



E-ISSN 2654-9948

ALGORITMA Journal of Mathematics Education (AJME)

<http://journal.uinjkt.ac.id/index.php/algorithm>

Vol. 8 No. 1 – 2026, hal. 1-17

MAPPING RESEARCH ON ABSTRACT ALGEBRA LEARNING; A BIBLIOMETRIC STUDY

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Abstract

Abstract algebra plays a crucial role in undergraduate mathematics education by fostering axiomatic reasoning, structural thinking, and formal proof construction. Comprehensive mappings of the field remain limited. This study aims to identify global research trends in abstract algebra learning through a bibliometric analysis of 75 Scopus-indexed publications published between 1973 and 2025. Using Biblioshiny in R, the analysis examined publication growth, influential sources, authorship patterns, geographical distribution, citation impact, and thematic structures. The results indicate a substantial increase in research activity, particularly after 2010, with strong contributions from mathematics education journals and research communities in the United States and Europe. Thematic analysis reveals that group theory, instructional approaches, and mathematics education are the dominant topics, whereas issues related to epistemological transitions and learning obstacles remain underexplored. These findings provide an overview of the development of abstract algebra learning research and highlight opportunities for future design-based studies aimed at addressing learning obstacles in advanced mathematics education.

Keywords: *abstract algebra learning; mathematics education; bibliometric analysis; learning obstacles; didactical design research*

Abstrak

Aljabar abstrak merupakan mata kuliah penting dalam pendidikan matematika karena mengembangkan penalaran aksiomatis, pemikiran struktural, dan kemampuan pembuktian formal. Kajian yang memetakan perkembangan bidang ini secara komprehensif masih terbatas. Penelitian ini bertujuan menganalisis tren penelitian global tentang pembelajaran aljabar abstrak melalui studi bibliometrik terhadap 75 publikasi terindeks Scopus periode 1973–2025. Analisis dilakukan menggunakan Biblioshiny pada perangkat lunak R untuk mengkaji pertumbuhan publikasi, sumber yang berpengaruh, pola kepenulisan, distribusi geografis, dampak sitasi, dan struktur tematik penelitian. Hasil menunjukkan peningkatan publikasi yang signifikan, terutama setelah 2010, dengan kontribusi dominan dari jurnal pendidikan matematika serta peneliti di Amerika Serikat dan Eropa. Analisis tematik mengungkap dominasi topik teori grup, pendekatan pembelajaran, dan pendidikan matematika, sementara isu transisi epistemologis dan learning obstacles masih relatif kurang diteliti. Temuan ini memberikan gambaran perkembangan bidang serta mengidentifikasi peluang penelitian lanjutan untuk mengatasi hambatan belajar pada pembelajaran matematika tingkat lanjut.

Kata kunci: pembelajaran aljabar abstrak; pendidikan matematika; analisis bibliometrik; learning obstacles; didactical design research

How to Cite: Antika, R., Turmudi, Rosjanuardi, R., & Asih, E.C.M. (2026). Mapping Research on Abstract Algebra Learning; A Bibliometric Study. *ALGORITMA Journal of Mathematics Education*, 8 (1), 1-17.

Permalink/DOI: <http://dx.doi.org/10.15408/ajme.v8i1.50754>

Received: April 2026; Accepted: June 2026; Published: June 2026

INTRODUCTION

Abstract algebra occupies a central position in undergraduate mathematics education because it introduces students to axiomatic reasoning, structural thinking, and formal proof. Unlike computational mathematics, abstract algebra requires learners to reason within formal structures such as groups, rings, and fields, where mathematical meaning emerges from relationships among axioms rather than numerical procedures (Harel, 2011; Larsen, 2013). Consequently, learning abstract algebra involves a transition from operational reasoning toward structural reasoning and abstract thinking (Tall, 2002). For prospective mathematics teachers, this course plays a crucial role in developing logical, analytical, and systematic thinking skills that are essential for understanding advanced mathematical concepts and for teaching mathematics effectively (Saragih, 2019; Saragih et al., 2025).

However, the intrinsic nature of abstract algebra, characterized by theorem proving and complex definitions, often becomes a major obstacle to achieving meaningful conceptual understanding (Saefudin, 2017; Manurung et al., 2024). This transition is often challenging for students. Many learners experience what has been described as the “transition problem,” namely the difficulty of moving from familiar procedures of elementary algebra to formal and abstract reasoning (Hausberger, 2015; Weber & Larsen, 2008). The challenge is further intensified by an epistemic shift from focusing on operations on elements to reasoning about properties of algebraic systems, reflecting the structural perspective introduced by Noether (Hausberger, 2016). Moreover, abstract algebra concepts are frequently taught without sufficient historical and epistemological context, making it difficult for students to understand the purpose and meaning of abstract structures (Hausberger et al., 2021).

