \frac{-m(xG(-wp+ur))}{I_z-N_{\dot{r}}} + \frac{myG(-vr+wq)}{I_z-N_{\dot{r}}}$$
(17)

Table 3. The Kalman Filter Algorithm [13].

1. Observation model

$x_{k+1} = C_k x_k + \varsigma_k$

$z_k = H_k x_k + \xi_k$

$x_0 \sim (x_0, P_{x_0}),\ \varsigma_k \sim (0, Q_k),\ \xi_k \sim (0, R_k)$

where,

x_k : variable state at the time- k

ς_k, ξ_k: Noise of system with mean=0 and covariance = Q_k, R_k, respectively

C_k, B_k, G_k: Coefficient matrix of each variable

z_k : Observation variable

H_k : Matrix observation

P_{x_0} : Initial covariance

2. Initialization

$P_0 = P_{x_0}, \hat{x}_0 = \bar{x}_0$

3. Time update (Prediction)

Covariance of Error: $P_{k+1}^- = C_k P_k C_k^T + Q_k$

Estimate $\hat{x}_{k+1}^- = C_k \hat{x}_k$

4. Measurement update (Correction)

Kalman Gain: $K_{k+1} = P_{\bar{k}+1}^- H_{k+1}^T (H_{k+1} P_{\bar{k}+1}^- H_{k+1}^T + R_{k+1})^{-1}$

Update of error covariance: $P_{k+1} = (I - K_{k+1}H_{k+1})P_{k+1}^-$

Estimation update: $\hat{x}_{k+1} = \hat{x}_{k+1}^- + K_{k+1}(z_{k+1} - H\hat{x}_{k+1}^-)$

From equation (12) – (17), we define the right side as B1, B2, B3, B4, B5, and B6. Meanwhile, we build a matrix for the left side.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & \frac{mzG}{m-X_{\dot{u}}} & -\frac{myG}{m-X_{\dot{u}}} \\ 0 & 1 & 0 & -\frac{mzG}{m-Y_{\dot{v}}} & 0 & \frac{(mxG-Y_{\dot{r}})}{m-Y_{\dot{v}}} \\ 0 & 0 & 1 & \frac{myG}{m-Z_{\dot{w}}} & \frac{(Z_{\dot{q}}+mxG)}{m-Z_{\dot{w}}} & 0 \\ 0 & -\frac{mzG}{I_x-K_{\dot{p}}} & \frac{myG}{I_x-K_{\dot{p}}} & 1 & 0 & 0 \\ \frac{mzG}{I_y-M_{\dot{q}}} & 0 & -\frac{(M_{\dot{w}}+xG)}{I_y-M_{\dot{q}}} & 0 & 1 & 0 \\ -\frac{myG}{I_z-N_{\dot{r}}} & \frac{(mxG-N_{\dot{v}})}{I_z-N_{\dot{r}}} & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} B1 \\ B2 \\ B3 \\ B4 \\ B5 \\ B6 \end{pmatrix}$$
(18)

In general, that matrix can be written as follows:

$$A\dot{x} = B,$$
(19)

where,

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & \frac{mzG}{m-X_{\dot{u}}} & -\frac{myG}{m-X_{\dot{u}}} \\ 0 & 1 & 0 & -\frac{mzG}{m-Y_{\dot{v}}} & 0 & \frac{(mxG-Y_{\dot{r}})}{m-Y_{\dot{v}}} \\ 0 & 0 & 1 & \frac{myG}{m-Z_{\dot{w}}} & \frac{(Z_{\dot{q}}+mxG)}{m-Z_{\dot{w}}} & 0 \\ 0 & -\frac{mzG}{I_x-K_{\dot{p}}} & \frac{myG}{I_x-K_{\dot{p}}} & 1 & 0 & 0 \\ \frac{mzG}{I_y-M_{\dot{q}}} & 0 & -\frac{(M_{\dot{w}}+xG)}{I_y-M_{\dot{q}}} & 0 & 1 & 0 \\ -\frac{myG}{I_z-N_{\dot{r}}} & \frac{(mxG-N_{\dot{v}})}{I_z-N_{\dot{r}}} & 0 & 0 & 0 & 1 \end{pmatrix}$$
(20)

$$\dot{x} = \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{pmatrix},$$
(21)

$$B = \begin{pmatrix} B1 \\ B2 \\ B3 \\ B4 \\ B5 \\ B6 \end{pmatrix}.$$
(22)

B is a nonlinear equation on the right side of the matrix model consisting of six equations, namely $B1$, $B2$, $B3$, $B4$, $B5$, and $B6$.

