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# TECHNIQUES OF SOLVING RATIONAL INEQUALITIES

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***Abstract***

The topic of rational inequalities is taught at high school and university mathematics. Both high school students and college students still have difficulty solving rational inequalities. This study aims to describe techniques for solving rational inequalities. To reach this aim, a qualitative study using the document analysis method was carried out. Analyzed documents in this study include secondary school mathematics textbooks, test preparation textbooks for university entrance, Calculus and Basic Mathematics for university students' textbooks, and relevant articles on rational inequalities. The results of this study included descriptions of techniques for solving rational inequalities, including the techniques of number line, of analysis, and of graph aided with the GeoGebra software. It can be concluded that the three techniques of solving rational inequalities complement each other for understanding the meaning and the process of solving rational inequalities.

**Keywords:** algebra education, rational inequalities, GeoGebra

**Abstrak**

Topik pertidaksamaan rasional diajarkan pada matematika tingkat sekolah menengah atas dan universitas. Baik siswa sekolah menengah atas maupun mahasiswa masih mengalami kesulitan dalam menyelesaikan pertidaksamaan rasional. Penelitian ini bertujuan untuk menguraikan teknik-teknik penyelesaian pertidaksamaan rasional. Untuk mencapai tujuan ini, studi kualitatif menggunakan metode analisis dokumen telah dilakukan. Dokumen yang dianalisis meliputi buku-buku pelajaran matematika tingkat sekolah menengah, buku-buku persiapan masuk perguruan tinggi, buku-buku Kalkulus dan Matematika Dasar untuk mahasiswa perguruan tinggi, dan artikel-artikel relevan mengenai pertidaksamaan rasional. Hasil penelitian ini berupa uraian mengenai teknik-teknik penyelesaian pertidaksamaan rasional yang meliputi teknik garis bilangan, teknik analisis, dan teknik grafik dengan bantuan perangkat lunak GeoGebra. Dapat disimpulkan bahwa tiga teknik penyelesaian pertidaksamaan rasional tersebut saling memperkuat satu sama lain untuk memahami makna dan proses penyelesaian pertidaksamaan rasional.

**Kata kunci:** pendidikan aljabar, pertidaksamaan rasional, GeoGebra

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## INTRODUCTION

Solving rational inequalities is one of the algebra topics in school mathematics taught for senior high school students over the world (Almog & Ilany, 2012; Tsamir & Almog, 2001; Tsamir & Bazzini, 2004) including in Indonesia (Kemendikbud, 2013; Pratiwi & Rosjanuardi, 2020). In Indonesia, this topic is given to secondary school students who choose a mathematics-and-science stream (Kemendikbud, 2013). As a consequence, test items on solving rational inequalities often appear in national examinations, in school mathematics competitions, and in university entrance tests (Aziz, 2014; Sembiring, 2002; Sobirin, 2009).

At the university level, the topic of rational inequalities is reviewed and taught again in the Calculus or Basic Mathematics course for undergraduate students of Mathematics, Science, and Engineering. The author's experiences in teaching Calculus, Basic Mathematics, and School Mathematics courses for mathematics education students, however, show that the students often encountered difficulties in dealing with rational inequalities. The difficulties can be observed from mistakes emerged in students' written work when solving rational inequalities. For example, in solving the rational inequality $\frac{2}{x} < \frac{1}{x-1}$, an incorrect algebraic technique for solving this inequality is as follow. A student often does a cross multiplication to obtain $2(x-1) < x$, without considering whether $x$ or $(x-1)$ is zero or not , which leads to $2x - x < 2$ and finally to $x < 2$. This technique, which is similar to an equation solving technique, leads to an incorrect solution for most cases of solving inequalities. The results of this limited observation for the case of mathematics education students' written work are in line with the results of other relevant studies in Indonesia (e.g., Anggoro & Prabawanto, 2019; Pratiwi & Rosjanuardi, 2020). Similar difficulties are also found in the results of relevant studies in other countries (e.g., Almog & Ilany, 2012; Tsamir & Almog, 2001).