Numerous studies have documented persistent difficulties experienced by students in learning abstract algebra. Students frequently encounter challenges in understanding identity elements, inverses, group structures, and proof construction (Larsen, 2013). Many students struggle to recognize concrete examples of abstract definitions and often develop misconceptions regarding fundamental concepts such as identity and inverse elements (Rahayu et al., 2020; Simamora et al., 2025). Research has also shown that students may produce formally correct proofs without fully understanding their underlying mathematical meanings (Stylianides et al., 2024). These difficulties are further exacerbated by students’ reliance on conventional computational procedures with limited engagement in axiomatic proof construction (Jamilah et al., 2017; Rubowo et al., 2017). In addition, students frequently experience technical difficulties in understanding logical language and the formal steps required to construct valid mathematical proofs (Hakim et al., 2025; Rahayu et al., 2021). Such learning difficulties have direct implications for students’ mastery of advanced mathematics, including calculus and structural analysis, which require

systematic reasoning (Ellu et al., 2022; Nasrullah et al., 2025). Internal factors and instructional approaches employed by lecturers have also been identified as important determinants of students' learning outcomes in abstract algebra (Hanifah & Abadi, 2019; Kusumawati & Kurniawan, 2020).

Various theoretical perspectives have been employed to investigate abstract algebra learning. APOS theory explains learning as the construction of mental structures connecting actions, processes, objects, and schemas (Arnon et al., 2014). Research on proof emphasizes the roles of explanation, justification, and communication in mathematical activity (Miyakawa & Shinno, 2020). From socio-cultural perspectives, mathematical understanding develops through representational mediation and participation in mathematical practices (Radford, 2000). Furthermore, the Anthropological Theory of the Didactic (ATD) suggests that learning difficulties may arise from mismatches between institutional mathematical organizations and learners' accessible reasoning systems (Chevallard, 2019). Such mismatches may generate learning obstacles in advanced mathematics learning (Brousseau, 1997; Suryadi, 2010).

Recent studies have highlighted the importance of instructional approaches in supporting students' engagement with abstract algebra. Inquiry-based learning and related pedagogical approaches have been suggested as ways to reduce cognitive tensions associated with idealization and thematization in abstract mathematical reasoning (Alam & Mohanty, 2024). Nevertheless, research on university algebra remains fragmented, and pedagogical frameworks for supporting proof and structural reasoning are not yet fully integrated (Veith et al., 2023). Studies employing concept inventories and quantitative measures have further demonstrated the need to connect observations of students' learning difficulties with empirical evaluations of instructional effectiveness (Veith et al., 2022).

Although research on abstract algebra learning has expanded over several decades, existing studies remain scattered across different theoretical perspectives and research topics. Most investigations focus on specific concepts, instructional interventions, or students' learning difficulties, while comprehensive mappings of the field's intellectual development remain limited. Consequently, little is known about publication trends, dominant themes, influential contributors, and emerging research directions in abstract algebra learning. Furthermore, previous studies have rarely distinguished between descriptive investigations that characterize university mathematics education and design-oriented research aimed at improving classroom practice. This situation indicates the need for a systematic synthesis of existing research to identify theoretical gaps and future directions for mathematics education research (Hausberger et al., 2021).

Accordingly, this study conducts a bibliometric analysis of abstract algebra learning research indexed in the Scopus database. Specifically, the study examines publication trends, thematic structures, and research gaps within the field. Through this mapping, the study seeks to provide

an intellectual overview of abstract algebra learning research and support future design-oriented investigations grounded in Didactical Design Research for addressing learning obstacles in advanced mathematics education. To achieve this objective, this study specifically addresses three fundamental research questions: (1) what the publication trends are in abstract algebra learning research; (2) what thematic structures characterize the field; and (3) what research gaps inform future investigations on abstract algebra learning

METHOD

Data Source and Search Strategy

This study employed a bibliometric mapping approach to examine global research trends in abstract algebra learning within mathematics education. Bibliographic data were retrieved from the Scopus database, which was selected because of its extensive international coverage and rigorous indexing of peer-reviewed scholarly publications. Scopus has been widely used in bibliometric studies due to its comprehensive metadata and broad disciplinary representation, making it suitable for mapping research developments across fields. Data collection was conducted in February 2026 using the Advanced Search feature in Scopus. A structured search strategy was developed to capture studies addressing abstract algebra within educational contexts while excluding publications devoted solely to pure mathematics. The search query combined keywords representing disciplinary content in abstract algebra with keywords related to educational processes and instructional practices. The following query string was applied: TITLE-ABS-KEY(("abstract algebra" OR "modern algebra" OR "group theory") AND ("mathematics education" OR "teaching" OR "learning" OR "instruction" OR "proof" OR "reasoning" OR "undergraduate mathematics")) AND (LIMIT-TO(DOCTYPE,"ar")) AND (LIMIT-TO(LANGUAGE,"English"))