$$B1 = \frac{X_{res} + X_{u|u|}u|u| + X_{wq}wq + X_{qq}qq}{m - X_{\dot{u}}} + \frac{X_{vr}vr + X_{rr}rr + X_{prop} - m(-vr+wq)}{m - X_{\dot{u}}} + \frac{-m(-xG(q^2+r^2)+yG(pq)+zG(pr))}{m - X_{\dot{u}}}$$
(23)

$$B2 = \frac{Y_{res} + Y_{u|v|}u|v| + Y_{r|r|}r|r| + Y_{ur}\,ur}{m - Y_{\dot{v}}} + \frac{Y_{wp}wp + Y_{pq}pq + Y_{uv}uv + Y_{uu\delta_r}u^2\delta_r}{m - Y_{\dot{v}}} - \frac{m\left(-wp + ur - y_G\left(r^2 + p^2\right)\right)}{m - Y_{\dot{v}}} +$$
$$\frac{-m(-z_G(qr) + x_G(pq))}{m - Y_{\dot{v}}}, \tag{24}$$

$$B3 = \frac{Z_{res} + Z_{w|w|}\,w|w| + Z_{q|q|}q|q| + Z_{uq}uq}{m - Z_{\dot{w}}} + \frac{Z_{vp}vp + Z_{rp}rp + Z_{uw}uw + Z_{uu\delta_s}u^2\delta_s}{m - Z_{\dot{w}}} + \frac{-m\left(-uq + vp - z_G\left(p^2 + q^2\right)\right)}{m - Z_{\dot{w}}} +$$
$$\frac{-m(-x_G(rp) + y_G(rq))}{m - Z_{\dot{w}}}, \tag{25}$$

$$B4 = \frac{K_{res} + K_{p|p|}\,p|p| + K_{prop} - (I_z - I_y)qr}{I_x - K_{\dot{p}}} + \frac{-m(y_G(-uq + vp) - z_G(-wp + ur))}{I_x - K_{\dot{p}}}, \tag{26}$$

$$B5 = \frac{M_{res} + M_{wIwI}\,wIwI + M_{q|q|}q|q|}{I_y - M_{\dot{q}}} + \frac{M_{uq}uq + M_{vp}vp + M_{rp}rp}{I_y - M_{\dot{q}}} + \frac{M_{uw}uw + M_{uu\delta_s}u^2\delta_s - (I_x - I_z)rp}{I_y - M_{\dot{q}}} - \frac{m(z_G(-vr + wq))}{I_y - M_{\dot{q}}} +$$
$$\frac{mxG(-uq + vp)}{I_y - M_{\dot{q}}}, \tag{27}$$

$$B6 = \frac{N_{res} + N_{vIvI}\,vIvI + N_{r|r|}r|r|}{I_z - N_{\dot{r}}} + \frac{N_{ur}ur + N_{wp}wp + M_{pq}pq}{I_z - N_{\dot{r}}} + \frac{N_{uv}uv + N_{uu\delta_r}u^2\delta_r - (I_y - I_x)pq}{I_z - N_{\dot{r}}} + \frac{-m(xG(-wp + ur))}{I_z - N_{\dot{r}}} \tag{28}$$

Equation (19) is a nonlinear model. Therefore, a linearization is carried out to get a linear form. The following is derived from the linearization more precisely [15].

$$\dot{x} = Cx + Dy. \tag{29}$$

Matrices C and D are measured by Jacobi Matrix application to the speed and control [12].

$$J_x = \left[\frac{\vartheta F}{\vartheta x}\right], \tag{30}$$
$$J_y = \left[\frac{\vartheta F}{\vartheta u}\right], \tag{31}$$

where $y = \left[X_{prop}, \delta_r, \delta_s, K_{prop}, \delta_s, \delta_R\right]^T$,

$$C = A^{-1}.J_x, \tag{32}$$
$$D = A^{-1}.J_y. \tag{33}$$

The equation of AUV motion must be changed into a discrete form since Kalman Filter can only be implemented on discrete systems. Discretization is applied using the Finite Difference method for forwarding difference. This method is obtained from the Taylor series in time-*t* to form the difference quotient [16].

$$\dot{x} = \frac{dx}{dt}, \tag{34}$$

$$\approx \frac{x_{k+1} - x_k}{\Delta t}, \tag{35}$$
$$\frac{x_{k+1} - x_k}{\Delta t} = Cx + Dy, \tag{36}$$

$$x_{k+1} = x_k + (Cx + Dy)\Delta t, \tag{37}$$

$$x_{k+1} = (C\Delta t + 1)x_k + D\Delta ty. \tag{38}$$

Equation (38) requires a system and observation model to estimate the AUV position before applying the Kalman Filter method. Firstly, we define the x-component and give the initial value for each component.

$$x = \begin{pmatrix} u \\ v \\ w \\ p \\ q \\ r \end{pmatrix}; x_0 = \begin{pmatrix} u_0 \\ v_0 \\ w_0 \\ p_0 \\ q_0 \\ r_0 \end{pmatrix}. \tag{39}$$

The state-space of the AUV motion is

$$x_{k+1} = (C\Delta t + 1)x_k + D\Delta ty.$$

That state space is implemented in the Kalman Filter to get an AUV position estimation. Adding noise in the system and observation model is needed, ς and ξ respectively. The estimation steps are as follows:

1. System Model
   System model from the state space added noise is written as follows:

   $$x_{k+1} = (C\Delta t + 1)x_k + D\Delta ty + \varsigma_k, \tag{40}$$

   $$\begin{pmatrix} u_{k+1} \\ v_{k+1} \\ w_{k+1} \\ p_{k+1} \\ q_{k+1} \\ r_{k+1} \end{pmatrix} = (C\Delta t + 1) \begin{pmatrix} u_k \\ v_k \\ w_k \\ p_k \\ q_k \\ r_k \end{pmatrix} + D\Delta t \begin{pmatrix} X_{prop} \\ \delta_r \\ \delta_x \\ K_{prop} \\ \delta_s \\ \delta_r \end{pmatrix} + \varsigma_k, \tag{41}$$

   where C and D result from the matrix Jacobi in the linearization process, and ς_k is a noise system generated by Gaussian distribution with mean 0 and covariance Q at the time-k.

2. Observation Model
   The observation data used in this estimation are $u, v, w,$ and r.

   $$z_k = Hx_k + \xi_k, \tag{42}$$

   $$z_k = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_k \\ v_k \\ w_k \\ p_k \\ q_k \\ r_k \end{pmatrix} + \xi_k, \tag{43}$$

   where ξ_k is a noise observation following a normal distribution with mean 0 and covariance R at the time-k.

3. Initialization
4. We give an initial value for:

   $$\hat{x}_0 = x_0, \tag{44}$$
   $$\hat{P}_0 = P_{x_0}. \tag{45}$$

5. Prediction Step

   $$\hat{x}_{\bar{k}+1} = (C\Delta t + 1)\hat{x}_k + D\Delta ty, \tag{46}$$

$$P_{\bar{k}+1} = (C\Delta t)P_k(C\Delta t)^T + Q_k, \tag{47}$$

where $\hat{x}_{\bar{k}-1}$ is an $n \times 1$ matrix, $P_{\bar{k}+1}$ and Q_k are a diagonal $n \times n$ matrix.

6. Correction Step
   Kalman Gain:

$$K_{k+1} = P_{\bar{k}+1}H_{k+1}^T(H_{k+1} + 1P_{\bar{k}+1}H_{k+1}^T + R_{k+1})^{-1}. \tag{48}$$

Estimation update:

$$\hat{x}_{k+1} = \hat{x}_{\bar{k}+1} + K_{k+1}(z_k - H\hat{x}_{k+1}). \tag{49}$$

$\hat{x}_{k+1}$ is the result of the AUV position estimation. Update of error covariance:

$$P_{k+1} = (I - K_{k+1}H_{k+1})P_{k+1}^-, \tag{50}$$

where K_k is an $n \times m$ matrix, R_k is an $m \times m$ matrix, z_k is an $m \times 1$ matrix, H_k is an $m \times n$ matrix, $\hat{x}_{k+1}$ is an $n \times 1$ matrix, I is an identity matrix.

The next step is a transformation of AUV by using equation (8) – (11).

3.2 AUV Position Estimation using ENKF and FKF

3.2.1   The Ensemble Kalman Filter (EnKF)

Evensen first constructed the Ensemble Kalman Filter (EnKF) method by generating or using several ensembles to estimate the error covariance at the prediction step [17]. It is one of the data assimilation methods widely used to estimate various problems in the nonlinear form. It is solved by a nonlinear model dynamical system and ample state space. Table 4 summarizes the EnKF algorithm for AUV position estimation. The AUV estimation using EnKF is implemented for the nonlinear equations of AUV's motion in equation (19) without the linearization process. That equations are the system model of x_{k+1} that is estimated using the EnKF algorithm. From the Kalman Filter algorithm, we get an estimation in the correction step as the result of estimation is $\hat{x}_k$.

3.2.2   The Fuzzy Kalman Filter

Fuzzy Kalman Filter is an algorithm for estimation using fuzzy set and Kalman Filter method [12]. From the linearization of state-space formation in equation (38), equation $x_{k+1} = (C\Delta t + 1)x_k + D\Delta ty$ is obtained, where the variable is used to be implemented in the Fuzzy Kalman Filter algorithm. The Fuzzy Kalman Filter steps contain a Fuzzification, Fuzzy Logic Rule Base, Defuzzification, and The Fuzzy Kalman Filter (FKF) algorithm [18]. The algorithms for estimation are described as follows[4].

1. **Linearization and Discretization**
   The first step in the Fuzzy Kalman Filter is a linearization for the nonlinear model of the AUV motion. The linearization results are performed using the Jacobian method or Taylor series in equation (29). The next step is discretization. The discretization results are shown in equation (34) – (38).

Table 4. The EnKF Algorithm [1].

---

1. **System model**
   $$x_{k+1} = f(x_k, u_k) + \varsigma_k.$$
   Observation model
   $$z_k = H_k x_k + \xi_k,$$
   $$\varsigma_k \sim N(0, Q_k), \xi_k \sim N(0, R_k),$$
   where
   x_k : variable state at the time- k,
   $f(x_k, u_k)$: the non-linear equations,
   ς_k, ξ_k: the noise of system with mean=0 and covariance $= Q_k, R_k$, respectively,
   z_k : observation variable,
   H_k : observation matrix.

