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Point and Figure Portfolio Optimization using Hidden Markov Models and Its Application on the Bumi Resources Tbk Shares

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Abstract

The problem of portfolio optimization is to select a trading strategy which maximizes the expected terminal wealth. Since the stocks are traded at discrete random times in a real-world market, we are interested in a time sampling method. The sampling of stock price is obtained from the process of time sampling which is used in a point and figure chart. Point and figure (PF) chart displays the up and down movements of unbalanced stock prices. The basic idea is to describe essential movements of the unbalanced stock prices using a hidden Markov model. The model parameters are transition probability matrices. They are estimated using maximum likelihood method and expectation maximization algorithm. The estimation procedure involves change of measure. The model is then applied to the stock price of Bumi Resources Tbk. collected on a daily basis. The estimated parameters are used to calculate the optimal portfolio using a recursive algorithm. The results show that the discrete hidden Markov model can be applied to describe essential movements of the stock price. The best result gives 93.63% accuracy of the estimate of observation sequence with mean absolute percentage error (MAPE) 3.63%. The numerical calculation shows that the optimal logarithmic PF-portfolio increases the wealth.
Keywords: point and figure portfolio; optimization portfolio; discrete hidden Markov model; expectation maximization algorithm; stock price of Bumi Resources Tbk.

Abstrak

Masalah pengoptimalan portofolio adalah pemilihan strategi perdagangan yang dapat memaksimalkan kekayaan terminal yang diharapkan. Karena di pasar dunia nyata, saham diperdagangkan pada waktu acak yang berbeda, sehingga kami tertarik pada metode pengambilan sampel waktu. Proses pengambilan sampel waktu diperoleh sampling harga saham yang digunakan dalam diagram point and figure *(PF-chart). Grafik point and figure hanya menampilkan pergerakan naik atau turun harga saham yang tidak seimbang. Ide dasarnya adalah untuk mendeskripsikan pergerakan esensial dari harga saham yang tidak seimbang menggunakan model* hidden Markov. *Parameter dari model ini adalah matriks probabilitas transisi. Parameter diestimasi menggunakan metode* maximum likelihood *dan algoritma expectation maximization. Prosedur estimasi melibatkan perubahan ukuran. Model ini kemudian diaplikasikan pada harga saham Bumi Resources Tbk. dari tanggal 2 Januari 2007 sampai dengan 31 Januari 2011. Hasil estimasi parameter tersebut digunakan untuk menghitung portofolio optimal menggunakan algoritma rekursif. Hasil penelitian ini menunjukkan bahwa model hidden Markov diskrit dapat diterapkan untuk menggambarkan pergerakan esensial dari harga saham. Model terbaik memberikan akurasi 93.63% dari estimasi deretan observasi dengan mean absolute percentage error (MAPE) 3,63% dan 5 faktor penyebab kejadian. Perhitungan numerik menunjukkan bahwa logaritma portofolio-PF yang optimal dapat meningkatkan kekayaan.*
Kata kunci: *portofolio* point and figure; *optimalisasi portofolio; model hidden Markov diskrit; algoritma* expectation maximization; *harga saham PT Bumi Resources.*

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1. INTRODUCTION

The investment decision basically concerns the issue of allocating assets for a certain period with the aim of maximizing the return with an acceptable level of risk. A collection of investments owned by individuals or institutions is called a portfolio [1]. In the investment, wealth is allocated by forming a portfolio. The portfolio consists of risk-free assets and risky assets (stocks). The problem of portfolio optimization is determining a trading strategy that maximizes returns at an acceptable risk level.

