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An Odd-Even Sum Labeling of Jellyfish and Mushroom Graphs

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Abstract

A graph $G(V,E)$ with p vertices and q edges called graph odd-even sum if there exists an injective function f from V to $\{\pm1,\pm2,\pm3,\dots,\pm(2p-1)\}$ such that induced a bijection $f*(uv)=f(u)+f(v)$ as label of edge and $u,v\in V$ forms the set $\{2,4,\dots,2q\}$, and f is called odd-even sum labeling. There are three criteria of graphs that can be labeled by this labeling, they are undirected, no loops, and finite for every edges and vertex. Jellyfish $J_{m,n}$ graph and Mushroom Mr_m graph have the criteria. So in this paper will be showed that the Jellyfish and Mushroom graphs can be labeled by this labeling.
Keywords: odd-even sum graph; odd-even sum labeling; Jellyfish and mushroom graphs.

Abstrak

Graf $G(V,E)$ dengan banyak titik p dan sisi q dikatakan graf jumlah ganjil-genap jika terdapat suatu fungsi injetif f dari V ke $\{\pm1,\pm2,\pm3,\dots,\pm(2p-1)\}$ sehingga bijektif $f*(uv)=f(u)+f(v)$ merupakan label sisi dengan $u,v\in V$ membentuk himpunan bilangan $\{2,4,\dots,2q\}$, dengan f merupakan pelabelan jumlah ganjil-genap. Kriteria graf yang dapat dilabeli oleh pelabelan jumlah ganjil-genap ada tiga, yaitu graf yang tidak berarah, tidak memiliki loop, dan terhingga, baik secara sisi maupun titik. Graf Jellyfish $J_{m,n}$ dan Mushroom Mr_m memenuhi ketiga kriteria tersebut. Pada tulisan ini akan ditunjukkan bahwa kedua graf tersebut dapat dilabeli dengan pelabelan jumlah ganjil-genap.
Kata kunci: graf jumlah ganjil-genap; pelabelan jumlah ganjil-genap; graf Jellyfish dan graf Mushroom.

1. INTRODUCTION

Let $G=(V,E)$ be simple and finite graph with p vertices and q edges. Grap G is said to be an odd-even sum graph if there exists an injective function $f:V\to\{\pm1,\pm2,\pm3,\dots,\pm(2p-1)\}$ such that induced a bijection $f^*:E\to\{2,4,6,\dots,2q\}$ with $f^*(uv)=f(u)+f(v),\forall uv\in E$. The function f is called an odd-even sum labeling of G [1].

Harary [2] defined a sum graph. Ponraj et al. [3] proved that a graph is pair sum labeling. Ramya et al. [4] proved that a graph is skolem even-vertex-odd difference mean labeling. Arockiaraj et al. [5] introduced the odd-sum labeling of sum subdivision graphs. Monika and Murugan [1] proved that a path $P_n, n\geq 2$, a star $K_{1,n}$, and caterpillar graph are odd-even sum labeling. In this paper, we proved that a Jellyfish and Mushroom graphs are odd-even sum graphs.

2. DEFINITION

In this section we define Jellyfish graph and Mushroom graphs. A Jellyfish $J_{m,n}$ graph for $m,n \geq 1$ is a graph with a set of vertex $V = \{u,x,v,y,x_i,y_j | \text{ for } i = 1,2,3,\ldots,m,\ j = 1,2,3,\ldots,n\}$, a set of edges $E = \{ux,uv,uy,vx,vy\} \cup \{xx_i | i = 1,2,\ldots,m\} \cup \{yy_j | j = 1,2,\ldots,n\}$ [6]. Jellyfish $J_{m,n}$ graph has no loop or double edges, and undirected. An example of Jellyfish $J_{m,n}$ graph is shown in Figure 1.

