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Traffic Model Based Predictive Control: A Piecewise-Affine using METANET

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Abstract

Traffic congestion on the freeway is a serious problem for modern society. Dynamic traffic management is a good alternative solution to improve efficiency on congestion problems. This article aims to analyze parts of freeway traffic network by using METANET model which is part of macroscopic traffic flow model that describes a set of parameters such as mean speed, traffic flow, and density of a traffic system. The piecewise-affine (PWA) approximation on METANET model is used to design traffic predictive controls and test them on a traffic model structure. This approach guarantees more intensive calculation for METANET traffic flow model in nonlinear form in the context of model predictive control (MPC). Some equations in the METANET model will be approximated by PWA function. With PWA-MPC approximation as direct calculation, equation of PWA model can be transformed into mixed-integer linear programming (MILP). Furthermore, to see the control of the model with MPC control, numerical simulations will be carried out on mean speed, traffic density, traffic flow, queue length, and MPC control. We use time 0 – 2.5 hours. Simulation result shows that the density of traffic, traffic flow, and queue length decreased in this time period, while the mean speed increased.
Keywords: traffic control; model predictive control; piecewise-affine model; METANET; mixed-integer linear programming (MILP).

Abstrak

Kemacetan lalu lintas di jalan bebas hambatan merupakan masalah yang sangat serius bagi masyarakat modern. Pengelolaan lalu lintas yang dinamis merupakan solusi alternatif yang baik untuk meningkatkan efisiensi pada masalah kemacetan. Artikel ini bertujuan untuk menganalisis bagian jaringan pada jalan bebas hambatan dengan mengkaji model METANET yang termasuk bagian dari model arus lalu lintas secara makroskopik yang menggambarkan kumpulan parameter seperti kecepatan rata-rata, arus lalu lintas, dan kepadatan. Pendekatan *piecewise-affine* (PWA) pada model METANET digunakan untuk mendesain kendali prediktif lalu lintas dan mengujinya pada suatu struktur model lalu lintas. Pendekatan ini menjamin penghitungan yang lebih intensif untuk model arus lalu lintas METANET yang berbentuk nonlinear dalam konteks kendali model prediktif (*model predictive control*/*MPC*). Beberapa persamaan pada model METANET akan didekati oleh fungsi PWA. Dengan pendekatan PWA-MPC sebagai perhitungan secara langsung, persamaan model PWA dapat diubah menjadi program linear bilangan bulat campuran (*mixed- integer linear programming*/MILP). Selanjutnya untuk melihat keterkendalian model dengan kendali MPC, simulasi numerik akan dilakukan terhadap kecepatan rata-rata, kepadatan lalu lintas, arus lalu lintas, panjang antrian, serta

kendali MPC. Waktu yang digunakan pada simulasi adalah 0 – 2.5 jam. Hasil simulasi menunjukkan bahwa kepadatan lalu lintas, arus lalu lintas, panjang antrian mengalami penurunan dalam kurun waktu tersebut, sedangkan kecepatan rata-rata mengalami peningkatan.
Kata Kunci: endali lalu lintas; model lalu lintas berbasis kendali prediktif; pendekatan model *piecewise-affine;* METANET; program linear bilangan bulat campuran.

1. INTRODUCTION

Model-based predictive control for traffic network is used for the needs to track traffic conditions and take into account optimization approaches such as speed limits that produce optimal control. In this article, models for traffic flow is selected macroscopically resulting a well accurate description to provide traffic demand, traffic conditions, and output in one side of traffic network [1][2][3]. A METANET-traffic model is used in this study to model the freeway traffic network [4][5].

Model predictive control (MPC) is a control model satisfying the above criteria so that this type of control is suitable to be applied in the traffic control problems, one of them is the METANET model. The problem of designing MPC control is an optimization control method applied on the horizon structure. This control is used on a model that is processed to obtained control signal by minimizing the objective function [6].

The previous MPC control is applied in the traffic control model such as Hegyi [7] producing nonconvex nonlinear optimization problems (METANET). Generally, the MPC nonlinear optimization problems are difficult to solve quickly in order to achieve optimal values. In the METANET model problem, a PWA approach is selected from the nonlinear function, thus it is possible to formulate MPC optimization problems as a mixed-integer linear programming (MILP) problem [8][9]. The PWA piecewise-affine formulation on the METANET model is used in the MPC network that has nontrivial solution, however, the formulation can yield best solution as compared to the initial nonlinear model [8][9][10].