Factors that caused the disappointing fact on students' difficulties above might include, for instance, lack of students' conceptual understanding on inequalities and of students' procedural fluency in manipulating algebraic expressions when dealing with the rational inequalities (Anggoro & Prabawanto, 2019; Pratiwi & Rosjanuardi, 2020). Other factors might come from an incomprehensive description of rational inequalities in school mathematics textbooks, from careless teaching and learning process for rational inequalities in secondary school level, and from misapplying equation solving techniques into inequality solving techniques (Almog & Ilany, 2012). Considering these factors into account, as one of the endeavours to prevent conceptual and procedural mistakes in dealing with inequalities, this study aims to describe techniques for solving rational inequalities. In this way, it is expected that readers, particularly teachers or students, will comprehensively understand on how to solve rational inequalities correctly and that the difficulties dealing with this type of inequality can be reduced in the future.

## METHOD

To reach the aim of this study, i.e., to describe techniques for solving rational inequalities, a qualitative study using document analysis method was carried out (Bowen, 2009; Sukmadinata, 2012). Analyzed documents in this study include secondary school textbooks (e.g., Kanginan, Akhmad, & Nurdiansyah, 2015; Sukino, 2014), test preparation textbooks for university entrance (e.g., Aziz, 2014; Foster & Herlin, 2005; Sembiring, 2012; Sobirin, 2009), Calculus and Basic Mathematics textbooks for university students (e.g., Martono, 1987; 1999; Wahyudin, 2012), and relevant articles on rational inequalities (e.g., McLaurin, 1985; Muksar, 2000; Tsamir & Almog, 2001). From these documents, three different techniques of solving rational inequalities are identified, i.e., the number line technique, the analysis technique, and the graph technique. For the graph technique, digital tools and mathematical softwares, such as GeoGebra, can be used as tools in drawing graphs.

## RESULTS AND DISCUSSION

General form for a rational inequality is the following: $\frac{f(x)}{g(x)} < 0$ [or $\leq 0$, or $> 0$, or $\geq 0$], where $g(x) \neq 0$. This type of inequality, from the document analysis, can be solved using three different techniques, i.e., the number line technique, the analysis technique, and the graph technique. The number line technique is the most commonly used in the three sources of documents: school mathematics textbooks, test preparation textbooks for university entrance, and Calculus or Basic Mathematics textbooks. The other two techniques are rarely addressed in school mathematics and test preparation textbooks.

### *The Number Line Technique for Solving Rational Inequalities*

From the analyzed documents, it can be summarized steps for solving a rational inequality using the number line technique, namely: (1) Transform a given rational inequality into the general form $\frac{f(x)}{g(x)} < 0$ [or $\leq 0$, or $> 0$, or $\geq 0$] where $g(x) \neq 0$; (2) if it is possible, write the numerator $f(x)$ and the denominator $g(x)$ each into linear factors; (3) Put on a number line critical values for $f(x)$ and $g(x)$, namely the values of $x$ such that $f(x) = 0$, and $g(x) = 0$; This action has caused the number line divided into several intervals; (4) Select a point from each interval, and test it in the $\frac{f(x)}{g(x)} < 0$ [or $\leq 0$, or $> 0$, or $\geq 0$] to see if it makes the inequality true or false; (5) Write down the interval, or intervals, that make the statement true. To illustrate how these steps are implemented, below we describe how to solve the inequality $\frac{8x^2-3x+10}{5x-2} < 2x - 1$.

*Step 1*. Transform the inequality $\frac{8x^2-3x+10}{5x-2} < 2x - 1$ into the general form $\frac{f(x)}{g(x)} < 0$ where $g(x) \neq 0$, as the following.

$$\frac{8x^2-3x+10}{5x-2} < 2x - 1.$$

$$\Leftrightarrow \frac{8x^2-3x+10}{5x-2} - (2x - 1) < 0.$$

$$\Leftrightarrow \frac{8x^2-3x+10}{5x-2} - \frac{(2x-1)(5x-2)}{5x-2} < 0.$$

$$\Leftrightarrow \frac{8x^2-3x+10}{5x-2} - \frac{(10x^2-9x+2)}{5x-2} < 0.$$

$$\Leftrightarrow \frac{-2x^2+6x+8}{5x-2} < 0.$$

*Step 2*. The numerator for the inequality $\frac{-2x^2+6x+8}{5x-2} < 0$ needs be to transformed into linear factors, namely $\frac{(8-2x)(x+1)}{5x-2} < 0$.

