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Another Antimagic Decomposition of Generalized Peterzen Graph

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Abstract

A decomposition of a graph P into a family $\mathbb{Q}$ consisting of isomorphic copies of a graph Q is $(\mathfrak{a}, b)$-Q-antimagic if there is a bijection $\varphi: V(P) \cup E(P) \rightarrow \{1, 2, 3, 4 \dots, v_P + e_P\}$ such that for all subgraphs *Q'* isomorphic to *Q*, the *Q*-weights

$$\varphi(Q') = \sum_{v \in V(Q')} \varphi(v) + \sum_{e \in E(Q')} \varphi(e)$$

constitute an arithmetic progression $\mathfrak{a}, \mathfrak{a} + b, \mathfrak{a} + 2b, \dots, \mathfrak{a} + (r - 1)b$ where $\mathfrak{a}$ and b are positive integers and r is the number of subgraphs of *P* isomorphic to *Q*. In this article we prove the existence of a $(\mathfrak{a}, b)$-P_4-antimagic decomposition of a generalized Peterzen graph $GPz(n, 3)$ for several values of *b*.
Keywords: covering; decomposition; antimagic; generalized Peterzen.

Abstrak

Suatu dekomposisi dari suatu graf P *ke dalam suatu famili* $\mathbb{Q}$ *yang terdiri dari salinan isomorfik dari graf* Q *dikatakan* $(\mathfrak{a}, b)$-Q-*antiajaib jika terdapat pemetaaan bijektif* $\varphi: V(P) \cup E(P) \rightarrow \{1, 2, 3, 4 \dots, v_P + e_P\}$ *sedemikian sehingga semua subgraf Q' yang isomorfik ke Q, dengan bobot-Q sebagai berikut*

$$\varphi(Q') = \sum_{v \in V(Q')} \varphi(v) + \sum_{e \in E(Q')} \varphi(e)$$

yang membentuk suatu barisan aritmatika yaitu $\mathfrak{a}, \mathfrak{a} + b, \mathfrak{a} + 2b, \dots, \mathfrak{a} + (r - 1)b$ *dengan* $\mathfrak{a}$ *dan b adalah bilangan bulat positif dan r adalah banyaknya subgraf dari P yang isomorfik ke Q. Pada artikel ini, kami membuktikan eksistensi* $(\mathfrak{a}, b)$-P_4-*antiajaib dekomposisi dari graf* generalized Peterzen $GPz(n, 3)$ *untuk beberapa nilai b.*
Kata kunci: *selimut; dekomposisi; antiajaib;* generalized Peterzen.

1. INTRODUCTION

All graphs are finite and simple graphs. The edge and vertex sets of a graph P are denoted by $V(P)$ and $E(P)$, respectively, where $|V(P)| = v_P$ and $|E(P)| = e_P$. A *graph labeling* of a graph P is a bijective function that carries a set of elements of P onto a set of labels, usually, a set of positive integers. If the domain and co-domain of this function are $V(P) \cup E(P)$ and the set $\{1, 2, 3, 4, \cdots, v_P + e_P\}$, respectively, then it is called a *total labeling*.

An edge-covering of P is a family of subgraphs $\mathbb{Q} = \{Q_1, Q_2, Q_3, \dots, Q_k\}$ such that each edge of $E(P)$ belongs to at least one of the subgraphs Q_i, $1 \leq i \leq k$. Then it is said that P admits an $(Q_1, Q_2, Q_3, \dots, Q_k)$-*(edge) covering*. If every Q_i is isomorphic to a given graph Q, then P admits an *Q*-covering [1]. An $(Q_1, Q_2, Q_3, \dots, Q_k)$-*(edge) covering* of *P* is called an $(Q_1, Q_2, Q_3, \dots, Q_k)$-

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decomposition, if $E(Q_i) \cap E(Q_j) = \emptyset$ for $i \neq j$. If every Q_i is isomorphic to a given graph Q, then P admits an *Q-decomposition*.