2. METHOD

MPC control is a control technique that uses a model to predict output along the horizon of future prediction, then applies the first element of the series of optimal control inputs that minimize the cost function. The basic idea of MPC is illustrated in Figure 1 [11][12]. There are two cases in MPC i.e. MPC without constraints and MPC with constraints. One model that are suitable for traffic control problem in MPC is METANET.

In traffic control problem, METANET model represents traffic network as direct graph with link denoted by index m that correspond to a segment. Segments are freeway that stretches and links are collection from several segments on freeway. In our research, every link is divided into N_m segments denoted by index i (Figure 2) where L_m represents the length of segment between 500-1000m. Based on [14], line is part of the road used for vehicle traffic which is physically in the form of road works.

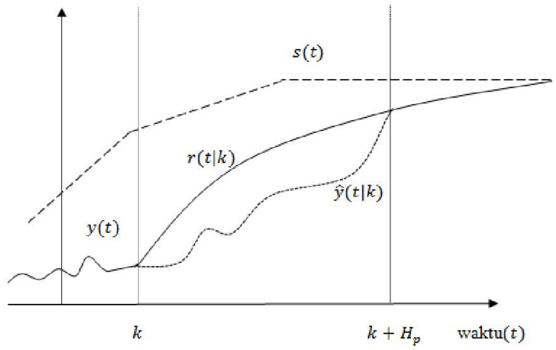

Figure 1. Model Predictive Control.

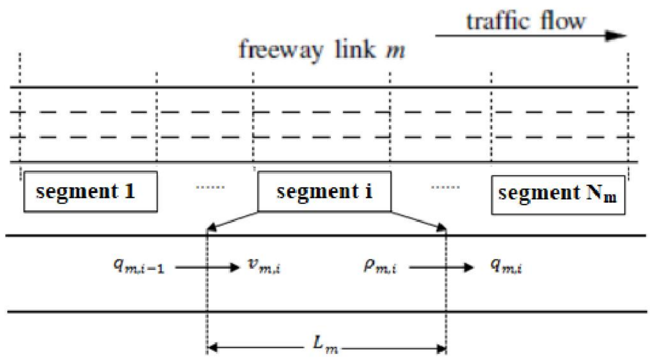

Figure 2. Freeway link divided into several segments in METANET model.

There are some related variables for traffic condition in every segment i of link i.e. m is traffic flow is denoted by $q_{m,i}(k)$ (vehicle/hour), traffic density is denoted by $\rho_{m,i}(k)$ (vehicle/km/lane), and mean speed is denoted by $v_{m,i}(k)$ (km/how). The amount of k represents the instant time $t = kT_s$, where T_s is time step used for traffic flow simulation (commonly $T_s = 10s$). For stability purpose, interval time step simulation for every link m must satisfy the following inequality equation $L_m > v_{free,m}\ T_s$ where $v_{free,m}$ is mean speed when a driver assumes that the traffic flows are free.

Traffic flow in each segment i of link m at time k is equal to traffic density multiply by the mean speed and number of lines on each segment that is denoted by λ_m, so that the following equation can be obtained:

$$q_{m,i}(k) = \rho_{m,i}(k) v_{m,i}(k) \lambda_m. \quad (1)$$

Traffic density on segment i of link m at time $k+1$ is equal to the traffic density at time k plus traffic flow change on the link for segment i at time k. Thus, the following equation can be obtained:

$$\frac{\lambda_m \left[\rho_{m,i}(k+1) - \rho_{m,i}(k) \right]}{\Delta T} + \frac{q_{m,i}(k) - q_{m,i-1}(k)}{\Delta L} = 0$$

$$\Leftrightarrow \frac{\lambda_m\left[\rho_{m,i}(k+1)-\rho_{m,i}(k)\right]}{T_s}=\frac{q_{m,i-1}(k)-q_{m,i}(k)}{L_m}$$
$$\Leftrightarrow \rho_{m,i}(k+1)=\rho_{m,i}(k)+\frac{T_s}{L_m\lambda_m}[q_{m,i-1}(k)-q_{m,i}(k)].$$

These equations describe traffic density changes by time.