2. **Initialization**
   Generate the n-ensembles of initial estimation $x_{0,1} = [x_{0,1}, x_{0,2}, x_{0,3}, \dots \dots x_{0,n}]$.
   With $x_{0,i} \sim N(\bar{x}_0, P_0)$ and P_0 is initial covariance.
   Mean of the initial estimation which generated: $\hat{x}_0 = \frac{1}{n}\sum_{i=1}^{N}\hat{x}_{0,1}.$

3. **The Prediction Step**
   $$\hat{x}_{\bar{k},i} = f(\hat{x}_{k-1,i}, u_{k-1}) + w_{K,I},$$
   with $w_{k,i} \sim N(0, Q_k)$ is the noise system.
   Mean: $\hat{x}_{\bar{k}} = \frac{1}{n}\sum_{i=1}^{N}\hat{x}_{\bar{k},i}.$
   Covariance of the Error: $P_{\bar{k}} = \frac{1}{n-1}\sum_{i=1}^{N}(\hat{x}_{\bar{k},i} - \hat{x}_{\bar{k}})(\hat{x}_{\bar{k},i} - \hat{x}_{\bar{k}})^T.$

4. **The Correction Step**
   $z_{k+1} = z_k + v_{k,i}$ with $v_{k,i} \sim N(0, R_k)$.
   Kalman Gain: $K_k = P_{\bar{k}} H^T (H P_{\bar{k}} H^T + R_k)^{-1}.$
   Update of error covariance: $P_{k+1} = (I - K_{k+1} H_{k+1}) P_{\bar{k}+1}.$
   Estimation correction $\hat{x}_{k,i} = \hat{x}_{k,i} + K_k(z_{k,i} - H\hat{x}_{\bar{k},i}).$
   Mean: $\hat{x}_k = \frac{1}{n}\sum_{i=1}^{N}\hat{x}_{k,i}$ with $P_{k=}[1 - K_k H]P_{\bar{k}}.$

---

2. **Fuzzification**

Fuzzification is a step to change a crisp form of input to be fuzzy. Variable x_{k+1} from equation (38) is changed as a membership function in fuzzy form.

$$x \in [x^-, x^+] \tag{51}$$

a. The membership function when the velocity is decreasingly to be minimum

$$\mu_x = \begin{cases} 1, & \text{when } x < x^-; \\ \dfrac{x - x^-}{x^+ - x^-} & \text{when } x^- \leq x \leq x^+; \\ 0, & \text{when } x^+ < x. \end{cases} \tag{52}$$

b. The membership function when the velocity is increasingly to be maximum

$$\mu_x = \begin{cases} 1, & \text{when } x < x^+; \\ \dfrac{x^+ - x}{x^+ - x^-} & \text{when } x^- \leq x \leq x^+; \\ 0, & \text{when } x^- < x. \end{cases} \tag{53}$$

3. **Fuzzy Logic Rule Base**
   The rule base IF-THEN fuzzy logic is below:

   $$\text{Rule}^i : \text{ IF c is } C_i \text{ then } x_{k+1}^i = \left(C^i \Delta t + 1\right)x_k + D\Delta ty,$$

   where,
   "c is C_i" stands for "c belongs to the interval C and has a membership value $\mu_C(c)$".
   Or
   c: $u^-, u^+, v^-, v^+, w^-, w^+, p^-, p^+, q^-, q^+, r^-, r^+$,
   0: Initial value for minimum velocity,
   1: Initial value for maximum velocity,
   C_i: $\mu_c(c)c$.
   The AUV motion has six variables: u, v, w, p, q, and r. After applying the Fuzzy Logic Rule Base, the state variables are 2^n or $2^6 = 64$.

4. **The Fuzzy Kalman Filter**
   The equation system of AUV motion is $x_{k+1} = (C\Delta t + 1)x_k + D\Delta ty$. Meanwhile, matrix C in the Fuzzy Kalman Filter Algorithm is transformed in C_i, derived from the Fuzzy Logic Rule Base. The equation is then implemented in the algorithm in Table 5. From the algorithm, we get an estimation correction below as the estimation result.

   $$\hat{x}_{\bar{k}+1} = \hat{x}_{\bar{k}+1} + K_{k+1}(Z_{k+1} - H\hat{x}_{\bar{k}+1}). \tag{54}$$

   Table 5. The FKF Algorithm.