The first group of keywords (“abstract algebra”, “modern algebra”, and “group theory”) was selected to represent major domains of abstract algebra commonly taught in undergraduate mathematics programs. The second group of keywords (“mathematics education”, “teaching”, “learning”, “instruction”, “proof”, “reasoning”, and “undergraduate mathematics”) was included to ensure that the retrieved publications explicitly addressed educational and learning-related issues rather than purely mathematical theory. To ensure relevance and academic quality, inclusion criteria were incorporated directly into the search query through restrictions on document type and language. Specifically, only peer-reviewed journal articles (DOCTYPE = "ar") published in English were included in the dataset. Furthermore, the combination of disciplinary and educational keywords served as an embedded filtering mechanism that excluded studies unrelated to mathematics education. Consequently, no separate manual screening procedure was conducted, as

the query design itself functioned as an initial selection process. The search yielded a final dataset of 75 documents. No temporal restriction was imposed during the search process. However, the earliest publication retrieved from the Scopus database that met the search criteria was published in 1973. Therefore, the final dataset spans the period from 1973 to 2025, enabling an examination of the long-term development of research in abstract algebra learning. The selected records were exported in CSV format and analyzed using the bibliometrix package in R through the Biblioshiny interface. The analysis integrated performance analysis and science mapping techniques to provide a comprehensive overview of the field. Specifically, the study examined publication productivity, citation impact, collaboration patterns, keyword co-occurrence, and thematic structures to identify the intellectual development and emerging research trends in abstract algebra learning.

Data Analysis

The data were analyzed using the bibliometrix package in R Studio through the Biblioshiny interface. The analysis integrated both performance analysis and science mapping techniques to provide a comprehensive overview of research development in abstract algebra learning. Performance analysis was conducted to examine publication productivity, influential journals, authors, countries, and citation impact. Citation analysis was employed to identify highly cited publications and influential scholarly contributions within the field. Collaboration analysis was used to investigate patterns of cooperation among authors, institutions, and countries. Furthermore, keyword co-occurrence analysis was performed to explore conceptual relationships among research topics, while thematic mapping analysis was used to identify the development, maturity, and interconnections of research themes in abstract algebra learning.

RESULTS AND DISCUSSION

General Characteristics of the Bibliometric Dataset

The main characteristics of the dataset analyzed in this study are presented in Table 1. The bibliometric dataset consists of 75 journal articles published between 1973 and 2025, distributed across 31 scholarly sources. This temporal span indicates that research related to abstract algebra learning has evolved over several decades, although its development appears gradual rather than rapidly expanding. The calculated annual growth rate of 3.51% suggests a steady but moderate increase in scholarly attention toward abstract algebra learning within mathematics education. The average document age of 10.7 years further indicates that the field includes both foundational contributions and relatively recent research developments, reflecting ongoing academic interest. In total, 135 authors contributed to the analyzed publications, with 2.6 co-authors per document, demonstrating moderate collaborative engagement among researchers. However, the proportion of international co-authorship remains relatively low (6.67%), suggesting that research on abstract

algebra learning is still largely conducted within national or regional academic communities rather than through extensive global collaboration.

Table 1. Main Information

DESCRIPTION	RESULTS
MAIN INFORMATION ABOUT DATA	
TIMESPAN	1973:2025
SOURCES (JOURNALS, BOOKS, ETC)	31
DOCUMENTS	75
ANNUAL GROWTH RATE %	3.51
DOCUMENT AVERAGE AGE	10.7
AVERAGE CITATIONS PER DOC	13.8
REFERENCES	0
MAIN INFORMATION ABOUT DATA	
KEYWORDS PLUS (ID)	46
AUTHOR'S KEYWORDS (DE)	254
DOCUMENT CONTENTS	
AUTHORS	135
AUTHORS OF SINGLE-AUTHORED DOCS	20
AUTHORS COLLABORATION	
SINGLE-AUTHORED DOCS	24
CO-AUTHORS PER DOC	2.6
INTERNATIONAL CO-AUTHORSHIPS %	6.667
DOCUMENT TYPES	
ARTICLE	75

From a conceptual perspective, the dataset includes 254 author keywords, indicating thematic diversity within the field. Such diversity suggests that abstract algebra learning research intersects with multiple theoretical and instructional perspectives, including proof, reasoning, conceptual understanding, and advanced mathematical thinking. Overall, these characteristics indicate that abstract algebra learning represents a developing yet increasingly recognized research domain within mathematics education.