Traffic speed and density are influenced by time and segment. However, mean speed is described as follows:

1. Relaxation indicates that a driver is trying to reach the desired speed, $V(\rho)$, relaxation time is denoted by τ.
2. Convection indicates unstable speed caused by vehicle inflows.
3. Anticipation indicates that a driver adjust the speed according to traffic condition to immediately pull over. The η shows the speed that is affected by anticipation time and κ is density involved in anticipation time.

Form the above definitions, we get:

$$\frac{v_{m,i}(k+1)-v_{m,i}(k)}{\Delta T}+v_{m,i}(k)\frac{v_{m,i}(k)-v_{m,i-1}(k)}{\Delta L}=\frac{1}{\tau}\left(V(\rho_{m,i}(k))-v_{m,i}(k)\right)-\frac{\eta}{\tau\left(\rho_{m,i}(k)+\kappa\right)}\times\frac{\rho_{m,i+1}(k)-\rho_{m,i}(k)}{\Delta L}$$
$$\Leftrightarrow \frac{v_{m,i}(k+1)-v_{m,i}(k)}{T_s-0}+v_{m,i}(k)\frac{v_{m,i}(k)-v_{m,i-1}(k)}{L_m-0}=\frac{1}{\tau}\left(V(\rho_{m,i}(k))-v_{m,i}(k)\right)-\frac{\eta}{\tau\left(\rho_{m,i}(k)+\kappa\right)}\times\frac{\rho_{m,i+1}(k)-\rho_{m,i}(k)}{L_m-0}$$
$$\Leftrightarrow \frac{v_{m,i}(k+1)-v_{m,i}(k)}{T_s}=v_{m,i}(k)\frac{v_{m,i-1}(k)-v_{m,i}(k)}{L_m}+\frac{1}{\tau}\left(V(\rho_{m,i}(k))-v_{m,i}(k)\right)-\frac{\eta}{\tau\left(\rho_{m,i}(k)+\kappa\right)}\times\frac{\rho_{m,i+1}(k)-\rho_{m,i}(k)}{L_m}$$
$$\Leftrightarrow v_{m,i}(k+1)=v_{m,i}(k)+\frac{T_s v_{m,i}(k)[v_{m,i-1}(k)-v_{m,i}(k)]}{L_m}+\frac{T_s}{\tau}\left(V(\rho_{m,i}(k))-v_{m,i}(k)\right)-\frac{T_s\eta[\rho_{m,i+1}(k)-\rho_{m,i}(k)]}{L_m\tau\left(\rho_{m,i}(k)+\kappa\right)} \quad (2)$$

where

$$V(\rho_{m,i}(k))=\min\left[v_{free,m}\exp\left[-\frac{1}{a_m}\left(\frac{\rho_{m,i}(k)}{\rho_{crit,m}}\right)^{a_m}\right],(1+\alpha)v_{control,m,i}(k)\right], \quad (3)$$

and a_m is a parameter on fundamental diagram, $\rho_{crit,m}$ is a critical density if traffic flow is maximal, $(1+\alpha)$ is a non-compliance factor in drivers with speed limit shown and $v_{control,m,i}(k)$ is speed limit [7][15].

The starting point or point of departure is modeled by a simple queuing model. Some involved variables in point of departure are length of queue $w_o(k)$ (vehicles), traffic demand $d_o(k)$ (vehicle/hour) and outflow $q_o(k)$ (vehicle/hour). Those variables are related so that the following equation can be formed:

$$\frac{w_o(k+1)-w_o(k)}{\Delta T}=d_o(k)-q_o(k)$$
$$\Leftrightarrow \frac{w_o(k+1)-w_o(k)}{T_s-0}=d_o(k)-q_o(k)$$
$$\Leftrightarrow w_o(k+1)=w_o(k)+T_s\left[d_o(k)-q_o(k)\right]$$