Suppose P admits an Q-*decomposition* and let $f : V(P) \cup E(P) \rightarrow \{1, 2, 3, 4 \ldots, v_P + e_P\}$ be a total labeling. An Q-weight of a subgraph Q of P under a total labeling is the sum of all edge and vertex labels on Q. If every subgraph $Q \in \mathbb{Q}$ has the same Q-weights, then it is called an *Q-magic decomposition* of P. If all $Q \in \mathbb{Q}$ have distinct Q-weights, then it is called an *Q-antimagic decomposition* of P. In particular, if Q-weights of all $Q \in \mathbb{Q}$ are an arithmetic sequence with the first term a and a common difference b then it is called an (a, b)-H-*antimagic decomposition* of P.

Inayah et al. [2] [3] introduced an $(\mathfrak{a}, b)$-*Q*-antimagic total labeling of a graph *P* admitting an *Q*-decomposition, denoted as an $(\mathfrak{a}, b)$-*Q*-antimagic decomposition as a bijective function ϕ: $V(P) \cup E(P) \rightarrow \{1, 2, 3, 4 \ldots, |V(P)| + |E(P)|\}$ such that for a subgraph *Q'* isomorphic to *Q*, the *Q*-weights $\varphi(Q') = \sum_{v \in V(Q')} \varphi(v) + \sum_{e \in E(Q')} \varphi(e)$ constitute an arithmetic progression $\mathfrak{a}, \mathfrak{a} + b, \mathfrak{a} + 2b, \ldots, \mathfrak{a} + (r-1)b$ where $\mathfrak{a}$ and b are positive integers and r is the number of all subgraphs of *P* isomorphic to *Q*. The recent results on this subject can be seen, as an example, in [4] and [5]. The complete results can be seen in a dynamic survey of graph labelings by Gallian [6].

In this article, we proved $(\mathfrak{a}, b)$-P_4-antimagic decompositions of generalized Peterzen graphs $GPz(n, 3)$. We show that the graphs admit $(\mathfrak{a}, b)$-P_4-antimagic decompositions for several values of b.

2. MAIN RESULTS

In this section, we prove the existence of the $(\mathfrak{a}, b)$-P_4-antimagic decomposition of the generalized Peterzen graph $GPz(n, 3)$ for $b \in \{1,2,3,4,5\}$. Watkins [7] defined the generalized Peterzen graph $GPz(n, 3)$ as a graph having vertex set

$$V(GPz(n,3)) = \{v_i, u_i : 0 \le i \le n-1\}$$

and edge set

$$\text{Outer Rim } E_O((GPz(n,k)) = \{u_i u_{(i+1) \bmod n}\}_{i=0}^{n-1},$$

$$\text{Inner Rim } E_I((GPz(n,k)) = \{v_i v_{(i+k) \bmod n}\}_{i=0}^{n-1},$$

$$\text{Spoke } E_S(n,k) = \{u_i v_i\}_{i=0}^{n+1}.$$

Let $\mathbb{Q} = \{P_4^0, P_4^1, \ldots, P_4^{n-1}\}$, where the edge and vertex sets of the subgraph P_4^i defined as follows: For $i \in [0, n-1]$,

$$V(P_4^i) = \{v_i, v_{(i+3) mod\, n}, u_i, u_{(i+1) mod\, n} : 0 \le i \le n-1\},$$

$$E(P_4^i) = \{v_i v_{(i+3) mod\, n}, u_i v_i, u_{i+1} u_{(i+1) mod\, n} : 0 \le i \le n-1\}.$$

It is not difficult to see that $\mathbb{Q} = \{P_4^0, P_4^1, \ldots, P_4^{n-1}\}$ is a P_4-decomposition of $GPz(n, 3)$. Figure 1 displays the generalized Peterzen Graph $GPz(n, 3)$.

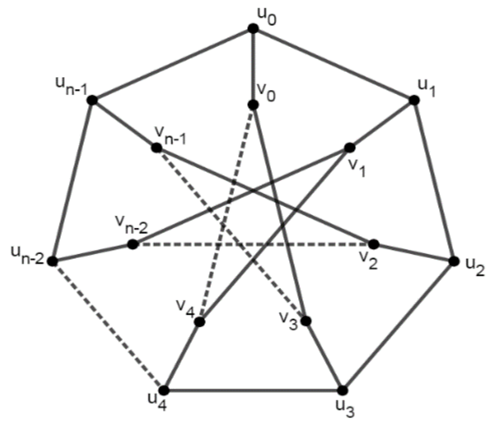

Figure 1. Generalized Peterzen Graph $GPz(n,3)$

Theorem 1. *For any integer* $n \geq 7$, *the graph* $GPz(n,3)$ *has a* $(20n+4,\ 1)$*-*P_4*-antimagic decomposition.*
Proof. Define a total labeling Ψ_q on the edges and vertices of the graph $GPz(n,3)$ in the following way