*Step 3*. Critical values for the inequality $\frac{(8-2x)(x+1)}{5x-2} < 0$ occur when $8 - 2x = 0, x + 1 = 0$, and $5x - 2 = 0$, which subsequently lead to $x = 4, x = -1$, and $x = 2/5$. These values should be put on a number line as shown in Figure 1. This number line is divided into four intervals, namely $x < -1, -1 < x < \frac{2}{5}, \frac{2}{5} < x < 4$, dan $x > 4$.

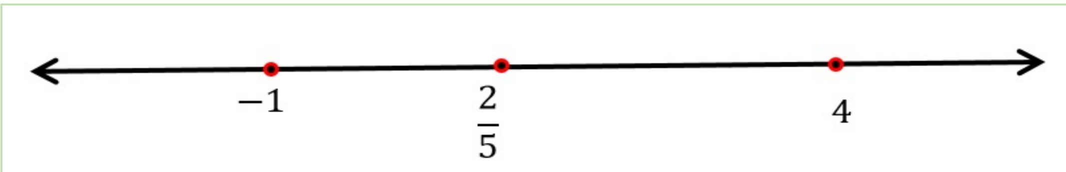



**Figure 1.** A Number Line with Four Intervals

*Step 4*. Select a point from each interval and next test it in the $\frac{(8-2x)(x+1)}{5x-2} < 0$ to see if it makes the inequality true or false. For the interval $x < -1$, for instance, choose $x = -2$ and substitute it into $\frac{(8-2x)(x+1)}{5x-2} < 0$ to obtain $\frac{(8-2(-2))(-2+1)}{5(-2)-2} = \frac{-12}{-12} < 0$. This makes the statement $\frac{(8-2x)(x+1)}{5x-2} < 0$ false because $\frac{-12}{-12}$ must be positive. For the interval $-1 < x < \frac{2}{5}$, for instance, choose $x = 0$ and substitute it into $\frac{(8-2x)(x+1)}{5x-2} < 0$ to obtain $\frac{(8-2(0))(0+1)}{5(0)-2} = \frac{8}{-2} < 0$. This makes the statement $\frac{(8-2x)(x+1)}{5x-2} < 0$ true. By doing similarly for other two intervals and recording all the results on the number line, the following diagram in Figure 2 is obtained.

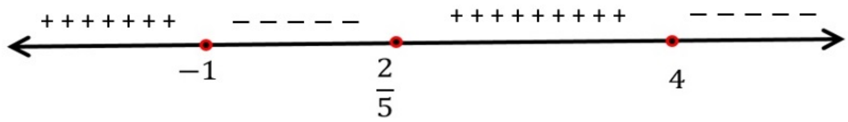
| | $x < -1$ | $-1 < x < \frac{2}{5}$ | $\frac{2}{5} < x < 4$ | $x > 4$ |
|---|---|---|---|---|
| $\frac{(8-2x)(x+1)}{5x-2} < 0$ | False | True | False | True |
| |  | | | |



**Figure 2.** Testing a Point for Each Interval on the Number Line

*Step 5.* From the Step 4, it can be concluded that the interval $-1 < x < \frac{2}{5}$ or $x > 4$ makes the inequality $\frac{(8-2x)(x+1)}{5x-2} < 0$ true. As the original inequality $\frac{8x^2-3x+10}{5x-2} < 2x - 1$ is equivalent to the inequality $\frac{(8-2x)(x+1)}{5x-2} < 0$, its solution is the same to the solution for $\frac{(8-2x)(x+1)}{5x-2} < 0$. Therefore, the solution for the original inequality is $-1 < x < \frac{2}{5}$ or $x > 4$.

The number line technique, according to McLaurin (1985), is actually a unified technique for solving any type of inequality. Therefore, it is not a surprise if this technique is addressed in textbooks of school mathematics, test preparation, basic mathematics, and Calculus; and is taught in the learning and teaching process of mathematics in the secondary school level. As another example, for the case of solving inequality $\frac{2}{x} < \frac{1}{x-1}$ mentioned in the Introduction, this inequality can be solved by following the steps of the number line technique above. After rewriting it into $\frac{x-2}{x(x-1)} < 0$ and using the number line technique, the solution for this inequality is $1 < x < 2$ or $x < 0$.

### *The Analysis Technique for Solving Rational Inequalities*

The first two steps for solving a rational inequality using the analysis technique are the same as the number line technique described in the previous sub-section. So, after obtaining $\frac{f(x)}{g(x)} < 0$ [or $\leq 0$, or $> 0$, or $\geq 0$] where $g(x) \neq 0$, in which the numerator and the denominator are in the form of linear factors, we analyze it by considering cases for the numerator and denominator such that the inequality is a true statement. To ilustrate this, let us solve the inequality $\frac{8x^2-3x+10}{5x-2} < 2x - 1$ using the analysis technique.

