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Modified Migrating Birds Optimization Algorithm: Multi-Depot Capacitated Vehicle Routing Problem

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Abstract

The multi-depot capacitated vehicle routing problem (MDCVRP) is a variation of the vehicle routing problem (VRP) modeled from distribution problems in the industrial world. This problem is a complex optimization problem in the field of operations research in applied mathematics. The MDCVRP is very interesting to discuss and find the best solution method. In this study, the authors apply the modified migrating birds' optimization (MMBO) algorithm, which is a hybrid of the migrating birds' optimization (MMBO) and iterated local search (ILS) algorithms. The purpose of this study is to analyze the results of applying the algorithm in solving MDCVRP. We used 20 MDCVRP data in simulation, grouped into four sizes (25, 50, 75, and 100 points). Based on the results of this research, it is known that the MMBO algorithm can produce the following solutions. First, on the data of 25 points, the experiment reaches the optimal value with small convergent iterations. Second, the best results on the data of 50 points have reached optimal value, but some other results have not been optimal. And, third, for data of 75 and 100 points, there is no optimal solution obtained by the MMBO algorithm. These results conclude that the MMBO algorithm effectively solves the MDCVRP problem with small data, but the bigger data, the more ineffective.
Keywords: MDCVRP; VRP; optimization; operation research; applied Mathematics; MMBO.

Abstrak

Multi-depot capacitated vehicle routing problem (MDCVRP) *adalah salah satu variasi dari* vehicle routing problem (VRP) *yang dimodelkan dari permasalahan distribusi di dunia industri. Permasalahan ini merupakan permasalahan optimasi kompleks dalam bidang riset operasi ilmu matematika terapan.* MDCVRP *sangat menarik untuk dibahas dan dicari metode penyelesaian terbaik. Dalam penelitian ini, penulis menerapkan algoritma* modified migrating birds optimization (MMBO) *yang merupakan hybrid* algoritma migrating birds optimization (MBO) *dan* iterated local search (ILS). *Tujuan penelitian ini adalah menganalisis hasil penerapan algoritma dalam menyelesaikan* MDCVRP. *Untuk simulasi, penulis menggunakan 20 data* MDCVRP *yang dikelompokkan menjadi empat ukuran (25, 50, 75, dan 100 titik). Berdasarkan hasil penelitian yang telah dilakukan, diketahui bahwa algoritma* MMBO *mampu menghasilkan solusi sebagai berikut. Pertama, Pada data 25 titik, percobaan mencapai nilai optimal dengan iterasi konvergen yang kecil. Kedua, Hasil terbaik pada data 50 titik telah mencapai nilai optimal namun sebagain hasil lainnya belum optimal. Dan ketiga, untuk data 75 dan 100 titik, tidak terdapat solusi optimal yang dihasilkan algoritma* MMBO. *Dari hasil tersebut dapat disimpulkan bahwa algoritma* MMBO *efektif untuk menyelesaikan* MDCVRP *data kecil, namun semakin besar datanya menjadi kurang efektif.*
Kata kunci: MDCVRP; VRP; *optimasi*; *riset operasi*; *matematika terapan*; MMBO.

1. INTRODUCTION

Distribution is one of the activities carried out by a company to distribute the products to consumers. Distribution process management will significantly affect the arrival of the product into

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the hands of customers. We, as well as possible, must distribute each product. Therefore, many companies are opening several distribution center points intended to reach more consumers and speed up the distribution process. However, in the distribution process, the company still considers the costs to be incurred. Distribution costs can be affected by the number of vehicles and fuel used. Vehicle trips with longer distances will require more fuel, whereas, on the contrary, short distances require less fuel. Thus, we can minimize fuel by finding the route with the shortest distance traveled. The problem of finding the shortest route in operations research in applied mathematics is commonly referred to as the vehicle routing problem (VRP).

The VRP, according to [1], was first introduced by Dantzig and Ramser in 1959. The VRP is one of the complex combinatorial optimization problems that is a combination of two problems, namely the traveling salesman problem (TSP) and the bin packing problem (BRP) [2]. VRP has been developed into many variations [3], one of which is the multi-depot vehicle routing problem (MDVRP) or multi-depot capacitated vehicle routing problem (MDCVRP). MDCVRP is adapted to the current situation of distribution problem where companies have more than one distribution center to serve their customers. In this MDCVRP, each vehicle that serves the customers departs and returns at several distribution centers. Each vehicle and distribution center has a maximum capacity. The products transported by each vehicle must not exceed that capacity. The purpose of the MDCVRP is to find a distribution route with a minimum distance and a minimum number of vehicles to minimize costs. Since the MDCVRP there are several distribution centers, it makes the problem more complex. Therefore, this problem is fascinating to discuss and find the best solution method.