$$\begin{aligned}
\Psi_q\left(v_i v_{(i+3)\ mod\ n}\right) &= \begin{cases} i+5 & \text{for } i \in [0, n-5] \\ -n+i+5 & \text{for } i \in [n-4, n-1] \end{cases} \\
\Psi_q(v_i u_i) &= \begin{cases} 2n & \text{for } i = 0 \\ n+i & \text{for } i \in [1, n-1] \end{cases} \\
\Psi_q\left(u_i u_{(i+1)\ mod\ n}\right) &= \begin{cases} 2n+1 & \text{for } i = 0 \\ 3n-i+1 & \text{for } i \in [1, n-1] \end{cases} \\
\Psi_q(u_i) &= \begin{cases} 4n-i-3 & \text{for } i \in [0, n-5] \\ 5n-i-3 & \text{for } i \in [n-4, n-1] \end{cases} \\
\Psi_q(v_i) &= 4n+i+1 \qquad \text{for } i \in [0, n-1]
\end{aligned}$$

It can be seen that the labeling ψ_q is a bijective function from $E(GPz(n,3)) \cup V(GPz(n,3))$ to $\{1,2,3,4,\ldots,3n\}$ and $\psi_q\big(V(GPz(n,3))\big) = \{1,2,3,4\ \ldots, n+1\}$. Furthermore, the P_4-weight under the labeling ψ_q are as follows.

$$w(P_4^i) = \begin{cases}
\Psi_q\left(v_{(i+3)}\right) + \Psi_q\left(v_i v_{(i+3)}\right) + \Psi_q(v_i) + \Psi_q(v_i u_i) + \Psi_q(u_i) \\ \quad +\Psi_q\left(u_i u_{(i+1)}\right) + \Psi_q\left(u_{(i+1)}\right), \text{for } i \in [0, n-4] \\[2ex]
\Psi_q(v_0) + \Psi_q(v_i v_0) + \Psi_q(v_i) + \Psi_q(v_i u_i) + \Psi_q(u_i) \\ \quad +\Psi_q\left(u_i u_{(i+1)}\right) + \Psi_q\left(u_{(i+1)}\right), \text{for } i = [n-3] \\[2ex]
\Psi_q(v_1) + \Psi_q(v_i v_1) + \Psi_q(v_i) + \Psi_q(v_i u_i) + \Psi_q(u_i) \\ \quad +f_q\left(u_i u_{(i+1)}\right) + f_q\left(u_{(i+1)}\right), \text{for } i = [n-2] \\[2ex]
\Psi_q(v_2) + \Psi_q(v_i v_2) + \Psi_q(v_i) + \Psi_q(v_i u_i) + \Psi_q(u_i) \\ \quad +\Psi_q(u_i u_1) + \Psi_q(u_0), \text{for } i = [n-1]
\end{cases}$$

For $i \in [0, n-1]$, under labeling ψ_q, we find

$$
\begin{aligned}
w(P_4^i) &= w(P_4^i) = \Psi_q(v_{(i+3)}) + \Psi_q(v_i v_{(i+3)}) + \Psi_q(v_i) + \Psi_q(v_i u_i) + \Psi_q(u_i) + \Psi_q(u_i u_{(i+1)}) + \\
&\quad \Psi_q(u_{(i+1)}) \\
&= (4n + (i+3) + 1) + (i+5) + (4n + i + 1) + (2n) + (4n - i - 3) + (2n+1) + \\
&\quad (4n - (i+1) - 3) \\
&= 20n + i + 4
\end{aligned}
$$

Since $w(P_4^{i+1}) - w(P_4^i) = 1$ and $w(P_4^0) = 20n + 4$, the generalized Peterzen $GPz(n,3)$ admits a $(20n+4,1)$-P_4-antimagic decomposition.■