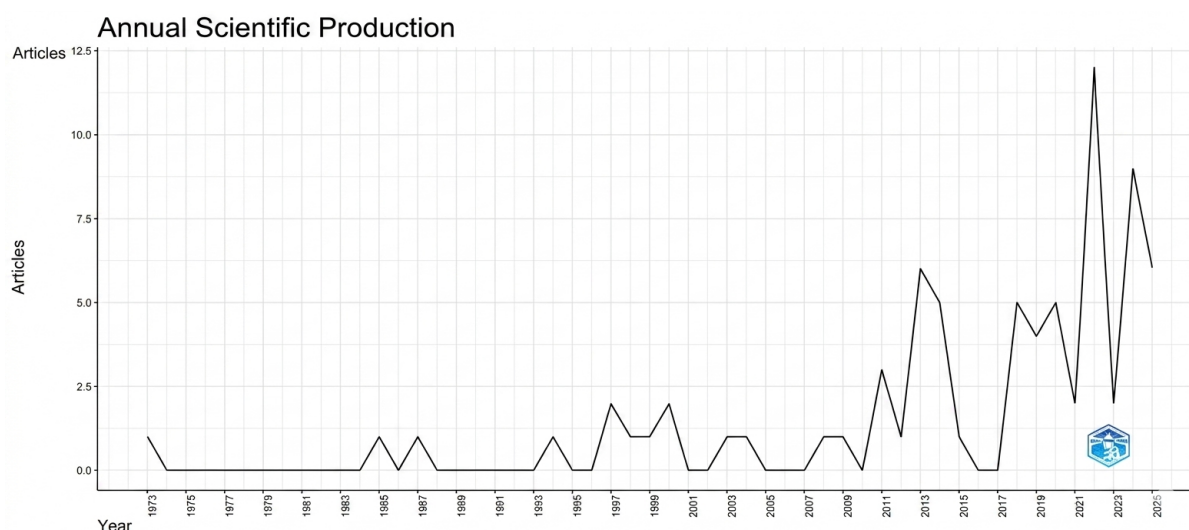


Figure 1. Annual scientific production

The temporal evolution of research on abstract algebra learning is illustrated in Figure 1, which presents the annual scientific production from 1973 to 2025. The distribution reveals that scholarly attention to abstract algebra learning developed slowly during the early decades, followed

by a noticeable increase in research activity in recent years. Between 1973 and the early 2000s, publications appeared sporadically, indicating that investigations related to abstract algebra learning were limited and not yet established as a distinct research area within mathematics education. During this period, studies addressing abstract algebra were typically embedded within broader discussions of advanced mathematical thinking rather than forming an independent line of inquiry.

A gradual increase becomes visible after approximately 2010, suggesting growing recognition of learning challenges associated with university-level mathematics. The most significant growth occurs after 2018, culminating in a peak publication output around 2021. This recent expansion indicates increasing international attention toward conceptual understanding, proof practices, and instructional approaches in abstract algebra courses. Despite fluctuations in yearly output, the overall trajectory demonstrates a transition from isolated contributions toward a more sustained research presence. The observed pattern suggests that abstract algebra learning is evolving into an emerging research domain rather than representing a mature and stabilized field. The increasing publication frequency in the last decade reflects broader developments in mathematics education research emphasizing advanced mathematical reasoning and proof-oriented learning.

Most Relevant Sources

The distribution of publications across scholarly journals is presented in Figure 2, which identifies the most relevant publication sources contributing to research on abstract algebra learning. The results indicate that research in this domain is primarily disseminated through specialized mathematics education journals rather than general education outlets.