Many studies have discussed solving the MDCVRP using various methods or algorithms, as follows: Ho et al. [4] proposed a hybrid genetic algorithm, Mirabi et al. [5] used hybrid heuristics algorithm, Yu et al. [6] tried to parallel improved ant colony optimization, Geetha dan Poonthalir [7] proposed nested particle swarm optimization, Juan et al. [8] applied a combination of biased randomized and iterated local search, and Oliveira et al. [9] offered cooperative coevolutionary algorithm. Based on these studies, the algorithms applied to the MDCVRP obtain fairly competitive solutions, which means that in some cases, those algorithms are practical and, in other cases, less effective. In addition, from all the studies mentioned, it is stated that the MDCVRP problem is crucial to continue to research and find the best solution method to obtain cost efficiency in distribution management.

Based on the description above, the authors are interested in further researching the MDCVRP by applying other optimization algorithms. The algorithm used in this study is the modified migrating birds' optimization (MMBO) algorithm introduced by Shen et al. [10]. The MMBO algorithm is obtained by combining the migrating birds' optimization (MBO) algorithm [11] with iterated local search (ILS) [12]. Based on the research results, the MMBO algorithm can find a much better solution than the original MBO algorithm for the university course-timetabling problem. The purpose of this study is to analyze the results of applying the algorithm in solving MDCVRP problems.

2. MULTI-DEPOT CAPACITATED VEHICLE ROUTING PROBLEM

The Multi-Depot Capacitated Vehicle Routing Problem (MDCVRP) is the same as the CVRP, except that it differs by the number of depots available. Due to more than one depot, the MDCVRP problem focuses on finding customer routes, scheduling them, and determining which depot customers should be served. In general, MDCVRP aims to find the optimum route to serve all customers from several depots. In this study, the objective function is the cost influenced by the number of vehicles operating, and the total distance traveled, modeled in Equation (1).

$$\text{Minimum } Z = BK \times \sum_{k\in K} m_k + BBM \times \sum_{i\in I\cup J}\sum_{j\in I\cup J}\sum_{k\in K} C_{ij}x_{ijk}, \tag{1}$$

with constraints:

$$\sum_{k\in K}\sum_{i\in I\cup J} x_{ijk} = 1 \;\; ; \;\; j\in J, \tag{2}$$
$$\sum_{j\in J} d_j \sum_{i\in I\cup J} x_{ijk} \le Q_k, \tag{3}$$
$$U_{lk} - U_{jk} + N x_{ljk} \le N-1 \;\; ; \;\; l,j\in J, \tag{4}$$
$$\sum_{j\in I\cup J} x_{ijk} - \sum_{j\in I\cup J} x_{jik} = 0 \;\; ; \;\; i\in I\cup J, \tag{5}$$
$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \le m_k, \tag{6}$$
$$-z_{ij} + \sum_{u\in I\cup J}\left(x_{iuk} + x_{ujk}\right) \le 1 \;\; ; \;\; i\in I; j\in J, \tag{7}$$
$$\sum_{j\in J} d_j z_{ij} \le V_i \;\; ; \;\; i\in I, \tag{8}$$
$$x_{ijk}\in\{0,1\} \;\; ; \;\; i,j\in I\cup J, \tag{9}$$
$$m_k\in\{0,1\}, \tag{10}$$
$$z_{ij}\in\{0,1\} \;\; ; \;\; i\in I; j\in J, \tag{11}$$
$$U_{lk}\ge 0 \;\; ; \;\; l\in J, \tag{12}$$

where I is the set of all depots, J is the set of all customers, K is the set of all vehicles, N is the number of customers, C_{ij} is the distance between points i to j, V_i is the maximum yield at depot i, d_j is the demand from customers j, Q_k is the capacity of k vehicles, BK is the vehicle cost and BBM is the fuel cost per kilometer. The decision variables: x_{ijk} is 1 if there is a trip from point i to point j by vehicle $k(i,j\in I\cup J)$ and 0 if there is no trip; m_k is 1 if k vehicles are operating and 0 if not operating; z_{ij} is 1 if the customer is allocated to the depot and 0 for the others; U_{lk} is the auxiliary variable for substructure elimination constraint in route k.

3. MODIFIED MIGRATING BIRD OPTIMIZATION

Algorithm Migrating Birds Modified Optimization (MMBO) MBO is a modification of the algorithm performed by Shen et al. [10], namely the MBO algorithm coupled with the algorithm Iterated Local Search (ILS). The purpose of this modification is to avoid trapped local optimum and increase the speed of convergence. There are four differences between the MBO and MMBO algorithms. First, an algorithm was Iterated Local Search (ILS) added to increase the power of the exploit (local search). Second, repeat tours are eliminated to reduce computation time. Third, the mechanism is sharing neighbors used for solutions that fail to update. And fourth, the alternation of the lead bird is changed from recursive to random between left or right alternation.