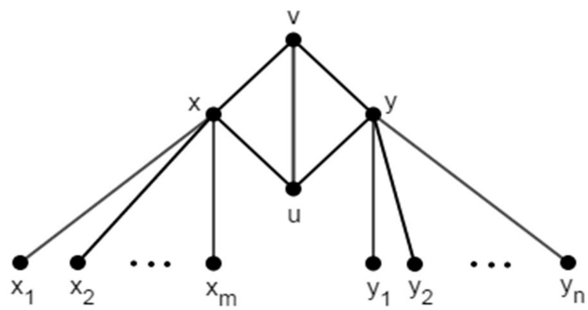

Figure 1. Jellyfish $\boldsymbol{J_{m,n}}$ graph.

Mushroom Mr_m graph for $m \geq 2$ is graph with a set of vertex $V = \{v_i, w, u_j | i = 1,2,3,\ldots,m, j = 1,2,3,\ldots,n\}$ and with a set of edges $E = \{wv_i | i = 1,2,\ldots,m\} \cup \{wu_j | j = 1,2,\ldots,m\} \cup \{v_i v_{i+1} | i = 1,2,\ldots,m-1\}$. This graph has no loop or double edges, and so undirected. An example of Mushroom Mr_m graph is shown in Figure 2.

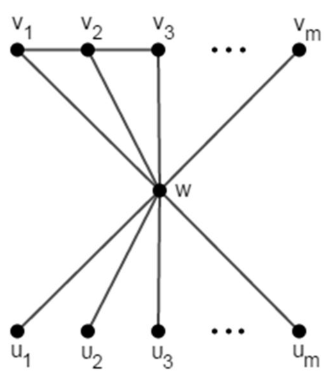

Figure 2. Mushroom $\boldsymbol{Mr_m}$ graph.

In this paper will be discussed regarding odd-even sum labeling for Jellyfish graph $J_{m,n}$ for $m,n \geq 1$ but $m = n$, and Mushroom graph Mr_m for $m \geq 2$.

3. RESULTS

In this section, we show that Jellyfish $J_{m,n}$ graph has an odd-even sum labeling. Two important results in this paper will be shown as follows.

Theorem 3.1. Jellyfish $J_{m,n}$ graph for $m, n \geq 1$ is an odd-even sum graph.

Proof. Let $V(J_{m,n}) = \{u, x, v, y, x_i, y_j | i = 1, 2, 3, \dots, m;\ j = 1, 2, 3, \dots, m\}$ and $E(J_{m,n}) = \{ux, uv, uy, vx, vy\} \cup \{xx_i | i = 1,2,3, \dots, m\} \cup \{yy_j | j = 1,2, \dots, n\}$. Then $|V(Jm, n)| = 2m + 4$ and $|E(Jm, n)| = 2m + 5$. Let $f : V(Jm, n) \to \{\pm 1, \pm 3, \dots, \pm 2(2m + 4) - 1\}$ be defined as follows:

$f(v) = -(2m + 1),$
$f(u) = 2m + 3,$
$f(x) = 2m + 5,$
$f(y) = 2m + 7,$
$f(x_i) = 2i + 1;$ $\quad i = 1, 2, 3, \dots, m;$
$f(y_j) = -(2j - 1);$ $\quad j = 1, 2, 3, \dots, n.$

Let f^* be the induced edge labeling of f, then

$f^*(vu) = 2;$
$f^*(vx) = 4;$
$f^*(vy) = 6;$
$f^*(ux) = 2(2m + 4);$
$f^*(uy) = 2(2m + 5);$
$f^*(xx_i) = 2m + 2i + 6;$ $\quad i = 1, 2, 3, \dots, m;$
$f^*(yy_j) = 2m - 2j + 8;$ $\quad j = 1, 2, 3, \dots, n.$

The induce edge labels are $2, 4, 6, \dots, 2(2m + 5)$ which are all distinct. Hence Jellyfish $J_{m,n}$ graph for m, $n \geq 1$ is an odd-even sum graph. ■

Theorem 3.2. Mushroom Mr_m graph for $m \geq 2$ is an odd-even sum graph.