The outflow of a starting point depends on traffic conditions of the corresponding mainstream segment and the existence of ramp metering control measures $r_o(k)$, where $r_o(k) \in [0,1]$. Thus, outflow equation can be expressed as $q_o(k) = \min\left[d_o(k) + \frac{w_o(k)}{T_s}, r_o(k)C_o, C_o\left(\frac{\rho_{jam,m} - \rho_{m,1}(k)}{\rho_{jam,m} - \rho_{crit,m}}\right)\right]$ where C_o is the capacity of the on-ramp under free flow traffic conditions and $\rho_{jam,m}$ is the maximum density from link m connecting to the on-ramp. The δ shows speed will decline caused by merging phenomena at on-ramps. If there is traffic flow from on-ramp to freeway then we define the following equation:

$$
\begin{aligned}
&\frac{v_{m,i}(k+1) - v_{m,i}(k)}{\Delta T} + v_{m,i}(k)\frac{v_{m,i}(k) - v_{m,i-1}(k)}{\Delta L} = \frac{1}{\tau}\left(V(\rho_{m,i}(k)) - v_{m,i}(k)\right) - \frac{\eta}{\tau\left(\rho_{m,i}(k)+\kappa\right)} \\
&\qquad \times \frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{\Delta L} - \frac{\delta q_o(k) v_{m,1}(k)}{L_m \lambda_m \left(\rho_{m,1}(k)+\kappa\right)} \\
&\Leftrightarrow \frac{v_{m,i}(k+1) - v_{m,i}(k)}{T_s} = v_{m,i}(k)\frac{v_{m,i-1}(k) - v_{m,i}(k)}{L_m} + \frac{1}{\tau}\left(V(\rho_{m,i}(k)) - v_{m,i}(k)\right) - \frac{\eta}{\tau\left(\rho_{m,i}(k)+\kappa\right)} \\
&\qquad \times \frac{\rho_{m,i+1}(k) - \rho_{m,i}(k)}{L_m} - \frac{\delta q_o(k) v_{m,1}(k)}{L_m \lambda_m \left(\rho_{m,1}(k)+\kappa\right)} \\
&\Leftrightarrow v_{m,i}(k+1) = v_{m,i}(k) + \frac{T_s v_{m,i}(k)[v_{m,i-1}(k) - v_{m,i}(k)]}{L_m} + \frac{T_s}{\tau}\left(V(\rho_{m,i}(k)) - v_{m,i}(k)\right) \\
&\qquad - \frac{T_s \eta [\rho_{m,i+1}(k) - \rho_{m,i}(k)]}{L_m \tau\left(\rho_{m,i}(k)+\kappa\right)} - \frac{T_s \delta q_o(k) v_{m,1}(k)}{L_m \lambda_m \left(\rho_{m,1}(k)+\kappa\right)}.
\end{aligned}
\tag{4}
$$

2.1. Piecewise Affine (PWA) Approximation

Given a function $f: R^n \to R$. Function f is called PWA function if consists of collection of affine functions that can be defined on polyhedral and can be expressed as $f(x) = a_i^T x + b_i$ if $x \in \Omega_i$. Some methods for PWA approximation are

1. Least Square Optimization

   The least square optimization method is well known method by taking the difference between initial function and the approximated curve. Nonlinear function of a single variable can be determined by one region of numbers of interval of PWA function. For example, the following PWA problem can be solved by the least square. Given function f that is defined on interval $[x_{\min}, x_{\max}]$ and function f_{PWA} is continuous on interval $\min_{\alpha,\beta,\gamma,\delta,\epsilon,\zeta} \int_{x_{min}}^{x_{max}} (f_{PWA}(x) - f(x))^2 dx$. Thus,

$$
f_{PWA}(x) = \begin{cases} \gamma + \dfrac{x - x_{min}}{\alpha - x_{min}}(\delta - \gamma), & \text{for } x_{min} \le x < \alpha \\ \delta + \dfrac{x - \alpha}{\beta - \alpha}(\epsilon - \delta), & \text{for } \alpha \le x < \beta \\ \epsilon + \dfrac{x - \beta}{x_{max} - \beta}(\zeta - \epsilon), & \text{for } \beta \le x \le x_{max}. \end{cases}
$$

2. PWA Identification

   Identification of PWA is a collection of algorithms that produces PWA approximation according to a set of data points. Therefore, this approach is very useful for complex multivariable function. There are three methods used in PWA identification, i.e. clustering, linear identification, and pattern recognition methods. These methods are not suitable to be used in this identification, thus the most appropriate method for bivariate identification is multicatgory robust linear programming (MRLP) algorithm [16].