As we already did in the previous sub-section, the inequalty can be transformed into $\frac{(8-2x)(x+1)}{5x-2} < 0$. From this inequality, through an analysis, we should consider four cases of inequalities below and should solve it for each case.

*Case 1.* $(8-2x)<0, (x+1)<0$, and $(5x-2)<0$.
From this case, we obtain $x>4$, $x<-1$ , and $x<\frac{2}{5}$. It is clear that there is no interval satisfying this condition.

*Case 2.* $(8-2x)<0, (x+1)>0$, and $(5x-2)>0$.
From this case, we obtain $x>4, x>-1$, and $x>\frac{2}{5}$. As a consequence, the interval $x>4$ satisfies the condition.

*Case 3.* $(8-2x)>0, (x+1)>0$, and $(5x-2)<0$.
From this case, we obtain $x<4, x>-1$, and $x<\frac{2}{5}$. As a consequence, the interval $-1<x<\frac{2}{5}$ satisfies this condition.

*Case 4.* $(8-2x)>0, (x+1)<0$, and $(5x-2)>0$.
From this case, we obtain $x<4, x<-1$, and $x>\frac{2}{5}$. As a consequence, there is no interval that satisfies this condition.

From the four cases above, by unifying intervals that satisfy the inequality, we conclude that the solution for the inequality is $-1<x<\frac{2}{5}$ or $x>4$.

Considering the above solution process, solving each case involves considerable work, in which many situations might produce mistakes. Therefore, we understand that this analysis technique is often not taught for secondary school students, and is rarely addressed in school mathematics textbooks.

### *The Graph Technique for Solving Rational Inequalities*

In principle, the graph technique is implemented by using a geometrical interpretation for an inequality. The technique is implemented by drawing graphs for the functions involved in the inequality. In the current digital era (Jupri, Drijvers, & Van den Heuvel-Panhuizen, 2015; 2016; Jupri & Sispiyati, 2020), digital tools and mathematical softwares, such as GeoGebra, can be used to help us in drawing graphs. Again, to illustrate this technique, let us solve the rational inequality $\frac{8x^2-3x+10}{5x-2}<2x-1$. Solving this inequality can be interpreted geometrically as finding intervals of $x$ values, such that the graph $y=\frac{8x^2-3x+10}{5x-2}$ is below the graph $y=2x-1$. Figure 3 shows these two graphs. From the Figure 3, it can be seen that the intervals satisfying the condition of the inequality $\frac{8x^2-3x+10}{5x-2}<2x-1$ include $x>4$ and $-1<x<\frac{2}{5}$. Therefore, the solution for the inequality is $x>4$ or $-1<x<\frac{2}{5}$.