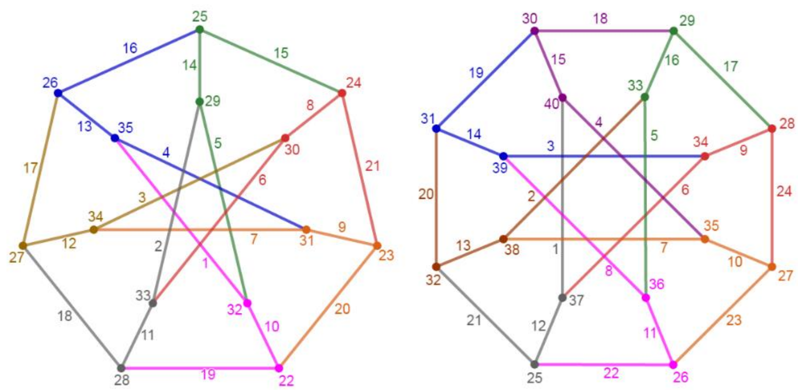

Figure 2. A $(144, 1) - P_4 -$Antimagic Decomposition of Generalized Peterzen Graph $GPz(7,3)$ (left), a $(164, 1) - P_4 -$ Antimagic Decomposition of Generalized Peterzen Graph $GPz(8,3)$ (right).

Theorem 2. *For any integer* $n \geq 7$, *the graph* $GPz(n,3)$ *has a* $(14n+4, 2)$-P_4-*antimagic decomposition.*
Proof. Define a total labeling Ψ_e on the edges and vertices of the graph $GPz(n,3)$ in the following way

$$
\begin{aligned}
\Psi_e(v_i v_{(i+3)\ mod\ n}) &= 4n + i + 1 && \text{for } i \in [0, n-1] \\
\Psi_e(v_i u_i) &= \begin{cases} 4n + i - 2 & \text{for } i \in [0,2] \\ 3n + i - 2 & \text{for } i \in [3, n-1] \end{cases} \\
\Psi_e(u_i u_{(i+1)\ mod\ n}) &= \begin{cases} 3n - 2i - 2 & \text{for } i \in [0, n-2] \\ 5n - 2i - 2 & \text{for } i = [n-1] \end{cases} \\
\Psi_e(u_i) &= n + 2i + 1 && \text{for } i \in [0, n-1] \\
\Psi_e(v_i) &= \begin{cases} -i + 3 & \text{for } i \in [0,2] \\ n - i + 3 & \text{for } i \in [3, n-1] \end{cases}
\end{aligned}
$$

It can be seen that the labeling ψ_e is a bijective function from $E(GPz(n,3)) \cup V(GPz(n,3))$ to $\{1, 2, 3, 4, \ldots, 3n\}$ and $\psi_e(V(GPz(n,3))) = \{1, 2, 3, 4, \ldots, n+1\}$. Furthermore, the P_4-weight under the labeling ψ_e are as follows

$$w(P_4^i) = \begin{cases} \Psi_e(v_{(i+3)}) + \Psi_e(v_i v_{(i+3)}) + \Psi_e(v_i) + \Psi_e(v_i u_i) + \Psi_e(u_i) \\ \quad +\Psi_e(u_i u_{(i+1)}) + \Psi_e(u_{(i+1)}), \text{for } i \in [0, n-4] \\ \Psi_e(v_0) + \Psi_e(v_i v_0) + \Psi_e(v_i) + \Psi_e(v_i u_i) + \Psi_e(u_i) \\ \quad +\Psi_e(u_i u_{(i+1)}) + \Psi_e(u_{(i+1)}), \text{for } i = [n-3] \\ \Psi_e(v_1) + \Psi_e(v_i v_1) + \Psi_e(v_i) + \Psi_e(v_i u_i) + \Psi_e(u_i) \\ \quad +\Psi_e(u_i u_{(i+1)}) + \Psi_e(u_{(i+1)}), \text{for } i = [n-2] \\ \Psi_e(v_2) + \Psi_e(v_i v_2) + \Psi_e(v_i) + \Psi_e(v_i u_i) + \Psi_e(u_i) \\ \quad +\Psi_e(u_i u_1) + \Psi_e(u_0), \text{for } i = [n-1] \end{cases}$$