3. Partial constant piecewise approximation
   An approximation of bivariate function using several complete variables consists of domain segment on one of the variables, where each region is a set at constant value. In general, bivariate function $f(x,y)$ can be approximated as follows: assuming variable x and y have relative distance $\frac{x_{\max}-x_{\min}}{x_{\max}}$ and $\frac{y_{\max}-y_{\min}}{y_{\max}}$, variable x is selected to be drawn from each region. To select variable x on interval $[x_i,x_{i+1}]$ for $i=1,2,...,N-1$ where $x_1=x_{\min}$ and $x_N=x_{\max}$ can be written as: $f(x,y)\approx f\left(\frac{x_i+x_{i+1}}{2},y\right)$ for $x\in\left[x_i,x_{i+1}\right]$.

2.2. Piecewise Affine Approximation in METANET Model

METANET model is a nonlinear equation that will be approximated by using PWA function [17]. This model is constructed by developing new model, i.e. with linearization on equations (1), (2), (3), and (4).

2.2.1. Nonlinear traffic flow equation

To model traffic flow equation, we can approximate using PWA identification and constant piecewise approximation on one of variables in equation (1). Speed variable $v_{m,i}(k)$ is chosen with the smallest domain as compared to traffic flow variables. This variable is replaced with mean value on each sub domain. Thus, equation (1) becomes $q_{m,i}(k)=\lambda_m\rho_{m,i}(k)\frac{v_j+v_{j+1}}{2}$ for $v_{m,i}(k)\in\left[v_j,v_{j+1}\right]$, where j = 1,2,…,n. Interval $\left[v_j,v_{j+1}\right]$ can be chosen one by one by considering the approximated function form or by determining more advanced method using optimization.

2.2.2. Speed equation

Several variables in equation (2) and (3) can be replaced by constant value based on fixed value determined by historical data. In equation (2), some problems must be related to the following descriptions. First, speed variable appears in exponential factor in equation (3). Density variable appears in exponential factor in the first term of function (3). This function represents fundamental diagram where speed is as traffic density function. Some variables in equation (3) are replaced by constant values, i.e. v_{free} equals to 102km/hour, a_m equals to 1.876, ρ_{crit} equals to 33.5 vehicle/km/line and α equals to 0.1. Thus, equation (3) becomes $V(\rho_{m,i}(k))=\min\left[102\exp\left[-\frac{1}{1.867}\left(\frac{\rho_{m,i}(k)}{33.5}\right)^{a_m}\right],(1.1)v_{control,m,i}(k)\right]$. Then, multiplication of speed variable in equation (2) is $v_{m,i}(k)\left(v_{m,i-1}(k)-v_{m,i}(k)\right)$, speed variable $v_{m,i-1}(k)$ is constant based on the value determined by the historical data. The error in this method is caused by multiplication between speed variables and constant value $\frac{T_s}{L_m}$. Association between these two causing relatively small error, the value $\frac{T_s}{L_m}$ is close to 2.78 $\times 10^{-3}$ hour/km.

Division between density variable with other densities is $\frac{\rho_{m,i+1}(k)-\rho_{m,i}(k)}{\left(\rho_{m,i}(k)+\kappa\right)}$, multiplication between $\frac{\rho_{m,i+1}(k)-\rho_{m,i}(k)}{\left(\rho_{m,i}(k)+\kappa\right)}$ and $\frac{T_s\eta}{L_m\tau}$ causing relatively small error, the value $\frac{T_s\eta}{L_m\tau}$ is close to 33.33km^2/km/line. Adding density on the denominator part in the function is constant denoted by κ, i.e. 40 *vehicle/km/line*. Thus, we obtain $33.33\frac{\rho_{m,i+1}(k)-\rho_{m,i}(k)}{\left(\rho_{m,i}(k)+\kappa\right)}$. The last function in equation (4) is $-\frac{T_s\delta q_o(k)v_{m,1}(k)}{L_m\lambda_m\left(\rho_{m,1}(k)+\kappa\right)}$. This function exists when there is speed decline from entrance deviation to freeway. Substituting the parameter value to the function, we can get $-\frac{1.13\times 10^{-5} q_o(k)v_{m,1}(k)}{\left(\rho_{m,1}(k)+40\right)}$.