   ---

   1. **System Model**
      $x_{k+1} = \left(C_k^i \Delta t + 1\right)x_k + D\Delta ty + \varsigma k$
      Observation Model
      $Z_k = H_k x_k + \varsigma_k$
      $\varsigma_k \sim N(0, Q_k), \varsigma_k \sim N(O, R_k)$
   2. **Initialization**
      $\hat{x}(0) = \hat{x}_0 ; \ P(0) = P_0$
   3. **The Prediction Step**
      $\hat{x}_{\bar{k}+1} = \left(C_k^i \Delta t + 1\right)\hat{x}_k + D\Delta ty$
      $P_{\bar{k}+1} = C_k^i P_k (C_k^i)^T + Q$
   4. **The Correction Step**
      $K_{k+1} = P_{\bar{k}+1} H_{k+1}^T (H_{k+1} P_{\bar{k}+1} H_{k+1}^T + R_{k+1}^T)^{-1}$
      The result estimation:
      $\hat{x}_{\bar{k}+1} = \hat{x}_{\bar{k}+1} + K_{k+1}(Z_{k+1} - H\hat{x}_{\bar{k}+1})$
      Error covariance: $P_{k+1} = (I - K_{k+1}H_{k+1})P_{\bar{k}+1}$

   ---

5. **Defuzzification**
   The estimation result from equation (54) is fuzzy from $\hat{x}_{\bar{k}+1}$. We need to transform to be crisp by using Defuzzification. The final result for Eq. (54) is calculated by using a weighted average formula,

   $$x_{k+1} = \frac{\rho^1 x_{k+1}^1 + \rho^2 x_{k+1}^2 + \rho^3 x_{k+1}^3 + \cdots + \rho^{64} x_{k+1}^{64}}{\rho^1 + \rho^2 + \rho^3 + \cdots + \rho^{64}}, \tag{55}$$

   where $\rho^i = \mu_C^i(c)$ represent the weight.

Finally, we get the result of AUV's velocity estimation of AUV. Afterward, we need to transform the result estimation of velocity in the position of AUV using equation (8) – (11).

4.   RESULTS AND DISCUSSIONS

This section shows the AUV position estimation computer simulations comparing KF, FKF, and EnKF algorithms in the two dimensions. The results explain the trajectory of AUV in the x, y, and z-axis. The nonlinear model of AUV motion is derived as the state variable in the estimation. We use the linear model system as the state variable for Kalman Filter and Fuzzy Kalman Filter methods, so we need to linearize the nonlinear model into a linear form. On the other hand, the state variable of the Ensemble Kalman Filter uses the nonlinear model without a linearization. The difference between Kalman Filter and Fuzzy Kalman Filter is based on the state space of each method, but the Filter algorithm is the same. The state variable of the Kalman Filter is a linear model of AUV motion from a linearization. At the same time, the state variable of the Fuzzy Kalman Filter is a linear form that is applied a fuzzy step before. After getting a state space from each Filter, we use the estimation algorithm

Firstly, we derive the trajectory of AUV, which is an actual number of estimations. This research has three trajectories: diving, turning, and straight trajectory. We give a diving trajectory as the estimated path in the first case. The path is given to the X-axis and Z-axis by assuming that the AUV position is the same as the Y-axis or the sway is constant. The given path is in the first 20 iterations made in a straight state, and then the path's slope changes gradually in the next iteration. The motion, in this case, tends to turn right with the depth of AUV constantly evolving in the deeper direction, which is then the AUV motion tends to be straight again. That turn is given by changing the slope gradually every iteration. The second and third cases of the trajectory are a turning and a straight path. The paths contain the X-axis and the Y-axis assuming the exact height of the AUV position, so the heave motion is considered the same and not changing. These trajectories are in Figure 3 – 5.

The rudder is 5°, and the angle is 5° with the change of time $\Delta t = 0.001$. The initial value are $x(0) = 0$ m, $y(0) = 0$ m and $z(0) = 0$ m, $u(0) = 0.1$ m/s, $v(0) = 0.1$ m/s, $w(0) = 0.1$ m/s, $p(0) = 0.1$ rad/s, $q(0) = 0.1$ rad/s, and $r(0) = 0.1$ rad/s with the covariance matrix is 10^{-6}. The RMSE of each method measures the comparison accuracy.

4.1.   Comparison Estimation

The estimation results are shown in Figures 6-8. We can compare the position estimation with the actual trajectories from these figures. The accuracy rate of each method is shown in Table 6-8, calculated using the RMSE. The RMSE calculates the error of each position estimation by comparing the position of the trajectory with the estimated result (e) from the starting point to the endpoint. The RMSE has been used to measure the performance of each algorithm to estimate the given trajectories using the following formula [19].
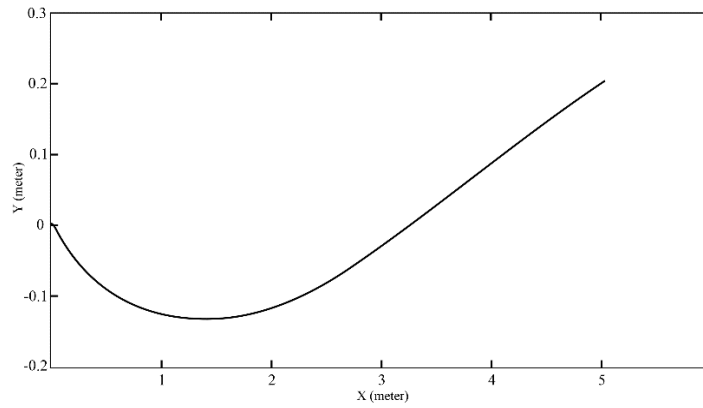
$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n} e_i^2} \, . \tag{56}$$
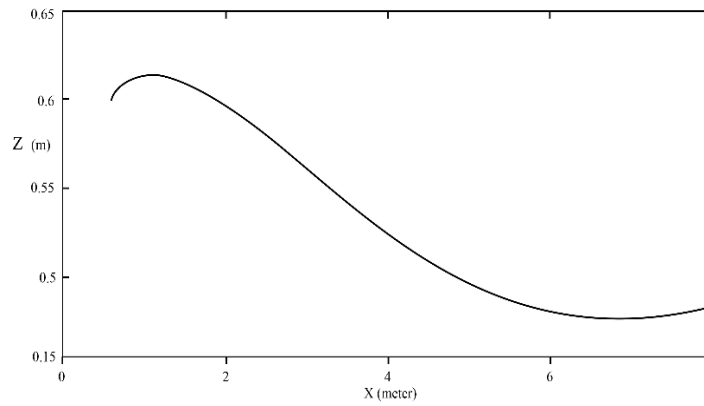
Figure 3. Turning trajectory.