For $i \in [0, n-1]$, under labeling ψ_e, we find

$$\begin{aligned} w(P_4^i) &= \Psi_e(v_{(i+3)}) + \Psi_e(v_i v_{(i+3)}) + \Psi_e(v_i) + \Psi_e(v_i u_i) + \Psi_e(u_i) + \Psi_e(u_i u_{(i+1)}) + \Psi_e(u_{(i+1)}) \\ &= (n-(i+3)+3) + (4n+i+1) + (-i+3) + (4n+i-2) + (n+2i+1) + (3n-2i- \\ &\quad 2) + (n+2(i+1)+1) \\ &= 14n + 2i + 4 \end{aligned}$$

Since $w(P_4^{i+1}) - w(P_4^i) = 2$ and $w(P_4^0) = 14n + 4$,, the generalized Peterzen $GPz(n, 3)$ admits a $(14n + 4, 2)$-P_4-antimagic decomposition.■

Theorem 3. *For any integer* $n \geq 7$, *the graph* $GPz(n, 3)$ *has a* $(19n + 5, 3)$-P_4*-antimagic decomposition.*
Proof. Define a total labeling Ψ_r on the edges and vertices of the graph $GPz(n, 3)$ in the following way

$$\begin{aligned} \Psi_r(v_i v_{(i+3) \bmod n}) &= i+1 && \text{for } i \in [0, n-1] \\ \Psi_r(v_i u_i) &= \begin{cases} 2n+i-1 & \text{for } i \in [0,1] \\ n+i-1 & \text{for } i \in [2, n-1] \end{cases} \\ \Psi_r(u_i u_{(i+1) \bmod n}) &= 2n+i+1 && \text{for } i \in [0, n-1] \\ \Psi_r(u_i) &= \begin{cases} 3n-i+3 & \text{for } i \in [0,2] \\ 4n-i+3 & \text{for } i \in [3, n-1] \end{cases} \\ \Psi_r(v_i) &= \begin{cases} 5n+i-2 & \text{for } i \in [0,2] \\ 4n+i-2 & \text{for } i \in [3, n-1] \end{cases} \end{aligned}$$

It can be seen that the labeling ψ_r is a bijective function from $E(GPz(n,3)) \cup V(GPz(n,3))$ to $\{1,2,3,4,\dots,3n\}$ and $\psi_r(V(GPz(n,3))) = \{1,2,3,4,\dots,n+1\}$. Furthermore, the P_4-weight under the labeling ψ_r are as follows

$$w(P_4^i) = \begin{cases} \Psi_r(v_{(i+3)}) + \Psi_r(v_i v_{(i+3)}) + \Psi_r(v_i) + \Psi_r(v_i u_i) + \Psi_r(u_i) \\ \quad + \Psi_r(u_i u_{(i+1)}) + \Psi_r(u_{(i+1)}), \text{for } i \in [0, n-4] \\ \Psi_r(v_0) + \Psi_r(v_i v_0) + \Psi_r(v_i) + \Psi_r(v_i u_i) + \Psi_r(u_i) \\ \quad + \Psi_r(u_i u_{(i+1)}) + \Psi_r(u_{(i+1)}), \text{for } i = [n-3] \\ \Psi_r(v_1) + \Psi_r(v_i v_1) + \Psi_r(v_i) + \Psi_r(v_i u_i) + \Psi_r(u_i) \\ \quad + \Psi_r(u_i u_{(i+1)}) + \Psi_r(u_{(i+1)}), \text{for } i = [n-2] \\ \Psi_r(v_2) + \Psi_r(v_i v_2) + \Psi_r(v_i) + \Psi_r(v_i u_i) + \Psi_r(u_i) \\ \quad + \Psi_r(u_i u_1) + \Psi_r(u_0), \text{for } i = [n-1] \end{cases}$$

For $i \in [0, n-1]$, under labeling ψ_r, we find

$$\begin{aligned} w(P_4^i) &= \Psi_r(v_{(i+3)}) + \Psi_r(v_i v_{(i+3)}) + \Psi_r(v_i) + \Psi_r(v_i u_i) + \Psi_r(u_i) + \Psi_r(u_i u_{(i+1)}) + \Psi_r(u_{(i+1)}) \\ &= (4n + (i+3) - 2) + (i+1) + (5n + i - 2) + (2n + i - 1) + (3n - i + 3) + (2n + \\ &\quad i + 1) + (3n - (i+1) + 3) \\ &= 19n + 3i + 5 \end{aligned}$$