Shares are securities as a proof of individual or institutional ownership of a company [2]. Investors who allocate their assets in a stock trading must consider the level of return and risk when choosing stocks. The rate of return is in the form of dividends and profits if the selling price of the shares exceeds the purchasing price. Meanwhile, the risk of investment in stocks is caused by fluctuations in stock prices which result in an uncertain rate of return. In world markets, stocks are traded in a continuous time but in reality, investors make decisions to sell or buy a stock in a discrete time point. Therefore, it is necessary to process time discretization, i.e. sampling time which results in unbalanced stock prices (up or down). Each time sample corresponds to the current stock price sample. The sample share prices can be described in a diagram called a point and figure chart (PF-chart).

The PF-charts only show the x symbol indicating a rising (up) in stock prices and the o symbol indicating a falling (down) in stock prices. The criteria for the rise and fall of share prices depend on a fixed interval of stock prices. A portfolio based solely on the information contained in a PF-chart is called a PF-portfolio. Investors who follow PF-portfolios will trade their shares on a time sample and investors' decisions are only based on the price sample associated to that particular time sample.

Portfolio optimization problems are an example of stochastic process problems, i.e. problems related to the probability of an event, where future events cannot be predicted with certainty. Every incident has a cause and sometimes the cause of the incident is not directly observed. The causes of an event can form various mathematical models, one of of them is called the Markov chain model. The pair of events and causes that are not observed (hidden) and form a Markov chain is called the hidden Markov model.

The hidden Markov model are characterized by several parameters, i.e. the transition probability matrix from the cause of the event and also several parameters from the observation process. These parameters are estimated using the Maximum Likelihood method and the Expectation Maximization (EM) algorithm. The result of parameter estimation is in the form of recursive estimation. The parameters obtained are then re-evaluated using parameters or with new data.

The hidden Markov model has many mathematical structures and can be well modeled in many important applications. Applications that have been studied include the problem of asset allocation [3], bond pricing [4], option pricing [5], portfolio optimization [6] [7] [8], finance [9] [10], medicine [11], psychology [12] and speech recognition [13]. In this study, the PF portfolio optimization is studied using the hidden Markov model and applied to the Bumi Resources Tbk. shares.

2. METHOD

2.1. Point and Figure Chart

One of important charts used in technical analysis [2] [14] [15] is called a point and figure chart (PF-chart). This diagram only shows significant change in stock price. This is based on the fact that investors only buy and sell their stocks when the prices experience up and down movements. Dorsey [16] stated that the rise and fall in prices is depicted using symbol x and o, respectively in the PF-chart. In other words, every column only contains x or o symbols. The symbol changes in a column indicate that there is change in the direction of stock price movement.

The construction of PF-chart from a stock price movement is carried out as follows [6]:

1. Set a value of $\Delta > 0$,
2. Start the observation at time τ_0,
3. Write one of the symbol (x or o) at time τ_1 if the stock price lies beyond the interval $[S^1(\tau_0) - \Delta, S^1(\tau_0) + \Delta]$. If the stock price goes above the upper limit of $S^1(\tau_0) + \Delta$, then write x symbol in the PF-chart and if the stock price goes below the lower limit of $S^1(\tau_0) - \Delta$ then write o symbol,
4. Repeat the same procedure for the next interval, i.e. $[S^1(\tau_0) - \Delta, S^1(\tau_0) + \Delta]$,
5. A stochastic process $\{X_k: k \in \mathbb{N}\}$ defined in a probability space $(\Omega, \mathcal{F}, P)$ is a set of random variables that maps a space Ω to a state space S. Thus, for every $k \in \mathbb{N}$, X_k is a random variable. In this case, $k \in \mathbb{N}$ is considered as time and value from a random variable X_k, i.e. as a state from a process at time *k*.

This procedure is done recursively until a time sample observation $\{\tau_k: k \in \mathbb{N}\}$ is obtained that indicates a stopping time from the stock prices. Every time sample observation τ_k is associated with a stock price $S^1(\tau_k)$.

A portfolio based on the information included in the PF-chart is called a PF-portfolio. Investor that follows a PF-portfolio will buy and sell the stocks at time $\{\tau_k: k \in \mathbb{N}\}$. Every time τ_k investor's decision is only based on the observation of $S^1(\tau_0)$, therefore the optimization of PF-portfolio is a problem of choosing a discrete portfolio.