The steps of the MMBO algorithm begin with initializing the population size and the number of iterations. The next step is to generate a feasible initial solution (which qualifies the problem constraints). After that, we calculate the objective function for each solution, sort each solution in a V formation, and namely solution improvement. Repair steps are repeated until the specified number of iterations is met. This step consists of three steps: updating the leader's solution, updating the non-leaders solution, and changing the leader's solution.

4. METHOD

The data used in this study is a combination of actual data and random data for the MDCVRP. This data consists of depot points, customer points, the distance between points, depot capacity, customer demand, vehicle capacity, and fuel costs. The data for the depot points and customers were taken from 199 villages in Jember Regency, which were then randomly selected into 20 simulation data with four sizes (25, 50, 75, and 100 points of customers). The distances between the depots and the customers' points were searched using the help of Google Maps. Depot capacity and customer demand are generated randomly. The products used are assumed to be packed in the box with 38×24×24 cm dimensions. In this study, we use a small box truck type of vehicle with the dimensions of the storage media, 237×155×129 cm. By calculating the dimensions of the products and the vehicle, the maximum number of items that can be entered is 180 boxes. Vehicle costs are assumed to be calculated from employee fees and vehicle maintenance costs. This study uses a vehicle cost of Rp75,000,00. Furthermore, the fuel cost required per km distance is assumed to be Rp1,000,00. The data of this research can be accessed via email diniwasilah98@gmail.com.

In this study, we make a simulation program for the MMBO algorithm to be used as a tool for the MMBO algorithm in solving the MDCVRP. The source code of the program can be accessed via email diniwasilah98@gmail.com. The program is run ten times for each data of the MDCVRP in the simulation process. Then, the results obtained are used as material for analysis. We analyze the algorithm effectiveness using two methods: the total cost of distribution and the illustration of the route. In the first method, the algorithm can be effective if the solution obtained for the MDCVRP problem has a smaller distribution cost. In addition, to test the algorithm's convergence, the simulation results calculated the percentage deviation between the average total cost and the best total cost. The percentage deviation is calculated using Equation (13).

$$PD = \frac{C_A - C_{best}}{C_{best}} \times 100\%, \tag{13}$$

where PD is the percentage deviation, C_A is the average total cost, and C_{best} is the best total cost. The smaller the value of the percentage deviation obtained indicates that the algorithm results have a slight difference or indicate the solutions are convergent. Furthermore, in the second method, this study refers to a theory in [13]. The optimal route is indicated by the absence of intersecting route lines for each vehicle or by forming a Hamiltonian cycle.

5. RESULTS AND DISCUSSION

The implementation of MMBO algorithm is made primarily on some combination of natural and random instances. In this section, we divide it into two subsections. In the first subsection, we give some information about the design and the results of the computational experimental. Then, in the second subsection, we provide analysis results illustrating distribution routes based on the best solution obtained.

4.1. Experimental Design and Computational Results

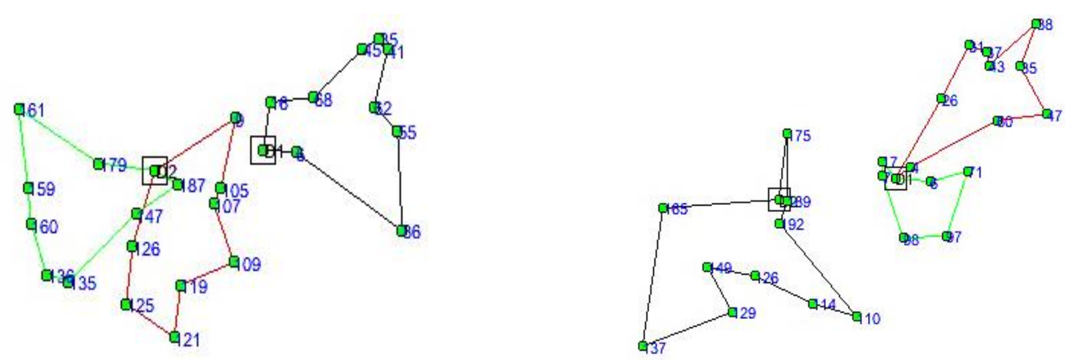
To explain the executive effect of the algorithm, the program of this article was compiled using Matlab 8.6. We run the program in the computer of Intel® Core™ i3-6006U CPU @ 2.00GHz RAM 4,00GB. The algorithm parameters values have been determined based on extensive numerical experiments. For the MDCVRP data with 25 and 50 points of customers, we use the population $N = 525$, the maximum iteration number of MMBO $Imax = 2000$, and the maximum iteration number of ILS $K = 25$. Meanwhile, for the MDCVRP data with 75 and 100 points of customers, we use the population $N = 1025$, the maximum iteration number of MMBO $Imax = 5000$, and the maximum iteration number of ILS $K = 25$. The computational results of the MMBO algorithm are shown in Table 1.