Since $w(P_4^{i+1}) - w(P_4^i) = 3$ and $w(P_4^0) = 19n + 5$, the generalized Peterzen $GPz(n,3)$ $a$dmits a $(19n + 5{,}3)$-P_4-antimagic decomposition.■

Theorem 4. *For any integer* $n \geq 7$, *the graph* $GPz(n,3)$ *has a* $(13n + 5, 4)$-P_4*-antimagic decomposition.*
Proof. Define a total labeling Ψ_t on the edges and vertices of the graph $GPz(n,3)$ in the following way

$$\begin{aligned} \Psi_t(v_i v_{(i+3)\ mod\ n}) &= \begin{cases} 4n - i + 1 & \text{for } i = 0 \\ 5n - i + 1 & \text{for } i \in [1, n-1] \end{cases} \\ \Psi_t(v_i u_i) &= \begin{cases} 4n + i - 3 & \text{for } i \in [0,3] \\ 3n + i - 3 & \text{for } i \in [4, n-1] \end{cases} \\ \Psi_t(u_i u_{(i+1)\ mod\ n}) &= n + 2 + 2i \quad \text{for } i \in [0, n-1] \\ \Psi_t(u_i) &= \begin{cases} 3n + 2i - 1 & \text{for } i = 0 \\ n + 2i - 1 & \text{for } i \in [1, n-1] \end{cases} \\ \Psi_t(v_i) &= \begin{cases} 4 - i & \text{for } i \in [0,3] \\ n + 4 - i & \text{for } i \in [4, n-1] \end{cases} \end{aligned}$$

It can be seen that the labeling ψ_t is a bijective function from $E(GPz(n,3)) \cup V(GPz(n,3))$ to $\{1, 2, 3, 4, \ldots, 3n\}$ and $\psi_t(V(GPz(n,3))) = \{1, 2, 3, 4, \ldots, n+1\}$. Furthermore, the P_4-weight under the labeling ψ_t are as follows

$$w(P_4^i) = \begin{cases} \Psi_t(v_{(i+3)}) + \Psi_t(v_i v_{(i+3)}) + \Psi_t(v_i) + \Psi_t(v_i u_i) + \Psi_t(u_i) \\ \quad +\Psi_t(u_i u_{(i+1)}) + \Psi_t(u_{(i+1)}), \text{for } i \in [0, n-4] \\ \Psi_t(v_0) + \Psi_t(v_i v_0) + \Psi_t(v_i) + \Psi_t(v_i u_i) + \Psi_t(u_i) \\ \quad +\Psi_t(u_i u_{(i+1)}) + \Psi_t(u_{(i+1)}), \text{for } i = [n-3] \\ \Psi_t(v_1) + \Psi_t(v_i v_1) + \Psi_t(v_i) + \Psi_t(v_i u_i) + \Psi_t(u_i) \\ \quad +\Psi_t(u_i u_{(i+1)}) + \Psi_t(u_{(i+1)}), \text{for } i = [n-2] \\ \Psi_t(v_2) + \Psi_t(v_i v_2) + \Psi_t(v_i) + \Psi_t(v_i u_i) + \Psi_t(u_i) \\ \quad +\Psi_t(u_i u_1) + \Psi_t(u_0), \text{for } i = [n-1] \end{cases}$$

For $i \in [0, n-1]$, under labeling ψ_t, we find

$$\begin{aligned} w(P_4^i) &= \Psi_t(v_{(i+3)}) + \Psi_t(v_i v_{(i+3)}) + \Psi_t(v_i) + \Psi_t(v_i u_i) + \Psi_t(u_i) + \Psi_t(u_i u_{(i+1)}) + \Psi_t(u_{(i+1)}) \\ &= (-(i+3)+4) + (4n-i+1) + (-i+4) + (4n+i-3) + (3n+2i-1) + (n+ \\ &\quad 2i+2) + (n+2(i+1)-1) \\ &= 13n + 4i + 5 \end{aligned}$$

Since $w(P_4^{i+1}) - w(P_4^i) = 4$ and $w(P_4^0) = 13n + 4$, the generalized Peterzen $GPz(n,3)$ admits a $(13n+4, 4)$-P_4-antimagic decomposition.■