2.2.3. From PWA to MPC

The results from PWA METANET model can be obtained when it is combined with MPC optimization method. PWA model can be written as a mixed-integer language program with several decision variables of integer and domain ratio [13]. Given dummy binary variable (denoted by $\delta, \delta_a, \delta_b \in \{0,1\}$) to show whether some regions apply associative rule with one of affine pieces in its PWA function (i.e. PWA function $y: \mathrm{X} \to R$ with one of affine pieces $f: \mathrm{X} \to R$ and $\mathrm{X} \subset R^n$). Next, c is an arbitrary constant and ε shows machine precision used to change perfect inequality to imperfect inequality. Thus, yielding the following properties:

(i) $f(x) \le c \Leftrightarrow \delta = 1$ is true if and only if

$$\left.\begin{aligned} f(x) &\le c + (M-c)(1-\delta) \\ f(x) &\ge c(1-\delta) + \varepsilon + (m-\varepsilon)\delta \end{aligned}\right\}$$

(ii) Variable $y = \delta f(x)$, if and only if

$$\left.\begin{aligned} y &\le M\delta \\ y &\ge m\delta \\ y &\le f(x) - m(1-\delta) \\ y &\ge f(x) - M(1-\delta), \end{aligned}\right\}$$

(iii) Binary value $\delta = \delta_a\delta_b$, if and only if

$$\left.\begin{aligned} -\delta_a + \delta &\le 0 \\ -\delta_b + \delta &\le 0 \\ \delta_a + \delta_b - \delta &\le 1. \end{aligned}\right\}$$

2.3. Controlling Predictive Model for Traffic Control Model

MPC control is used based on the size of state variable at time *k*, performing state prediction along prediction horizon denoted by N_p. One of many possible optimization methods for traffic model is by maximizing traffic flow, the spread of traffic density, and minimizing the difference of control variables. The most used objective function is minimizing the total times spend (TTS) in the system, where TTS is the total waiting vehicles at on-ramps to freeway added by waiting time in that road, and the penalty equation in the changes of decision variables. MPC algorithm is used to determined control signal that minimizing the following objective function $J(k) = J_{TTS}^{MPC}(k) + J_{MPC}(k)$. This objective function minimizes vehicle waiting time at on-ramp and along the queue in the mainstream segment at point of departure before entering the freeway, thus we get:

$$J_{TTS}(k) = T_s \sum_{k=1}^{N_{sim}} \left(\sum_{(m,i)\in I_{all}} L_m \lambda_m \rho_{m,i}(k) + \sum_{o\in O_{all}} w_o(k) \right),$$

where N_{sim} represents optimal time simulation, I_{all} is a set of pair (*m,i*) from every link and segment in that network and O_{all} is a set of index from all points of departure. Next, the main objective of MPC control is to reduce TTS on prediction horizon, i.e. $J_{TTS}^{MPC}(k) = T_s \sum_{j=k}^{k+N_p} \left(\sum_{(m,i)\in I_{all}} L_m \lambda_m \rho_{m,i}(j) + \sum_{o\in O_{all}} w_o(j) \right)$ where $j \in \{1, 2, ..., N_p\}$. Given penalty equation in input deviation and horizon control $N_c < N_p$, then control signal is assumed to be constant, i.e.

$$J_{MPC}(k) = \sum_{j=k}^{k+N_c-1} \left(a_{ramp} \sum_{o\in O_{all}} \left(r_o(j) - r_o(j-1)\right)^2 + a_{speed} \sum_{(m,i)\in C_{all}} \left(v_{control,m,i}(j) - v_{control,m,i}(j-1)\right)^2 \right),$$

where a_{ramp} and a_{speed} are load coefficients and C_{all} is all sets of pair index from link and segments.

3. RESULTS AND DISCUSSIONS

The case study of METANET model simulation in freeway traffic network is depicted in Figure 3. In segment 3 and 4, speed limits are given. Here, O_i is a set of origin of intersection *i*, L_i is the speed limit, and D_1 is the destination.