2.2. Point and Figure Portfolio

A portfolio $\Theta(.)$ is a pair of $(\Theta^0(.), \Theta^1(.))$ Measured and adapted process from $\{\mathcal{F}_t: t \geq 0\}$ with $\int_0^t |\Theta^i(s)|^2\, ds < \infty$ where $(\mathrm{i} = 0,1), \forall \mathrm{t} \geq 0$. In this case, $\Theta^i(t)$ shows the i-th total asset in time *t*. A portfolio $\Theta(.)$ is called a self-financed portfolio at time t if:

$$X^{\Theta}(t) = X^0(t) + \sum_{i=1}^{1} \int_0^t \Theta^i(u) dS^i(u)\,, \forall \mathrm{t} \geq 0 \tag{1}$$

A self-financed portfolio is a trading strategy when buying on a number of assets are funded only from the selling of portfolio assets. A self-financed portfolio is called a PF-portfolio if $\forall \mathrm{t} \geq 0$:

$$\Theta^i(t) = \Theta_0^i I_{[\tau_0,\tau_1]}(t) + \sum_{k=1}^{\infty} \Theta_k^i I_{[\tau_k,\tau_{k+1}]}(t)\,, \mathrm{i} = 1,2 \tag{2}$$

where $\{\Theta_k^0: k \in N\}$ and $\{\Theta_k^1: k \in N\}$ are adapted-Υ with:

$$\Upsilon = \{\sigma(S^1(\tau_j): j \leq k): k \in N\} = \{\sigma(Y_j: j \leq k): k \in N\} \tag{3}$$

$I_A(t)$ is an indicator function in a set A and $\sigma(Y_1, Y_2, .., Y_j)$ is a full σ-field generated by $Y_1, Y_2, .., Y_j$. Next, if $\Theta(.)$ is a self-financed, then the wealth $\mathrm{X}^{\Theta}(.)$ satisfies:

$$\mathrm{X}^{\Theta}(\tau_{k+1}) - \mathrm{X}^{\Theta}(\tau_k) = \sum_{i=0}^{1} \Theta_k^i \left(S^i(\tau_{k+1}) - S^i(\tau_k)\right), \forall \mathrm{k} \in \mathrm{N} \tag{4}$$

2.3. Hidden Markov Models

Let $X = \{X_k : k \in \mathbb{N}\}$ be a Markov chain defined in a probability space $\{\Omega, \mathcal{F}, P\}$ with finite and homogenous state and suppose it is assumed the states are not directly observed. A process $Y = \{Y_k : k \in \mathbb{N}\}$ is an observable process in a discrete range data. The process $\{X_k, Y_k\}$ is called a discrete hidden Markov model. The hidden Markov model discusses in this study is in the form [3]:

$$X_{k+1} = AX_k + V_{k+1}$$
$$Y_{k+1} = CX_k + W_{k+1}, k \in N \tag{4}$$

where:
1. $\{X_k : k \in N\}$ is a homogenous and unobservable Markov chain with state space $S_X = \{e_1, e_2, .., e_N\}$ where e_i is a unit vector in $\mathbb{R}^N$,
2. $\{Y_k : k \in N\}$ is an observable process in a discrete range data with state space $S_Y = \{f_1, f_2, .., f_M\}$ where f_j is a unit vector in $\mathbb{R}^M$,
3. $A = (a_{ji})_{N \times N}$ is a transition probability matrix with $a_{ji} = P(X_{k+1} = e_j | X_k = e_i)$ which satisfies $\sum_{j=1}^{N} a_{ji} = 1$ and $a_{ji} \geq 0$,
4. $C = (c_{ji})_{M \times N}$ is a transition probability matrix with $c_{ji} = P(Y_{k+1} = f_j | X_k = e_i)$ which satisfies $\sum_{j=1}^{M} c_{ji} = 1$ and $c_{ji} \geq 0$.