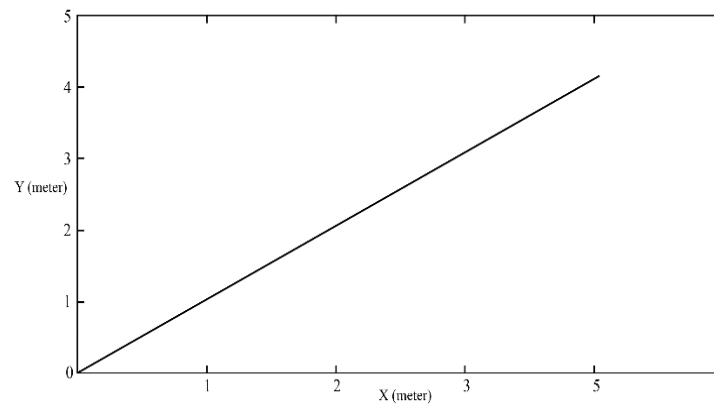
Figure 4. Diving trajectory.
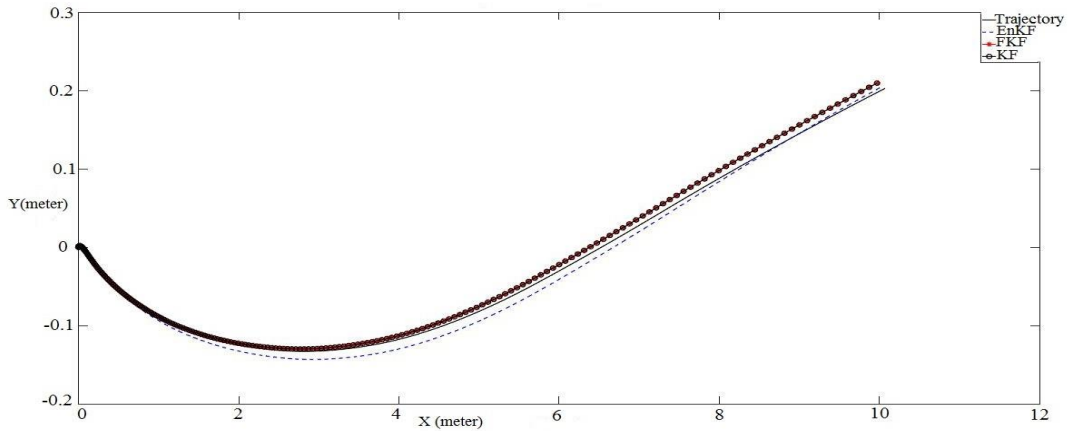

Figure 5. Straight trajectory.

Figure 6. Estimation of trajectory 1.

We can see the estimation result of the first trajectory in Figure 6. It shows the EnKF, FKF, and KF estimations are almost the same as the actual trajectory. We can see the accuracy comparison of each method from RMSE in Table 6. The EnKF has a more accurate estimation in the X position than estimation from FKF and KF. On the other hand, the FKF and KF give similar error/RMSE. It is shown from Figure 7 that the path estimation for both KF and FKF are almost the same. The error margin between KF and FKF is 3×10^{-3} for the X position, 8×10^{-6} for the Y position, and 2×10^{-6} for the angle. Their errors are almost the same because the algorithm of FKF is built from the Kalman Filter algorithm with fuzzy modification in the state space for FKF.