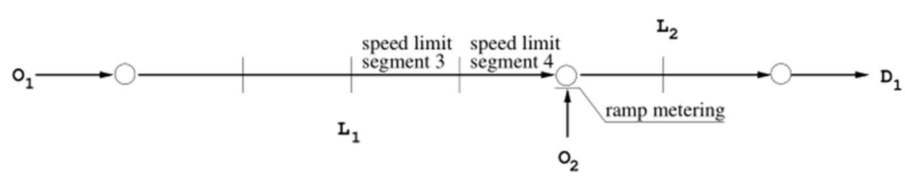

Figure 3. Simulation model for case study.

To simulate the model, we use some parameters value i.e. $v_{free} = 102\frac{km}{hour}, T_s = 10\ s, \tau = 18\ s, \kappa = 40\frac{vehicle}{km\ line}$, $\eta = 60\frac{km^2}{hour}$, $\rho_{jam} = 180\frac{vehicle}{km\ line}$, $\rho_{crit} = 33.5\frac{vehicle}{km\ line}$, $C_{o1} = 4000\frac{vehicle}{hour}$, $C_{o2} =$

$2000 \frac{vehicle}{hour}$, $L_m = 1\ km$, $N_{sim} = 40\ s$, $\lambda_m = 3\ line$, $r_o(k) = 1$, $\alpha = 0.1$, $\delta = 0.0122$, and $a_m = 1.867$. We use Matlab to simulate this model. The MPC control graph for METANET model with PWA approximation is shown in Figure 4. This figure shows that traffic flow experiences a decrease to zero for time between 0 to 2.5. Traffic flow in link 1 of segment 2 to link 2 of segment 2 experiences an increase and decline at the end. Figure 5 shows the traffic density in each link and segment. This figure show that the traffic density will decline to zero. The lines describe every link and segment of the traffic flow.

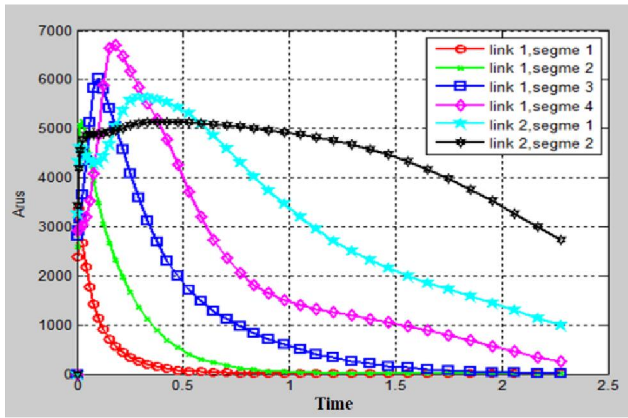

Figure 4. Traffic flow (in hour).

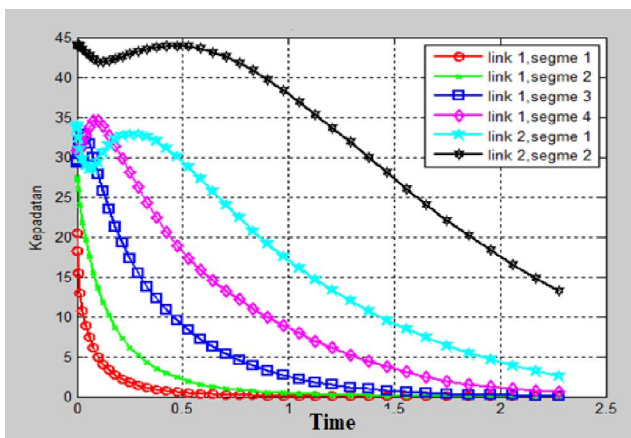

Figure 5. Traffic density.

Figure 6 displays mean speed in every link and segment. From this figure, we can see that the mean speed tends to increase. In this figure, we also show that the speed in link 1 of segment 3 and 4 approaches 100 causing by speed limit. Figure 7 shows the length of queue of the traffic network. According to this figure, length of queue in link 1 of segment 1 and link 2 of segment 1 experiences a decrease to zero.