Table 1. The computational results

No	Data	Average Total Cost	Minimum Total Cost	Percentage Deviation (%)	Running Time
1	Data25a	474,400	474,400	0	4,762.0130
2	Data25b	480,500	480,500	0	4,855.3510
3	Data25c	484,300	484,300	0	4,909.4866
4	Data25d	496,400	496,400	0	4,993.6554
5	Data25e	511,300	511,300	0	4,929.4647
6	Data50a	908,530	903,700	0.5345	8,900.5890
7	Data50b	853,860	845,900	0.9410	8,907.0584
8	Data50c	790,380	784,200	0.7881	8,876.8263
9	Data50d	806,150	798,500	0.9580	8,513.7786
10	Data50e	751,970	745,500	0.8679	8,527.6312
11	Data75a	1,194,380	1,173,900	1.7446	59,499.1524
12	Data75b	1,186,110	1,166,300	1.6985	59,501.5998
13	Data75c	1,208,485	1,186,550	1.8486	59,806.4649
14	Data75d	1,149,150	1,128,000	1.8750	59,518.5389
15	Data75e	1,251,060	1,229,500	1.7536	54,482.2878
16	Data100a	1,587,490	1,555,100	2.0828	79,456.8229
17	Data100b	1,536,020	1,373,100	11.8651	85,344.3872
18	Data100c	1,548,770	1,513,300	2.3439	76,398.0325
19	Data100d	1,614,920	1,582,000	2.0809	70,735.7049
20	Data100e	1,526,910	1,490,900	2.4153	77,596.0364