Table 6. The RMSE of the first trajectory estimation

Method	X Position (m)	Y Position (m)	Angle (rad)
EnKF	0.040570	0.008348	0.002545
FKF	0.044340	0.003524	0.000663
KF	0.044343	0.003516	0.000661

The error for the second trajectory is shown in Table 7. The best estimation for X position and angle is from the FKF and KF methods, but the best estimate for Z position is from the FKF method.
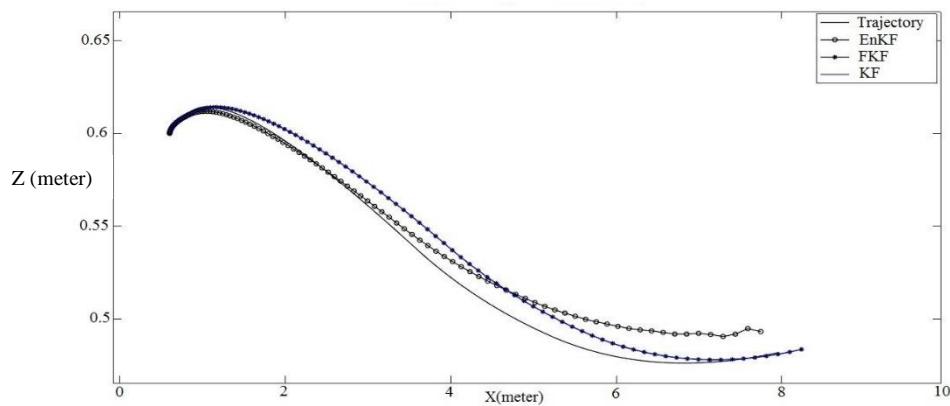

Figure 7. The estimation of trajectory 2

Table 7. The RMSE of the second trajectory estimation

Method	X Position (m)	Z Position (m)	Angle (rad)
EnKF	0.098755	0.009732	0.043939
FKF	0.032415	0.003101	0.002848
KF	0.032415	0.015067	0.002849

The third trajectory is a straight line. The best estimation for this trajectory is from the FKF and KF methods. We can see the difference error between KF and FKF is from the angle estimation. For the explanation that KF and FKF give the best estimate for the third trajectory because the straight trajectory given is a linear form, and the state space of KF and FKF is linear.
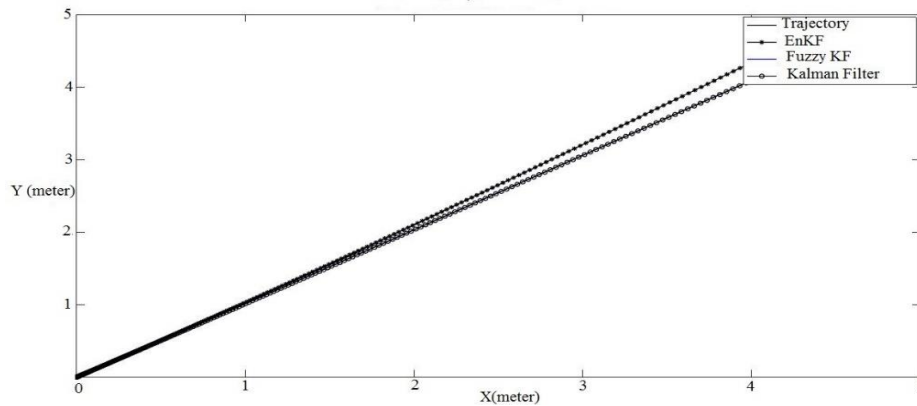

Figure 8. Estimation of trajectory 3.

Table 8. The RMSE of the third trajectory estimation.

Method	X Position (m)	Y Position (m)	Angle (rad)
EnKF	0.018777	0.070146	0.010562
FKF	0.001888	0.001736	0.00005
KF	0.001888	0.001736	0.000000

The research from Ngatini et al., 2017 explained the AUV estimation comparison between Ensemble Kalman Filter (EnKF) and Fuzzy Kalman Filter (FKF) with the dynamical system of AUV as the trajectory. That estimation result indicated that the Ensemble Kalman Filter has a better estimation than the FKF. The EnKF was reported to give the best estimation because the state space in EnKF was the dynamical system of the AUV equation without any linearization. In this research, we consider different trajectories built not from the dynamical system of AUV but form our path representing the diving, turning, and straight path. The final results show that the FKF and KF estimate the first trajectory best than the EnKF. The best estimation for the second trajectory is given by the FKF method. And the KF estimation provides the best estimation for the third trajectory. It means that different methods give the best estimate of every trajectory.

5. CONCLUSIONS

The Kalman Filter (KF), Fuzzy Kalman Filter (FKF), and Ensemble Kalman Filter (EnKF) give the estimation of the results of AUV under the specified trajectory. The estimated trajectories are the diving, straight, and turning paths which are actual trajectories. The estimation comparison is based on the simulation and the RMSE. The first trajectory estimation shows that the KF method gives the

best result for the Y position and Angle of AUV motion. The RMSE of KF for Y Position and Angle has the slightest error, 0.0035 m and 0.0006 rad, respectively.