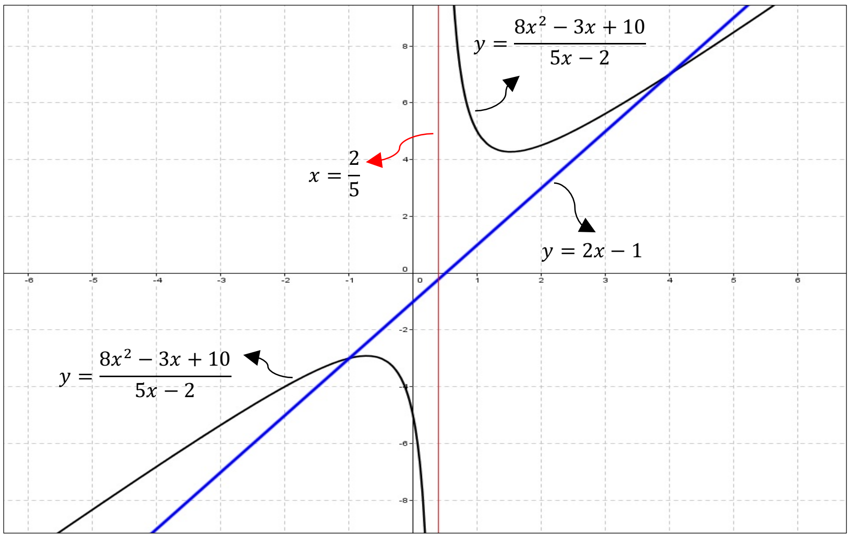



**Figure 3.** Graphs for $y = \frac{8x^2-3x+10}{5x-2}$ and $y = 2x - 1$

We have already known that the inequality $\frac{8x^2-3x+10}{5x-2} < 2x - 1$ is equivalent to the inequality $\frac{(8-2x)(x+1)}{5x-2} < 0$. These two inequalities therefore have the same solutions. Using the graph technique, as shown in Figure 4, we observe that the interval $x > 4$ or $-1 < x < \frac{2}{5}$ satisfies the inequality $\frac{(8-2x)(x+1)}{5x-2} < 0$. Therefore, the solution for the inequality $\frac{8x^2-3x+10}{5x-2} < 2x - 1$ is $-1 < x < \frac{2}{5}$ or $x > 4$.

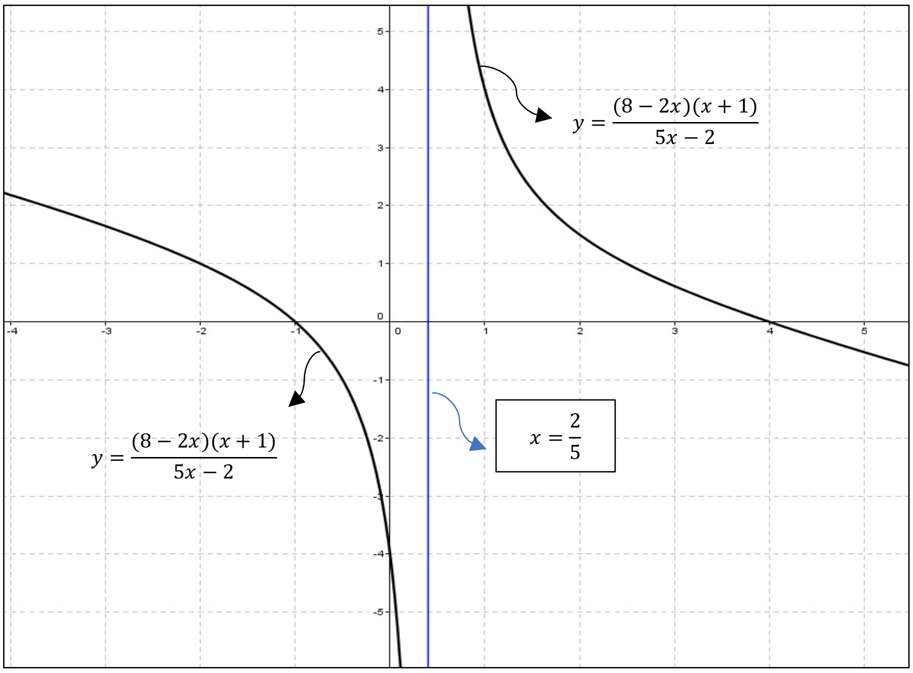



**Figure 4.** The Graph for $y = \frac{(8-2x)(x+1)}{5x-2}$

From the description above, we observe that the graph technique is actually a geometrical version for the number line technique. In the author's view, therefore, this graph technique makes the number line technique more meaningful to understand: Both techniques complement each other in giving more comprehensive understanding on the solution process of an inequality.

## CONCLUSION

From the description in the previous section, the following conclusions can be drawn. Three techniques for solving rational inequalities, i.e., the number line technique, the analysis technique, and the graph technique, are identified from schoool mathematics textbooks, test preparation textbooks, Calculus and Basic Mathematics textbooks, and relevant research articles. The number line technique is commonly used for solving rational inequalities and is considered to be a unified technique for solving inequalities in general. The analysis technique is rarely addressed in school mathematics textbooks and is seldom taught for secondary school students because it tends to make careless mistakes in the solution process. The graph technique is in principle implemented by using a geometrical interpretation for an inequality. In the current digital era, mathematical softwares, such as GeoGebra, are helpful for use for drawing graphs of functions involved in an inequality. Also, it can be observed that the graph technique makes the number line technique more meaningful to understand. Overall, it can be concluded that the three techniques of solving rational inequalities complement each other in understanding the meaning and the process of solving rational inequalities. For future research, it might be fruitful to investigate, for instance, the effect of teaching the three techniques of solving inequalities to students' ability in solving inequalities in general, and rational inequalities in particular.

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