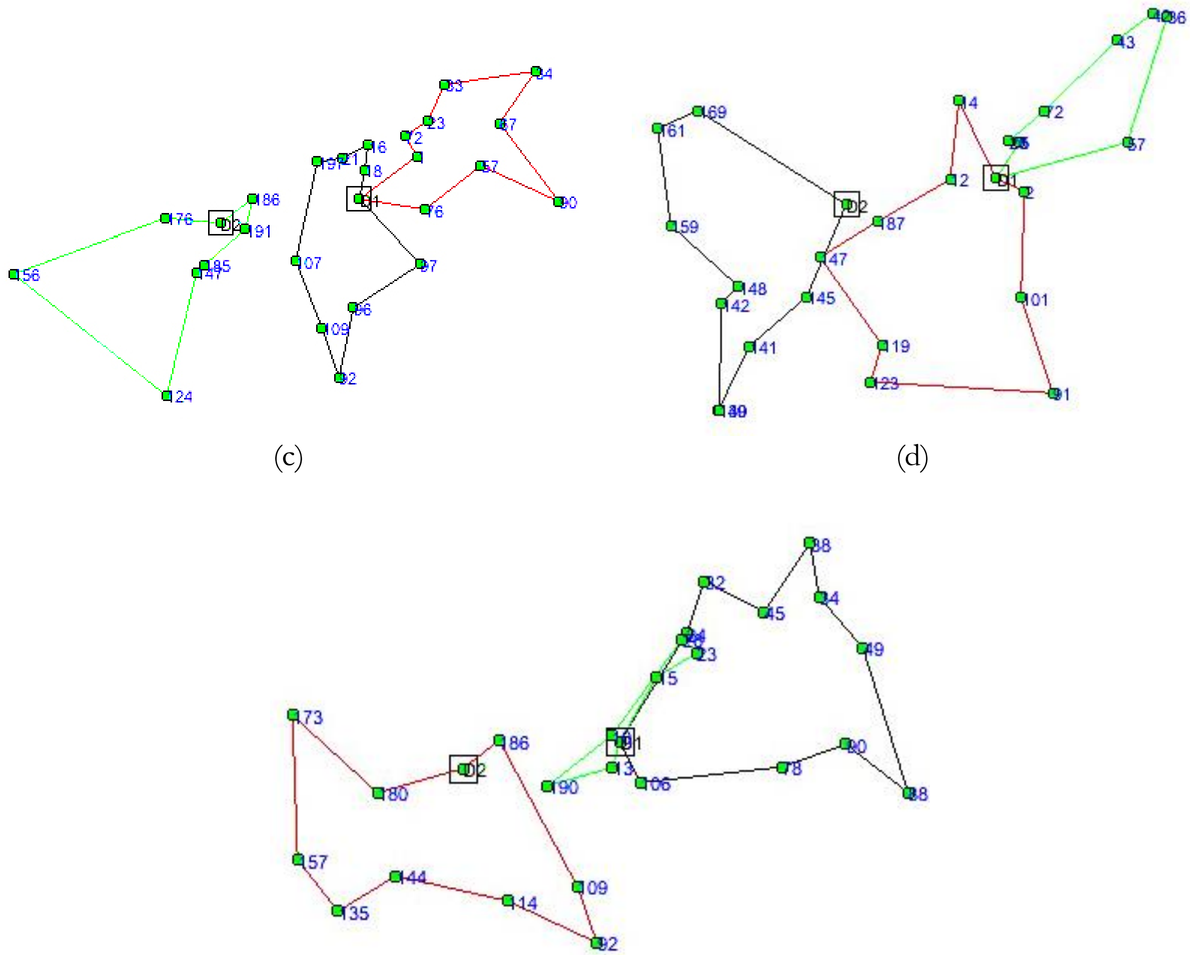
4.2. Analysis and Discussion

In Table 1, we can see that the values of percentage deviation for data25a – data25e are equal to 0%. The percentage deviation of 0% means that all trials in each data were solved with the same total cost. These results indicate that the solution is convergent. For further analysis, it is based on the illustration of the distribution route of each vehicle. We took the best solution from the ten trials to make the illustration. The route illustration is presented in Figure 1.



(a) (b)

Figure 1. Distribution routes illustration of 25 points of customers. (**a**) data25a; (**b**) data25b; (**c**) data25c; (**d**) data25d; (**e**) data25e



(c) (d)

(e)

Figure 1. Distribution routes illustration of 25 points of customers. (**a**) data25a; (**b**) data25b; (**c**) data25c; (**d**) data25d; (**e**) data25e (continued)

Based on the distribution route illustration in Figure 1, we can see that the five data with 25 points resulted in a solution with three vehicles routes. Each route obtained shows that its form is a Hamiltonian cycle or no intersecting route lines. Thus, from this second analysis method, it is known that the distribution route taken from the best solution of ten trials for each data is optimal. Since the two analysis methods show that the results indicate optimal solutions, the MMBO algorithm on 25 customer points is effective.

In Table 1, we can see that the values of percentage deviation for data50a – data50e are less than 1%. This percentage deviation means several different solutions from ten trials but with a relatively small difference in total cost. Since the percentage deviation does not reach 0%, the optimal solution of the ten experiments is not yet known. For further analysis, it is based on the illustration of the distribution route of each vehicle. We took the best solution from the ten trials to make the illustration. The route illustration is presented in Figure 2.