Meanwhile, the smallest RMSE of X position is given by the EnKF method, i.e., 0.04 m. The estimation for the second trajectory shows FKF provides the smallest error for X position, Z position, and Angle, 0.032 m, 0.003 m, and 0.003 rad, respectively. The estimation for the last trajectory shows Kalman Filter gives the best estimate with an error of 0.002 m for X-position, 0.00174 m for Y-position, and less from 5×10^{-5} rad for Angle. Every trajectory has a different best method for estimation. Hence, in this case, the best estimate is given by other estimation methods. Fuzzy Kalman Filter gives the best estimation result for the first trajectory (Y-position and angle) and the second trajectory. Ensemble Kalman Filter gives the best estimation result for the X-position in the first trajectory. At the same time, Kalman Filter gives the best estimation result for the third trajectory. Future research can be developed using other data assimilation methods to make the estimation more accurate.

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REFERENCES

[1] J. M. Lewis, S. Laksmivarahan, and S. Dhall, *Dynamic Data Assimilation, A Least Square Approach*. New York: Press Taylor and Francis Group, 2006.

[2] E. Apriliani, D. K. Arif, and B. A. Sanjoyo, "The Square Root Ensemble Kalman Filter to Estimate the Concentration of Air Pollution," *IEEE, International Conference on Mathematical Application in Engineering* (ICMAE'10), Kuala Lumpur, Malaysia 2010.

[3] Ngatini and H. Nurhadi, "Estimasi Lintasan AUV 3 Dimensi (3D) Dengan Ensemble Kalman Filter," vol. 4, no. 1, 2019.

[4] Ngatini, E. Apriliani, and H. Nurhadi, "Ensemble and Fuzzy Kalman Filter for Position Estimation of an Autonomous Underwater Vehicle Based on Dynamical System of AUV Motion," *Expert Systems With Applications*, vol. 68, no. 7, pp. 29–35, 2017.

[5] J. Yuh, "Learning Control for Underwater Robotic Vehicles," *IEEE Control Systems Magazine*, vol. 14, no. 2, pp. 39–46, 1994.

[6] C. von Alt, *Autonomous Underwater Vehicles. The Autonomous Underwater Lagrangian Platforms and Sensors Workshop*. United States: Woods Hole Oceanographic Institution, 2003.

[7] B. Allota, R. Costanzi, F. Fanelli, N. Monni, L. Paolucci, and A. Ridolfi, "Sea Currents Estimating During AUV Navigation Using Unscented Kalman Filter," *IFAC (International Federation of Automatic Control)*, vol. 50, no. 1, pp. 13668–13673, 2017.

[8] C. Yang, "Modular Modeling and Control for Autonomous Underwater Vehicle (AUV)," Master's Thesis, National University of Singapore, 2007.

[9] T. I. Fossen, *Guidance and Control of Ocean Vehicles*. Chichester, England: John Wiley and Sons Ltd., 1994.

[10] M. Ataei and A. Y. Koma, "Three-Dimensional Optimal Path Planning for Waypoint Guidance of an Autonomous Underwater Vehicle," *Robotics and Autonomous Systems* Vol. 67, 2014.

[11] T. Herlambang, E. B. Djatmiko, and H. Nurhadi, "Ensemble Kalman Filter with a Square Root Scheme (EnKF-SR) for Trajectory Estimation of AUV SEGOROGENI ITS," *Journal of International Review of Mechanical Engineering*, vol. 9, no. 6, pp. 553–560, 2015.

[12] Z. Ermayanti, E. Apriliani, H. Nurhadi, and T. Herlambang, "Estimate and Control Position on The Autonomous Underwater Vehicle Based on Determined Trajectory using fuzzy Kalman filter method," *IEEE-International Conference on Advanced Mechatronics, Intelligent Manufacture, and Industrial Automation* (ICAMIMIA), Surabaya, Indonesia, 2015.

[13] F. L. Lewis, L. Xie, and D. Popa, *Optimal and Robust EstimationWith an Introduction to Stochastic Control Theory*, 2nd Edition. CRC Press, 2017.

[14] A. Hommels, A. Murakami, and S. Nishimura, "Comparison of the Ensemble Kalman Filter with the Unscented Kalman Filter: Application to the Construction of a Road Embankment," Proceeding of 19th European Young Geotechnical Engineers' Conference, Hungary, 2008.

[15] Subiono, *Sistem Linear dan Kontrol Optimal 2.1.1*. Surabaya, Indonesia: Department of Mathematics, Institut Teknologi Sepuluh Nopember, 2013.

[16] L. B. Burden and J. D. Faires, *Numerical analysis*, Ninth edition. Brooks/Cole Cengage Learning. Youngstown State University, United State of America, 2010.

[17] G. Evensen, "The Ensemble Kalman Filter: Theoretical Formulation and Practical Implementation," *Ocean Dynamics*, vol. 53, no. 4, pp. 343–367, 2003.

[18] G. Chen, Q. Xie, and L. S. Shieh, "Fuzzy Kalman Filtering," *Information Sciences*, vol. 109, pp. 197–209, 1998.

[19] T. Chai and R. R. Draxler, "Root mean square error (RMSE) or mean absolute error (MAE)? – Arguments against avoiding RMSE in the literature," *Geoscientific Model Development*, vol. 7, pp. 1247–1250, 2014.