Proof. Let $V(Mr_m) = \{v_i, w, u_j | i = 1,2,3, \dots, m,\ j = 1,2,3, \dots, m\}$ and $(Mr_m) = \{ wv_i \mid i = 1, 2, 3, \dots, m\} \cup \{wu_j \mid j = 1, 2, 3, \dots, m\} \cup \{v_i v_{i+1} \mid i = 1, 2, 3, \dots, (m-1)\}$. Then $|V(Mrm)| = 2m + 1$ and $|E(Mrm)| = 3m - 1$. Let $f : V(Mrm) \to \{\pm 1, \pm 3, \dots, \pm(2(2m + 1) - 1)\}$.

Case (i) m is odd,

$f(v_{2i-1}) = m + 2i - 2;\ \ i = 1, 2, 3, \dots, \frac{m+1}{2},$
$f(v_{2i}) = 3m + 2i;$ $\quad i = 1, 2, 3, \dots, \frac{m-1}{2}$, for $m \geq 3,$
$f(w) = -1,$
$f(u_{2j-1}) = 2m + 2j - 1;$ $\quad j = 1, 2, 3, \dots, \frac{m+1}{2},$
$f(u_{2j}) = 2j + 1;$ $\quad j = 1, 2, 3, \dots, \frac{m-3}{2}$, for $m \geq 5,$
$f(u_{m-1}) = 4m + 1.$

Let f^* be the induced edge labeling of f, then

$f^*(wv_{2i-1}) = m + 2i - 3;$ $\quad i = 1, 2, 3, \dots, \frac{m+1}{2},$
$f^*(wv_{2i}) = 3m + 2i - 1;$ $\quad i = 1, 2, 3, \dots, \frac{m-1}{2}$, for $m \geq 3,$
$f^*(wu_{2j-1}) = 2m + 2j - 2;$ $\quad j = 1, 2, 3, \dots, \frac{m+1}{2},$
$f^*(wu_{2j}) = 2j;$ $\quad j = 1, 2, 3, \dots, \frac{m-3}{2}$, for $m \geq 5,$
$f^*(wu_m) = 4m,$

$f^*(v_i v_{i+1}) \ = 4m + 2i;\ i = 1, 2, 3, \ldots, (m-1).$

Case (ii) m is even,

$f(v_{2i-1}) = m + 2i - 1;\ i = 1, 2, 3, \ldots, \frac{m}{2},$

$f(v_{2i}) \ = 3m + 2i - 1; i = 1, 2, 3, \ldots, \frac{m}{2},$

$f(w) \ = -1,$

$f(u_{2j-1}) = 2m + 2j - 1; \quad j = 1, 2, 3, \ldots, \frac{m}{2},$

$f(u_{2j}) \ = 2j + 1; \quad j = 1, 2, 3, \ldots, \frac{m-2}{2},$ for $m \geq 4,$

$f(u_m) \ = 4m + 1.$

Let f^* be the induced edge labeling of f, then

$f^*(wv_{2i-1}) = m + 2i - 2; \quad i = 1, 2, 3, \ldots, \frac{m}{2},$

$f^*(wv_{2i}) \ = 3m + 2i - 2; \quad i = 1, 2, 3, \ldots, \frac{m}{2},$

$f^*(wu_{2j-1}) = 2m + 2j - 2; \quad j = 1, 2, 3, \ldots, \frac{m}{2},$

$f^*(wu_{2j}) \ = 2j; \quad j = 1, 2, 3, \ldots, \frac{m-2}{2},$ for $m \geq 4,$

$f^*(wu_m) \ = 4m,$

$f^*(v_i v_{i+1}) \ = 4m + 2i; i = 1, 2, 3, \ldots, (m-1).$

The induce edge labels are $2, 4, 6, \ldots, 2(3m-1)$ which are all distinct. Hence Mushroom Mr_m graph for $m \geq 2$ is an odd-even sum graph. ■

4. CONCLUSION

Based on the results of subsection before, can be obtained two theorems that the authors call Theorem 3.1, states that Jellyfish $J_{m,n}$ graph for $m, n \geq 1$ is an odd-even sum graph and Theorem 3.2 states that Mushroom Mr_m graph for $m \geq 2$ is an odd-even sum graph.

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