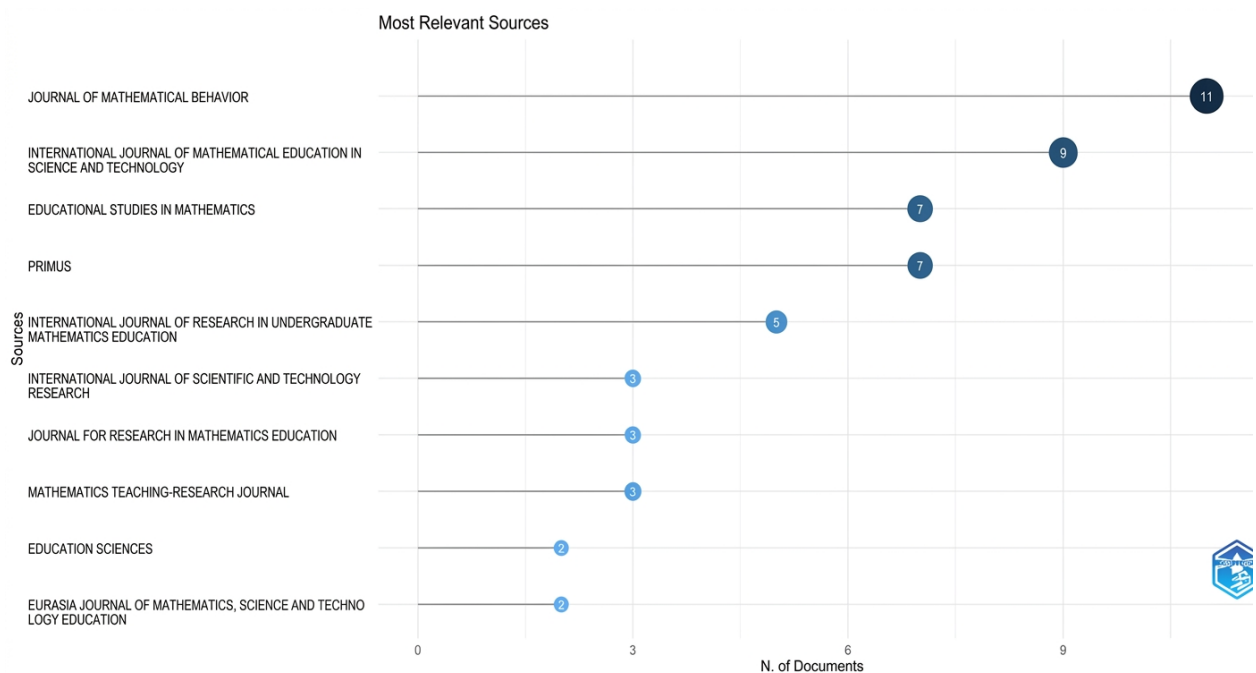


Figure 2. Most Relevant Sources

The Journal of Mathematical Behavior emerges as the leading publication source, contributing the highest number of documents within the dataset. This dominance reflects the journal’s long-standing focus on students’ mathematical reasoning, conceptual development, and advanced mathematical thinking, which closely align with research concerns in abstract algebra learning. Other prominent sources include the International Journal of Mathematical Education in Science and Technology, Educational Studies in Mathematics, and PRIMUS, all of which are internationally recognized journals emphasizing university-level mathematics teaching and learning. The presence of these journals suggests that abstract algebra learning research is strongly situated within discussions of undergraduate mathematics education and proof-oriented instruction.

Additionally, contributions appearing in journals such as the Journal for Research in Mathematics Education and the International Journal of Research in Undergraduate Mathematics Education indicate increasing engagement with theoretically grounded investigations addressing learning processes in advanced mathematics contexts. Overall, the distribution across multiple high-impact mathematics education journals suggests that abstract algebra learning research is institutionally anchored within established scholarly communities. At the same time, the absence of a single overwhelmingly dominant outlet indicates thematic dispersion, reinforcing the characterization of the field as developing rather than fully consolidated.

Most Relevant Authors

The most influential contributors to research on abstract algebra learning are presented in Figure 3, which displays authors with the highest number of publications within the analyzed dataset. The results reveal that research development in this field is shaped by a relatively small group of recurring scholars who contribute consistently to discussions on advanced mathematical learning.

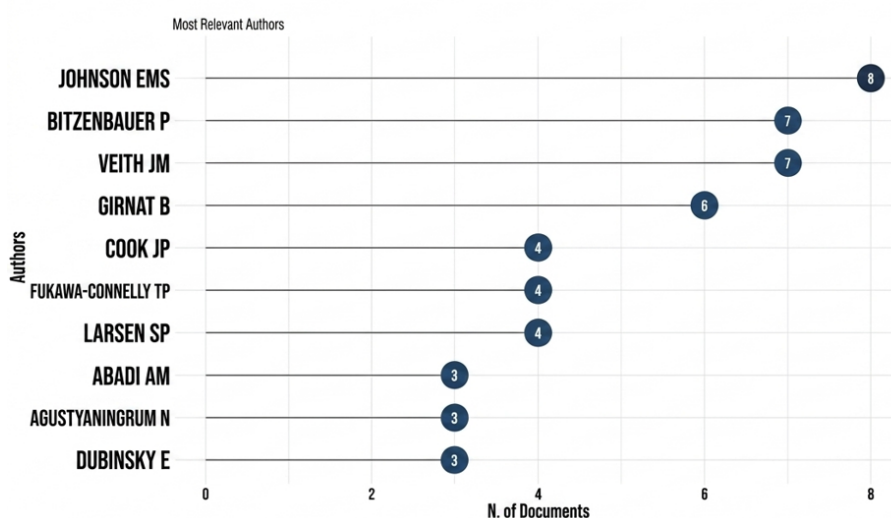


Figure 3. Most Relevant Author

Among these contributors, Johnson emerges as the most productive author, followed by Bitzenbauer and Veith, indicating sustained scholarly engagement in investigating instructional and conceptual aspects of abstract algebra. Other prominent contributors include Girnat, Cook, Fukawa-Connelly, and Larsen, whose work has significantly influenced research on undergraduate mathematics education and students' understanding of algebraic structures.