The parameters of discrete hidden Markov model are the transition probability matrices A and C as well as the expected value from the observable process. These parameters are estimated using Maximum Likelihood estimation and Expectation Maximization (EM) algorithm. The algorithm to estimate the parameters are:
1. Estimate the states:
$$\gamma_{k+1}(X_{k+1}) = q_{k+1} = \sum_{j=1}^{N} c_j(Y_{k+1}) \langle q_k, e_j \rangle a_j. \tag{5}$$
2. Estimate the number of jumps:
$$\gamma_{k+1,k+1}(\mathcal{J}_{k+1}^{rs}) = \sum_{j=1}^{N} c_j(Y_{k+1}) \langle \gamma_{k,k}(\mathcal{J}_{k+1}^{rs}), e_j \rangle a_j + c_r(Y_{k+1}) \langle q_k, e_r \rangle a_{sr} e_s. \tag{6}$$
3. Estimate the duration of the event:
$$\gamma_{k+1,k+1}(O_{k+1}^{r}) = \sum_{j=1}^{N} c_j(Y_{k+1}) \langle \gamma_{k,k}(O_{k+1}^{r}), e_j \rangle a_j + c_r(Y_{k+1}) \langle q_k, e_r \rangle a_r. \tag{7}$$
4. Estimate the observable process
$$\gamma_{k+1,k+1}(\mathcal{T}_{k+1}^{rs}) = \sum_{j=1}^{N} c_j(Y_{k+1}) \langle \gamma_{k,k}(\mathcal{T}_{k+1}^{rs}), e_j \rangle a_j + M \langle q_k, e_r \rangle \langle Y_{k+1}, f_s \rangle c_{sr} a_s, \tag{8}$$

where $c_j = Ce_j = (c_{1j}, c_{2j}, \dots, c_{Mj})^T$ is the j-th column of matrix $C = (c_{ji})$ and $a_j = Ae_j = (a_{1j}, a_{2j}, \dots, a_{Nj})^T$ is the j-th column of matrix $A = (a_{ji})$. The estimated model parameters are:

$$\hat{a}_{sr}(k+1) = \frac{\gamma_{k+1}(\mathcal{J}_{k+1}^{rs})}{\gamma_{k+1}(O_{k+1}^{rs})} \text{ and } \hat{c}_{sr}(k+1) = \frac{\gamma_{k+1}(\mathcal{T}_{k+1}^{rs})}{\gamma_{k+1}(O_{k+1}^{rs})}. \tag{9}$$

Next, the expected conditional value of Y_{k+1} given $\mathcal{Y}_k$ is known is:

$$\hat{Y}_{k+1} = E[Y_{k+1}|\mathcal{Y}_k] = \sum_{j=1}^{M}\sum_{i=1}^{N} c_{ji} q_k(e_i) f_i. \tag{10}$$

3. RESULT & DISCUSSION

The PF-chart construction is applied to the stock prices of Bumi Resources Tbk collected from January 4th, to March 31th 2010 for illustration. A total of 61 stock prices is observed from this timeframe with initial observation value $S^1(\tau_0) = 2454$. Figure 1 displays the trend of these stock prices collected on a daily basis.