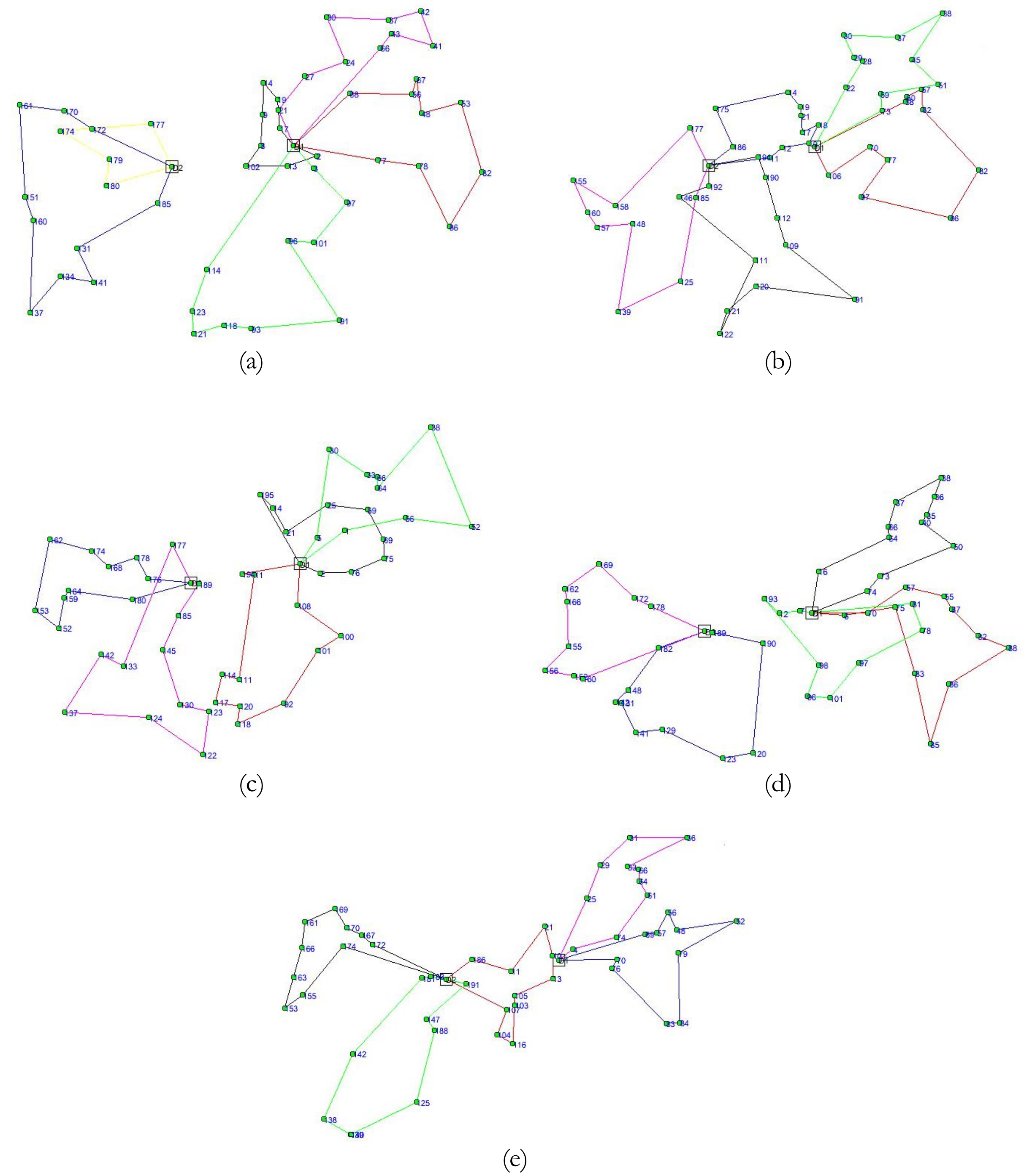


Figure 2. Distribution routes illustration of 50 points of customers. (**a**) data50a; (**b**) data50b; (**c**) data50c; (**d**) data50d; (**e**) data50e.

Based on the distribution route illustration in Figure 2, the distribution route that has been illustrated from the best results of data50a to data50e, there are no intersecting lines except for vehicle route 1 in data50b, which is in the sequence of points 120-121-122-111. Thus, for the data of 50 points, the MMBO algorithm is quite effective. However, after being analyzed by looking at the actual route, the path from point 122 to point 111 is not a straight line, and the path requires vehicles to pass through point 121. Therefore, in this case, the lines that intersect in that route can be ignored because the actual routes do not intersect but coincide so that the distribution route can still be said to be optimal. Based on the results of the analysis of the first and second methods, it is known that the best solution produced from ten trials has been optimal, but several other solutions are less than optimal.

In Table 1, we can see that the values percentage deviation for data75a – data75e is in the interval 1.6% – 1.9%, and the values percentage deviation for data100a – data100e are greater than 2%. This percentage deviation means that there are several different solutions from ten trials, so the optimal solution of the ten experiments is not yet known. For further analysis, it is based on the illustration of the distribution route of each vehicle. We took the best solution from the ten trials to make the illustration. The route illustration is presented in Figure 3 (75 points) and Figure 4 (100 points).

Based on the distribution route illustration in Figure 3 dan Figure 4, the distribution route that has been illustrated from the best results of data75a to data100e several vehicle routes have intersecting lines. These results show that the best solution from the data with 75 and 100 customer points does not represent the Hamiltonian cycle. Therefore, the best solutions found by the MMBO algorithm on the data of 75 and 100 customer points are not optimal. Thus, the MMBO algorithm is less effective for big-scale data.