Notably, the presence of scholars such as Dubinsky, associated with the development of APOS theory, highlights the strong influence of constructivist cognitive frameworks in shaping research on abstract algebra learning. This observation suggests that investigations in the field remain closely connected to theoretical traditions emphasizing conceptual construction and mental structures in advanced mathematics learning. The concentration of publications among a limited number of authors indicates that abstract algebra learning research is sustained by specialized research communities rather than broadly distributed scholarly participation. Such a pattern is characteristic of emerging research domains in which theoretical development is driven by key contributors before wider expansion occurs. Overall, the authorship distribution suggests that the intellectual growth of abstract algebra learning research is strongly anchored in established mathematics education scholars focusing on proof, reasoning, and undergraduate mathematical thinking.

Country Scientific Production

The geographical distribution of research productivity in abstract algebra learning is illustrated in Figure 4, while the corresponding publication frequencies are summarized in Table 1. The results reveal a highly uneven global distribution of scholarly contributions across countries.

Country Scientific Production

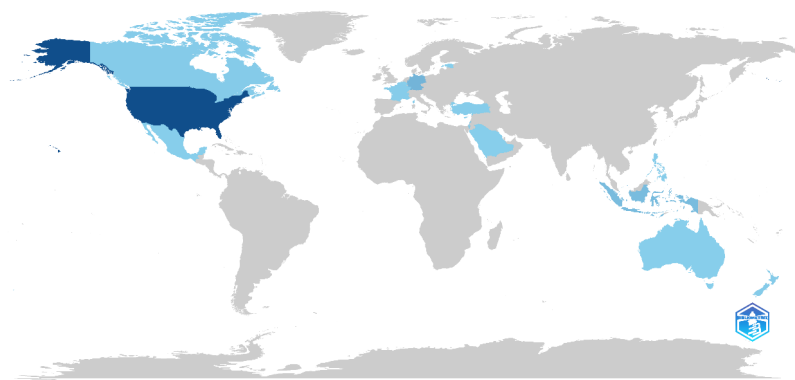


Figure 4. Country Scientific Production

The United States emerges as the dominant contributor, accounting for the largest proportion of publications within the dataset. This prominence reflects the long-standing development of undergraduate mathematics education research communities in North America, particularly those focusing on advanced mathematical thinking, proof, and inquiry-oriented

instruction. Following the United States, Germany represents the second most productive country, indicating strong European engagement in theoretical and instructional investigations related to abstract algebra learning. Notably, Indonesia ranks among the leading contributors, demonstrating growing participation from Southeast Asia in research addressing university-level mathematics education. Additional contributions originate from countries such as Israel, Canada, New Zealand, France, the Philippines, Turkey, and Australia, suggesting that research on abstract algebra learning has gradually expanded across multiple continents. However, the relatively small number of contributing countries compared to the global higher education landscape indicates that scholarly activity remains concentrated within specific academic regions. Overall, the geographical pattern suggests that abstract algebra learning research is internationally distributed but institutionally concentrated. Such a configuration is characteristic of emerging research domains in which intellectual leadership is established within a limited number of countries before broader global diffusion occurs.

Thematic Structure

The thematic map provides particularly strong theoretical implications. The positioning of: 1) mathematics education, algebra, and group theory as *motor themes*, and; 2) undergraduate mathematics education and structure sense as emerging or weakly developed themes, indicates that although abstract algebra occupies a central position in mathematics education discourse, research addressing students' conceptual restructuring remains underdeveloped. The thematic structure is illustrated in Figure 5.