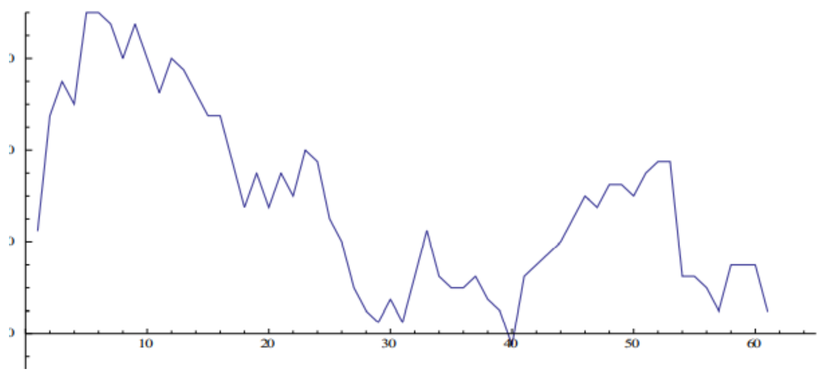

Figure 1. Stock prices of Bumi Resources from January 4th to March 31th 2010.

The following summarizes the steps in construction the PF-chart:

1. set $\Delta = 100$,
2. $S^1(\tau_0)$ is the initial value of the stock prices, i.e. $S^1(\tau_0) = 2454$,
3. for interval $[S^1(\tau_0) - \Delta, S^1(\tau_0) + \Delta] = [2325,2525]$ and according to Figure 1, the stock prices move up and exceed the interval. Thus, at time τ_1, the stock price is at the level of 2525 and $S^1(\tau_1) = 2525$. The first column in PF-chart is denoted by x symbol.
4. for the next interval $[S^1(\tau_1) - \Delta, S^1(\tau_1) + \Delta] = [2425,2625]$ and according to Figure 1, the stock prices still move up and exceed this interval. Thus, $S^1(\tau_2) = 2625$ and still on the first column of PF-chart is denoted by x symbol.
5. repeat the same process for interval $[S^1(\tau_k) - \Delta, S^1(\tau_k) + \Delta]$ for $k = 2,3, \dots ,18$ until we obtain a time sequence $\{\tau_k, k = 1,2, \dots ,19\}$ showing as a time sample.

The obtained PF-chart is shown in Figure 2. Based on the PF-chart construction, a small and insignificant stock price, i.e. a stock price within the interval $[S^1(\tau_k) - \Delta, S^1(\tau_k) + \Delta]$, can be eliminated from the PF-chart. Technical analysis indicates that PF-chart can be used as a filter to show only important information from the stock prices. This is in line with the market reality, although stocks are traded in a continuous time, investor only trade them at discrete time point, which is the time when the stock price rises or falls.

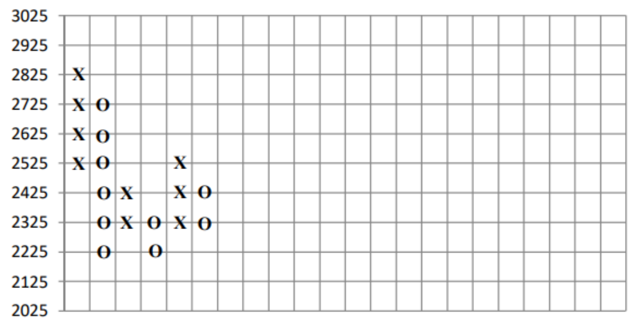

Figure 2. PF-chart of Bumi Resources Tbk. from January 4th to March 31th 2010.

Application of hidden Markov model to the stock prices of Bumi Resources Tbk. is by considering a daily stock prices (close-to-close) from January 2nd 2007 to January 31th 2011 with a total of 993 observations. Figure 3 displays these observed stock prices.

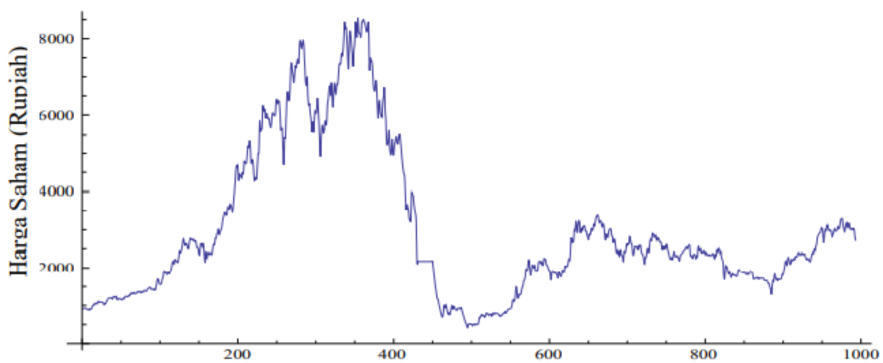

Figure 3. Stock prices of Bumi Resources Tbk. from January 2nd 2007 to January 31th 2011.