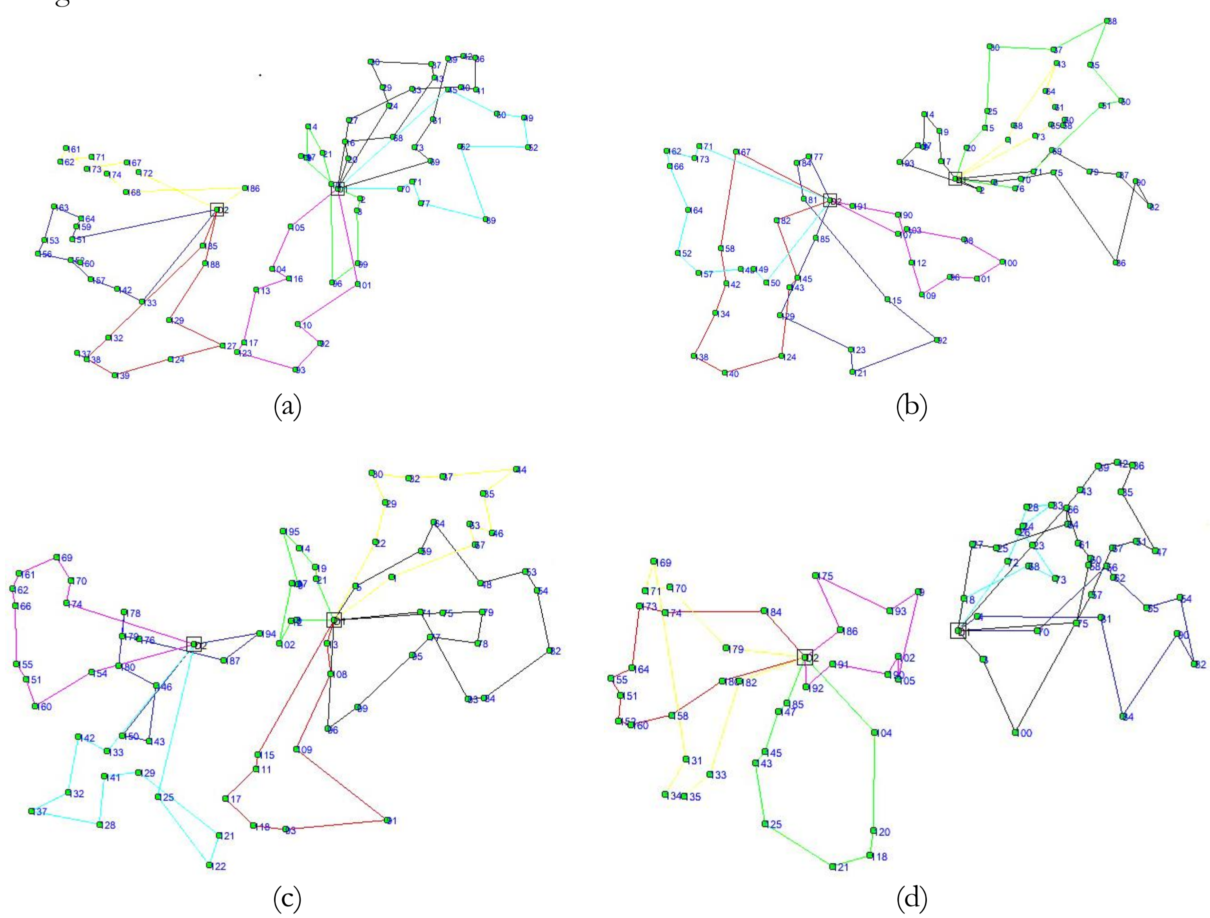
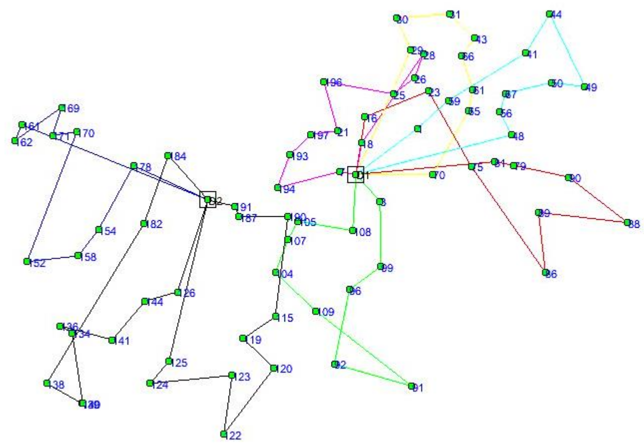
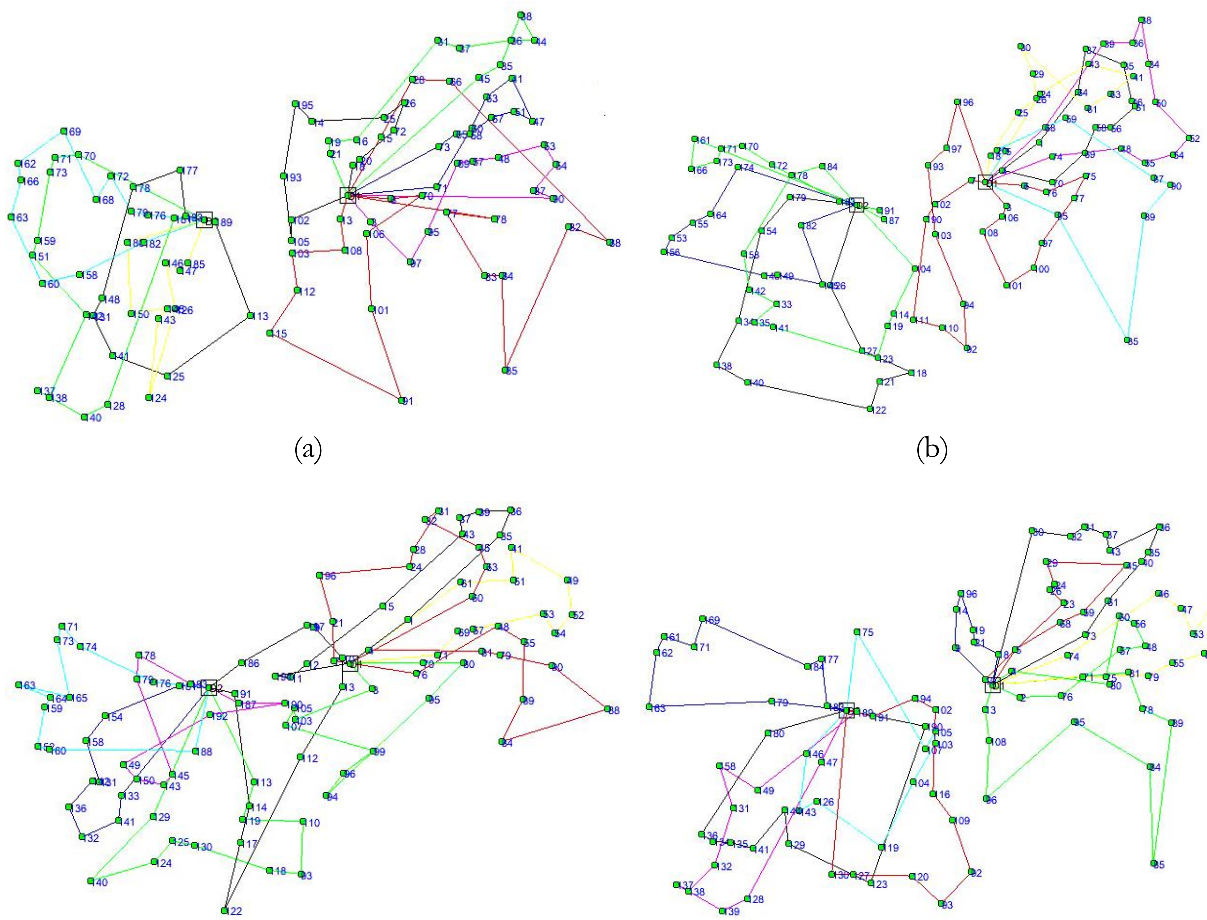