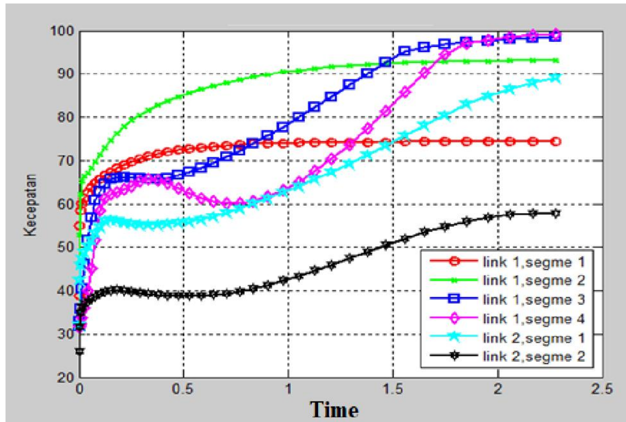

Figure 6. Mean speed.

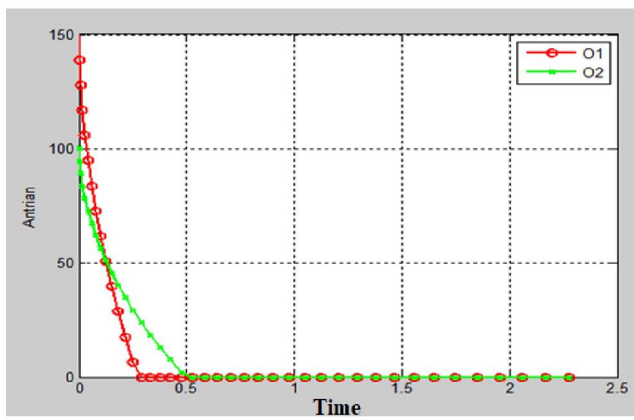

Figure 7. Length of queue.

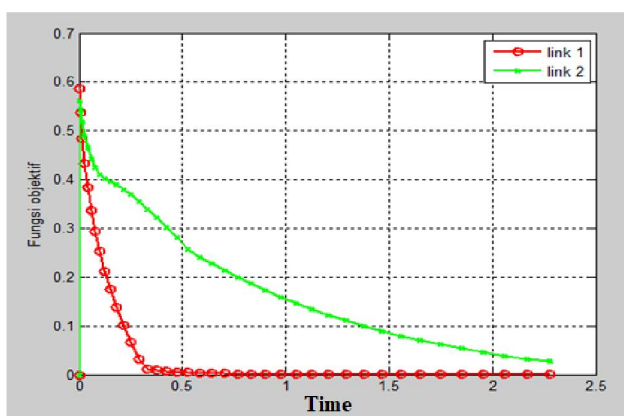

Figure 8. Objective function.

Figure 8 describes the objective function to be controlled. In this figure we can see that objective function decrease to zero in all links. Figure 9 shows objective function given MPC control. It also shows a decreasing trend to zero.

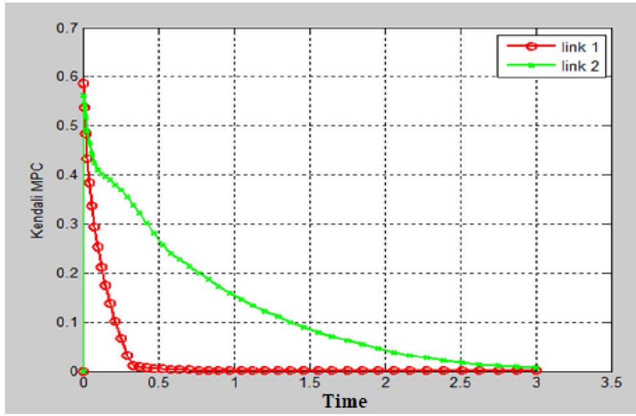

Figure 9. MPC control.

4. CONCLUSION

METANET model is a nonlinear model. To transform the model into linear form then PWA approximation is considered. PWA approximation consists of least square optimization method, PWA identification, and partial constant piecewise affine approximation. METANET model is approximated by PWA by substituting parameter values to the model. Based on study case, only the mean speed shows increasing trend and the density of traffic, traffic flow, and queue length tend to decrease in this time period. Mean speed in link 1 of segment 3 and 4 approaches 100. It's caused by the speed limit.

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