Using the basic concept of PF-chart, a time discretization scheme for stock trading results in unbalanced stock prices. The initial observation of stock price is $S_0^1 = 910$ and we obtained a time sampling and its associated stock price sample equal to 455 observations. The result of time sampling process yields a sequence of observations $\{Y_1, Y_2, \dots, Y_{455}\}$ that takes values $\{d, u\}$ where d and u represents the up and down in stock prices, respectively. To explain the behavior of these movement (up and down) in stock prices of Bumi Resources Tbk., a hidden Markov model is developed to obtain the best estimate of the sequence $\{\hat{Y}_1, \hat{Y}_2, \dots, \hat{Y}_{455}\}$ so that $Y_k = \hat{Y}_k$ is maximum.

It is assumed that $\{Y_1, Y_2, \dots, Y_{455}\}$ is generated by an observable process which is only influenced by the process of caused events forms a homogeneous Markov chain that cannot be observed directly. The number of caused events (N) is the input that is determined and selected $N = 2,3, \dots, 10$. Based on the algorithm (5)-(10), a computational program of algebra mathematics using Mathematica 8.0 software is developed. Table 1 presents the computational results. The notation $u \rightarrow d$ in the table indicates that $Y_k = u$ and $\hat{Y}_k = d$. Based on Table 1, we can plot the estimation accuracy as shown in Figure 4. This indicates that the best hidden Markov model that can explained the behavior of the sequence of the observation process is when the caused events is N=5.

Table 1. The computational results for the estimation of observation sequence.

N	The number of accuracy estimation				Percentage of accuracy estimation			MAPE (%)
	$u \to u$	$u \to d$	$d \to u$	$d \to d$	u	d	Sequence of observation	
2	241	4	59	151	98	72	86.15	7.36
3	225	20	55	155	92	74	83.52	10.44
4	234	11	35	175	96	83	89.89	6.26
5	241	4	25	185	98	88	93.63	3.63
6	229	16	41	169	93	80	87.47	8.02
7	199	46	17	193	81	92	86.15	11.98
8	217	28	50	160	89	76	82.86	11.65
9	220	25	55	155	90	74	82.42	11.54
10	227	18	71	139	93	66	80.44	11.76

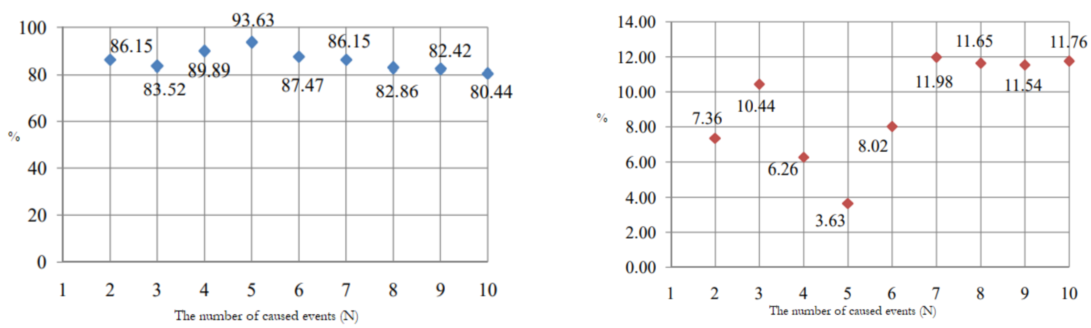

Figure 4. Plot of estimation accuracy based on the observation sequence (left) and MAPE (right).