Figure 3. Distribution routes illustration of 75 points of customers. (**a**) data75a; (**b**) data75b; (**c**) data75c; (**d**) data75d; (**e**) data75e.



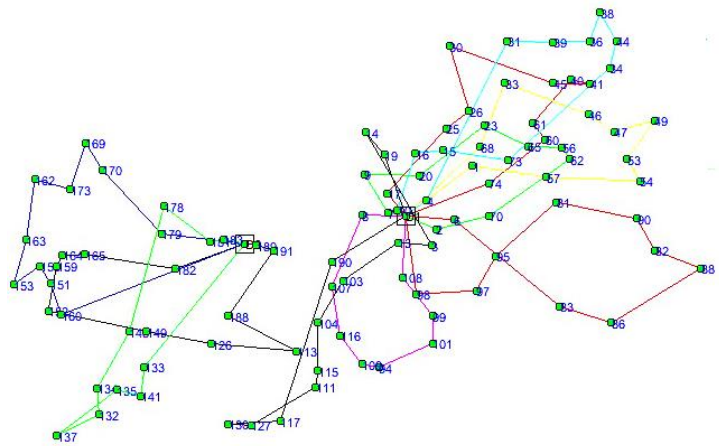
(e)

Figure 3. Distribution routes illustration of 75 points of customers. (**a**) data75a; (**b**) data75b; (**c**) data75c; (**d**) data75d; (**e**) data75e. (continued)



(a) (b) (c) (d)

Figure 4. Distribution routes illustration of 100 points of customers. (**a**) data100a; (**b**) data100b; (**c**) data100c; (**d**) data100d; (**e**) data100e.



(e)

Figure 4. Distribution routes illustration of 100 points of customers. (**a**) data100a; (**b**) data100b; (**c**) data100c; (**d**) data100d; (**e**) data100e (continued)

6. CONCLUSION

Based on the results and discussion described previously, we conclude that the MMBO algorithm is more effective than larger data. This effectiveness is because the MMBO algorithm produced optimal solutions from 10 program trials carried out in small data. Meanwhile, the larger the data, the MMBO algorithm had slower convergence, and it wasn't easy to find the optimal solution. Thus, we can see that the MMBO algorithm has almost the same performance as several other algorithms that have been used in previous studies. From the results of the analysis that has been carried out, we see that the weakness of the MMBO algorithm in solving the MDCVRP is the lack of exploration capabilities. The MMBO algorithm focuses on local search or exploitation capabilities, making it less effective for large data. Therefore, we suggest that further research can improve the MMBO algorithm by adding exploration capabilities or a heuristic algorithm result as an initial solution of the MMBO algorithm.

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