Theorem 5. *For any integer* $n \geq 7$, *the graph* $GPz(n,3)$ *has a* $(18n+6, 5)$-P_4*-antimagic decomposition.*
Proof. Define a total labeling Ψ_y on the edges and vertices of the graph $GPz(n,3)$ in the following way

$$\begin{aligned} \Psi_y(v_i v_{(i+3) \bmod n}) &= 2i+1 && \text{for } i \in [0, n-1] \\ \Psi_y(v_i u_i) &= 2i+2 && \text{for } i \in [0, n-1] \\ \Psi_y(u_i u_{(i+1) \bmod n}) &= \begin{cases} 3n+i-3 \\ 2n+i-3 \end{cases} && \begin{matrix} \text{for } i \in [0,3] \\ \text{for } i \in [4, n-1] \end{matrix} \\ \Psi_y(u_i) &= \begin{cases} 4n+i \\ 3n+i \end{cases} && \begin{matrix} \text{for } i = 0 \\ \text{for } i \in [1, n-1] \end{matrix} \\ \Psi_y(v_i) &= \begin{cases} 4n-i+4 \\ 5n-i+4 \end{cases} && \begin{matrix} \text{for } i \in [0,3] \\ \text{for } i \in [4, n-1] \end{matrix} \end{aligned}$$

It can be seen that the labeling ψ_y is a bijective function from $E(GPz(n,3)) \cup V(GPz(n,3))$ to $\{1,2,3,4,\dots,3n\}$ and $\psi_y(V(GPz(n,3))) = \{1,2,3,4,\dots,n+1\}$. Furthermore, the P_4-weight under the labeling ψ_y are as follows

$$w(P_4^i) = \begin{cases} \Psi_y(v_{(i+3)}) + \Psi_y(v_i v_{(i+3)}) + \Psi_y(v_i) + \Psi_y(v_i u_i) + \Psi_y(u_i) \\ \quad +\Psi_y(u_i u_{(i+1)}) + \Psi_y(u_{(i+1)}), \text{for } i \in [0, n-4] \\ \Psi_y(v_0) + \Psi_y(v_i v_0) + \Psi_y(v_i) + \Psi_y(v_i u_i) + \Psi_y(u_i) \\ \quad +\Psi_y(u_i u_{(i+1)}) + \Psi_y(u_{(i+1)}), \text{for } i = [n-3] \\ \Psi_y(v_1) + \Psi_y(v_i v_1) + \Psi_y(v_i) + \Psi_y(v_i u_i) + \Psi_y(u_i) \\ \quad +\Psi_y(u_i u_{(i+1)}) + \Psi_y(u_{(i+1)}), \text{for } i = [n-2] \\ \Psi_y(v_2) + \Psi_y(v_i v_2) + \Psi_y(v_i) + \Psi_y(v_i u_i) + \Psi_y(u_i) \\ \quad +\Psi_y(u_i u_1) + \Psi_y(u_0), \text{for } i = [n-1] \end{cases}$$

For $i \in [0, n-1]$, under labeling ψ_y, we find

$$\begin{aligned} w(P_4^i) &= \Psi_y(v_{(i+3)}) + \Psi_y(v_i v_{(i+3)}) + \Psi_y(v_i) + \Psi_y(v_i u_i) + \Psi_y(u_i) + \Psi_y(u_i u_{(i+1)}) + \\ &\quad f_y(u_{(i+1)}) \\ &= (4n - (i+3) + 4) + (2i+1) + (4n - i + 4) + (2i+2) + (4n+i) + (3n+i-3) + \\ &\quad (3n + (i+1)) \\ &= 18n + 5i + 6 \end{aligned}$$

Since $w(P_4^{i+1}) - w(P_4^i) = 5$ and $w(P_4^0) = 18n + 6$, the generalized Peterzen $GPz(n,3)$ admits a $(18n+6,5)$-P_4-antimagic decomposition.■

3. CONCLUSION

In this article, we proved the existence of $(\mathfrak{a}, b)$-P_4-antimagic decompositions of the generalized Peterzen graph $GPz(n,3)$ for (i) every integer $n \geq 7$ and odd positive integers $b \in \{1,3,5\}$; and (ii) every integer $n \geq 7$ and even positive integers $b \in \{2,4\}$.

The open problems related to these results are as follows:
For every integer $6 \geq n$ and positive integers b, find $(\mathfrak{a}, b)$-P_4-antimagic decompositions of the generalized Peterzen graph $GPz(n,3)$.

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