From this computational result where N=5, the estimation accuracy for the observation sequence is 93.63% and MAPE 3.63%. The transition probability matrix is:

$$A = \begin{pmatrix} 1.03 \times 10^{-57} & 1.72 \times 10^{-57} & 9.99 \times 10^{-60} & 7.25 \times 10^{-58} & 1.33 \times 10^{-57} \\ 3.51 \times 10^{-48} & 5.31 \times 10^{-49} & 8.09 \times 10^{-47} & 2.67 \times 10^{-46} & 8.56 \times 10^{-47} \\ 1 & 1 & 1 & 1 & 1 \\ 2.23 \times 10^{-13} & 1.76 \times 10^{-13} & 5.65 \times 10^{-13} & 1.59 \times 10^{-12} & 2.88 \times 10^{-13} \\ 9.25 \times 10^{-58} & 4.11 \times 10^{-58} & 1.01 \times 10^{-59} & 6.19 \times 10^{-57} & 5.19 \times 10^{-57} \end{pmatrix},$$

$$C = \begin{pmatrix} 0.1855 & 0.4678 & 0.9687 & 0.9609 & 0.0988 \\ 0.8145 & 0.5321 & 0.0312 & 0.0390 & 0.9011 \end{pmatrix}.$$

In addition, we also obtained the expected value from the Markov chain, i.e.:

$$E[X] = \boldsymbol{\pi} = (6.25 \times 10^{-8}, 6.25 \times 10^{-8}, 6.25 \times 10^{-8}, 6.25 \times 10^{-8})^{\boldsymbol{T}}.$$

The wealth process from the optimal portfolio with utility logarithm function of Bumi Resources Tbk. shares for the period of January 2nd, 2007 to January 31th, 2011 is shown in Table 2 and Figure 5.

Table 2. The wealth process of PF-portfolio for Bumi Respurces Tbk. Shares.

k	τ_k	S_k^1	$\gamma_{k-1}(1)$	π_k^*	$\frac{\tilde{S}_k^1 - \tilde{S}_{k-1}^1}{\tilde{S}_{k-1}^1}$	$\tilde{X}_{k+1}^{x,v*}$
1	12.59	950.95	1.00000	6.09031	0.04500	1.27406
2	14.09	993.74	0.90164	5.89674	0.04500	1.51332
3	16.46	1038.46	0.76088	4.97345	0.04500	1.68361
4	20.17	1085.19	0.62357	4.81281	0.04500	1.81866
:	:	:	:	:	:	:
454	991.80	2905.61	4.33927×10^{-10}	5.97214	-0.04500	2.83840
455	992.67	2774.86	4.55628×10^{-10}	6.31857	-0.04500	2.83840

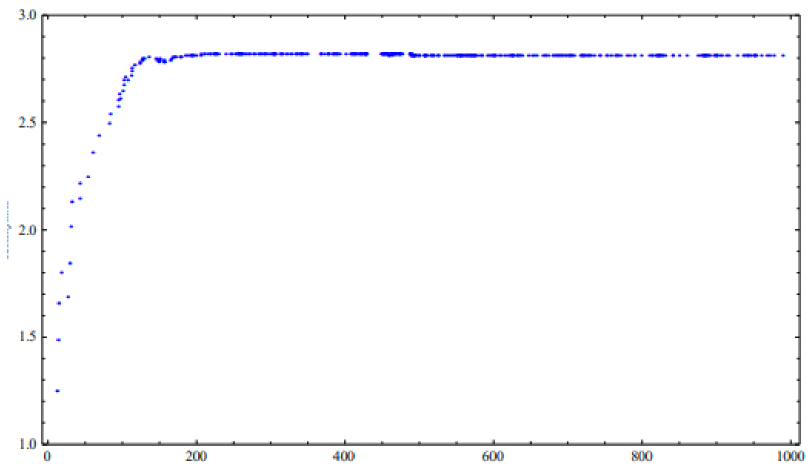

Figure 5. Plot of wealth process of PF-portfolio for Bumi Respurces Tbk. shares for the period of January 2nd, 2007 to January 31th, 2011.