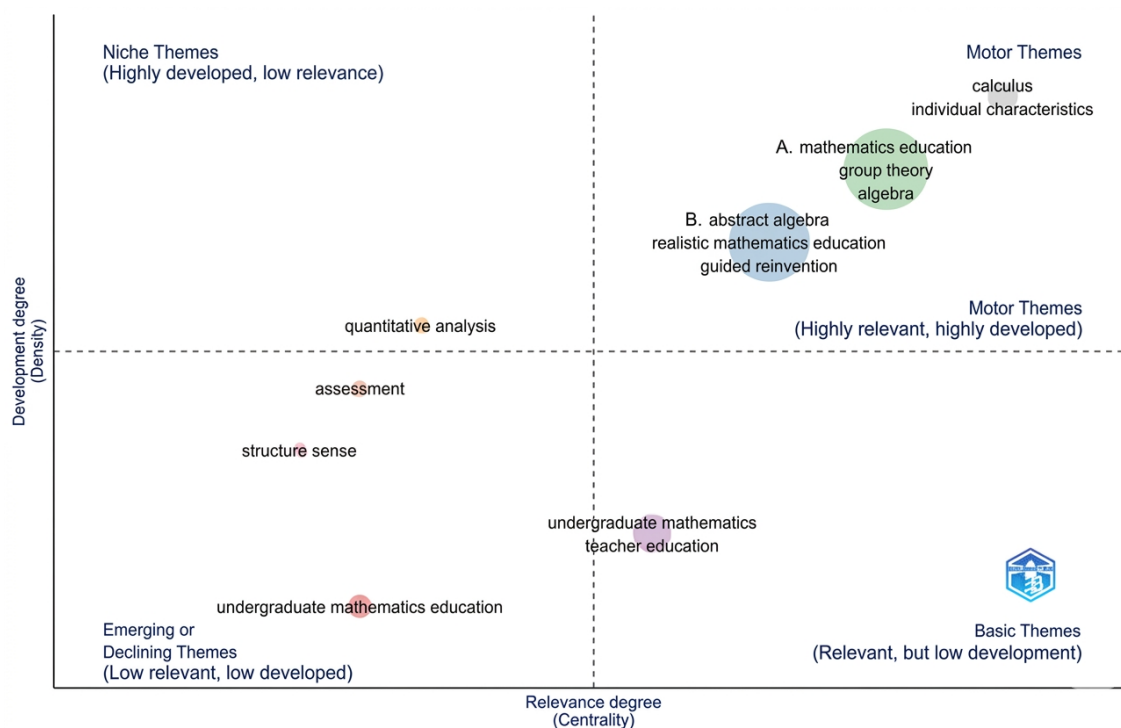


Figure 5. Thematic Structure

Within ATD terminology, this situation suggests the persistence of institutional mismatches between scholarly knowledge and taught knowledge, generating conditions under which learning obstacles naturally emerge. These obstacles can be interpreted as: 1) Epistemological obstacles, students encounter difficulties transitioning from operational algebraic reasoning toward structural reasoning required in group theory; 2) Didactical obstacles, teaching practices emphasize formal proof production rather than the epistemic function of proof. 3) Ontological obstacles, learners struggle to reconceptualize algebraic objects as abstract structures rather than computational entities.