4. CONCLUSIONS

The discrete hidden Markov model can be well used to explain the behavior in a sequence of stock price movements of Bumi Resources Tbk. The accuracy of the predicted sequence of observations depends on the initial value of the model parameters. The results obtained for the number of causes events N = 5 are sufficient to explain these behaviors, because adding more numbers of events does not show any significant effect. It is shown that the best fitted model yields a very high accuracy of 93.63% and with the mean absolute percentage error (MAPE) of 3.63%. In addition, the estimation of model parameters in the hidden Markov model combined with the martingale method for PF-portfolio can optimize the wealth in random time of stock trading.

REFERENCES

[1] Z. Bodie, A. Kane and A. J. Marcus, Investments, 5th ed., Boston : Mc Graw Hill, 2005.

[2] L. Salim, Analisis Teknikal dalam Perdagangan Saham, Jakarta: PT Elex Media Komputindo Kelompok Gramedia, 2003.

[3] R. J. Elliot and J. Van-der Hoek, "An application of hidden markov model to asset allocation problems," *Journal of Finance and Stochastics,* vol. 1, pp. 229-238, 1997.

[4] C. Landen, "Bond pricing in a hidden markov model of the short rate," *Journal Finance and Stochastics,* vol. 4, pp. 371-389, 2000.

[5] S. D. Campbell, Regime Switching in Economics, University of Pennsylvania: Dissertation, 2002.

[6] R. J. Elliot and J. Hinz, "Portfolio optimization, hidden markov models, and technical analysis of P&F-charts," *International Journal of Theoretical and Applied Finance,* vol. 5, pp. 385-399, 2011.

[7] H. Y. and X. Wang, "Portfolio selection with a hidden markov model," *Quality Technology and Quantitative Managmenet,* vol. 11, pp. 167-174, 2014.

[8] E. Canakoglu and S. Ozekici, "Portfolio selection with imperfect information: a hidden markov model," *Applied Stochastic Models in Business and Industry,* vol. 27, pp. 95-114, 2011.

[9] N. Nguyen, "Hidden markov model for stock trading," *International Journal of Financial Studies,* vol. 6, no. 2, pp. 1-17, 2018.

[10] R. Sasikumar and A. S. Abdullah, "Forecasting the stock market values using hidden markov model," *International Journal of Business Analytics and Intelligence,* vol. 4, no. 1, pp. 17-21, 2016.

[11] R. Langrock, T. A. Marques, R. W. Baird and L. Thomas, "Modeling the diving behavior of whales: a latent-variable approach with feedback and semi-markovian components," *Journal of Agriculture Biological and Environmental Statistics,* vol. 19, no. 1, pp. 82-100, 2013.

[12] I. Visser, M. E. J. Raijmakers and P. C. M. Molenaar, "Fitting hidden markov models to psychological data," *Scientific Programming,* vol. 10, pp. 185-199, 2002.

[13] L. R. Rabiner, "A tutorial on hidden markov models and selected applications on speech recognition," *Proceedings of the IEEE,* vol. 77, no. 2, 1989.

[14] I. Ramlall, Applied Technical Analysis for Advanced Learners and Practitioners: In Chart We Trust, UK: Emerald, 2017.

[15] Nasdaq, "Point and Figure Basics," [Online]. Available: https://www.nasdaq.com/docs/DWA-Point-Figure-Basics_0.pdf. [Accessed 2 February 2020].

[16] T. J. Dorsey, Point and Figure Charting, 3th ed., New York: John Willey & Sons, 2007.