Trend Topics

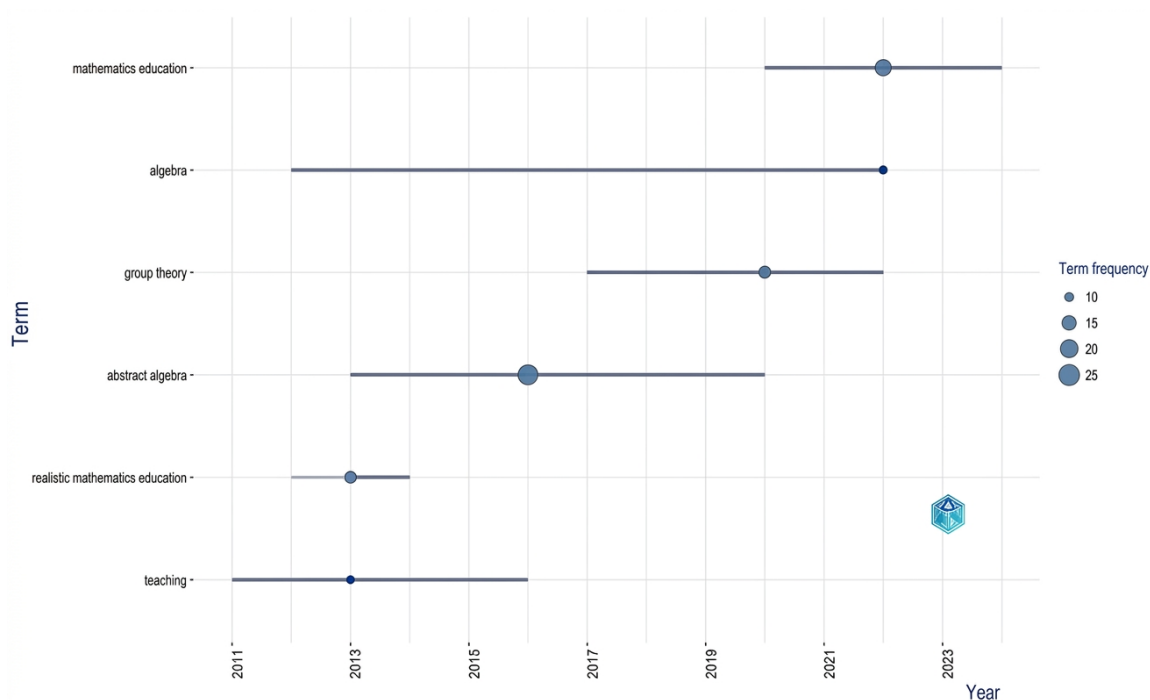


Figure 6. Trend Topic

The trend topics analysis reveals the temporal evolution of dominant research themes in abstract algebra learning as shown in Figure 6. Early studies were primarily associated with general instructional concerns such as *teaching* and *realistic mathematics education*, which appeared prominently around 2010–2014. Beginning in the mid-2010s, research attention shifted toward abstract algebra as a specific disciplinary domain, indicating increasing recognition of conceptual challenges inherent in advanced mathematical structures. Subsequently, *group theory* emerged as a sustained topic between approximately 2017 and 2021, reflecting growing interest in students' understanding of algebraic structures at the undergraduate level. More recent years demonstrate the consolidation of *mathematics education* as a dominant and continuing research theme extending into the 2020s. This pattern suggests a movement from localized pedagogical approaches toward broader theoretical investigations of learning processes in advanced mathematics.

learning environments that explicitly anticipate epistemological, ontological, and didactical learning obstacles in abstract algebra. In this sense, the present bibliometric mapping functions not merely as a literature survey but as a diagnostic analysis of the institutional knowledge structure informing subsequent didactical design development. The thematic configuration identified through the treemap analysis positions learning obstacle investigation in group theory as a theoretically grounded continuation of global research development, thereby establishing the present bibliometric study as the epistemic foundation for subsequent Didactical Design Research in abstract algebra learning.

CONCLUSION

The findings of this bibliometric study suggest that research on abstract algebra learning has reached an important developmental threshold. While the field has achieved considerable growth in terms of publication volume, theoretical diversity, and international participation, its dominant orientation remains largely descriptive and instructional. Existing studies predominantly investigate how abstract algebra is taught or understood, yet comparatively fewer investigations examine how mathematical knowledge itself is reorganized within instructional institutions to enable meaningful access to axiomatic reasoning.

This situation signals a broader transition currently emerging in mathematics education research—from documenting learning difficulties toward systematically redesigning learning environments capable of transforming students' mathematical activity. Abstract algebra occupies a fundamental position in undergraduate mathematics education because it develops students' abilities in axiomatic reasoning, proof construction, and structural thinking. However, learning abstract algebra remains challenging for many students due to its highly abstract nature and its reliance on formal definitions and deductive reasoning. Empirical studies have consistently reported students' difficulties in understanding concepts such as groups, identity elements, inverses, and mathematical proof. In classroom practice, students often rely on procedural approaches acquired from previous mathematics courses and struggle to transition toward structural and theoretical reasoning. Consequently, the persistent learning difficulties observed across different educational contexts cannot be interpreted solely as individual cognitive challenges but should also be understood as manifestations of tensions between institutional mathematical organizations and students' evolving ways of thinking.

From the perspective of the Anthropological Theory of the Didactic, these tensions may emerge when institutional mathematical knowledge is not fully aligned with learners' accessible reasoning systems, thereby generating learning obstacles in abstract algebra. Viewed through this perspective, bibliometric mapping does not merely summarize past research trajectories; rather, it

reveals the necessity of a new generation of studies that integrate theoretical analysis with design-oriented inquiry. Approaches grounded in the Anthropological Theory of the Didactic and operationalized through Didactical Design Research offer promising pathways for advancing this transformation. By explicitly identifying learning obstacles and using them as resources for instructional redesign, future research may move beyond remediation toward the reconstruction of epistemic access to abstract mathematical structures.

An additional implication emerging from this bibliometric mapping concerns the role of mathematical proof in abstract algebra learning. Although proof construction is widely recognized as a defining feature of abstract algebra, relatively few studies explicitly investigate how students understand the purposes that proof serves within mathematical activity. Existing research tends to focus on proof production, correctness, or problem-solving performance, while questions concerning proof as a means of explanation, justification, and mathematical communication remain comparatively underexplored. This gap suggests the need for future studies that examine not only whether students can produce formally acceptable proofs but also whether they understand the epistemic functions that make proof meaningful. Such investigations may provide deeper insight into forms of apparent competence that conceal limitations in students' understanding and may help explain why proof-related activity sometimes becomes procedural or ritualized rather than genuinely mathematical.

Consequently, the future development of abstract algebra education lies not only in improving teaching strategies but in rethinking how learners encounter mathematical structures as meaningful objects of reasoning, justification, and inquiry. Advancing this agenda requires sustained efforts to connect theoretical frameworks, empirical classroom investigation, and iterative didactical design. In this sense, the present study contributes by establishing an intellectual map from which design-based and theory-driven research programs can emerge, supporting a shift from understanding difficulties in abstract algebra learning toward transforming the conditions under which mathematical understanding becomes possible

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