
Quantum Ericson Engine with Multiple States in One Dimensional Potential Well

Nailul Hasan ^{1,*}

¹Program Studi Teknik Industri, Fakultas Teknik, UPN "Veteran" UPN Jatim Fisika FST
Jl. Rungkut Madya No.1, Gn. Anyar, Kec. Gn. Anyar, Kota SBY, Jawa Timur 60294

*nailul.hasan.fisika@upnjatim.ac.id

Submitted : August 2021; Revised : October 2021; Approved : December 2021; Available Online :
December 2021

Abstrak. Sebuah mesin kuantum ericson yang terdiri dari satu partikel dengan banyak keadaan, telah dieksplorasi. Ini adalah analogi kuantum untuk mesin ericson klasik yang terdiri dari kombinasi dua proses isothermal dan dua proses isobarik. Dengan menganalogikan partikel dalam potensial tak terbatas satu dimensi serta gas dalam silinder piston, proses termodinamika kuantum dalam mesin ericson kuantum dapat dijelaskan dari analogi proses klasik seperti isothermal, dan isobarik. Hasil penelitian menunjukkan bahwa efisiensi termal mesin ericson kuantum memiliki kemiripan dengan mesin ericson klasik.ta.

Kata Kunci: mesin ericson, mesin kuantum ericson, proses isothermal, proses isobarik

Abstract. It has been investigated a quantum ericson-engine consisting of a single particle with numerous states. It's a quantum equivalent of the classical Ericson engine, which is made up of two isothermal and two isobaric processes. Quantum thermodynamic processes in a quantum ericson-engine can be explained using analogies of classical processes such as isothermal and isobaric by comparing a particle in a one-dimensional infinite potential well to a gas in a piston-cylinder. According to the findings, the thermal efficiency of the quantum ericson-engine is comparable to that of the classical ericson-engine.

Keywords: ericson engine, quantum ericson-engine, isothermal process, isobaric process

DOI : 10.15408/fiziya.v4i2.22169

INTRODUCTION

A heat engine is a machine that can transform heat energy into mechanical energy. Heat energy cannot be turned totally into work in a cycle, according to the second rule of thermodynamics. The thermal efficiency of a heat engine is defined as the ratio of work to heat energy entering the heat engine. In general, the efficiency of a traditional heat engine is greater than 50%. It means that no traditional heat engine, even the basic dual-engine, can convert all of the heat energy flowing into the system into work [1, 2]. Bender et al. successfully constructed a quantum heat engine in the form of a single particle in a one-dimensional infinite potential well, indicating the presence of a quantum heat engine. A gas system in a piston tube is analogous to this arrangement [3]. Bender's research was successful in articulating the efficiency of the quantum Carnot engine, paving the way for further quantum heat engine research. [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14].

A single particle trapped in a one-dimensional infinite potential well with one wall can move freely like a piston in this research, which looks at the quantum ericson-engine using a comparable setup. It is necessary to determine the quantum ericson-engine efficiency formulation and compare it to the classical version [15]. This study is likely to pave the way for more quantum mechanics-based research, particularly in the realm of quantum ericson engines.

Our paper is organized as follows: at first, we define quantum isochoric and isothermal processes based on quantum identification of work performed and heat exchange. Second, we study the cycle of the quantum stirling engine, then we discuss the efficiency and compare it with the classical stirling engine. At last, we make a short discussion according to the topic.

RESEARCH METHOD

A single particle having mass m obeys non-relativistic wave equation, so called schrodinger equation. The particle is confined in a one dimensional box of width L with infinite potential walls.

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \varphi}{\partial x^2} = E\varphi$$

Or we can rewrite it

$$\frac{\partial^2 \varphi}{\partial x^2} = -\frac{2mE}{\hbar^2} \varphi = -k^2 \varphi$$

Or where the constant is $k^2 = \frac{2mE}{\hbar^2}$. It is related to energy of particle $E = \frac{\hbar^2 k^2}{2m}$. The solution of the equation is

$$\varphi(x) = A \sin \sin kx + B \cos \cos kx$$

The value B from boundary condition of wave function, $\varphi(0) = B = 0$. The other end $\varphi(L) = A \sin \sin kL = 0$ make the value

$$kL = n\pi$$

Where quantum number $n=1,2,3,4, \dots$ ect. The energy of particle now is quantized

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2}$$

The value A can be derived from normalization conditions for the wave function.

$$\varphi_n(x) = \sqrt{\frac{2}{L}} \sin \sin \frac{n\pi}{L} x$$

The absolute square of the wave function tells us the probability density of the particle for a given quantum number n. The general solution for the equation is just linear combination for every possible solution

$$\psi(x) = \sum_{n=1}^{\infty} a_n \varphi_n(x)$$

The total energy of the system is given by

$$E = \sum_{n=1}^{\infty} |a_n|^2 E_n$$

Where $|a_n|^2$ tell us about the probability of the particle in state n and energy E_n . The Normalization condition for wave function give us

$$\sum_{n=1}^{\infty} |a_n|^2 = 1$$

The force exerted by the particle to the wall is given by

$$F = -\frac{\partial E}{\partial L} = \sum_{n=1}^{\infty} |a_n|^2 \frac{\pi^2 \hbar^2}{mL^3}$$

As the opposite, the work of the the force can be calculated by equation

$$W = \int F dL$$

As in the classical, the quantum stirring engine we have two isothermal processes and two isochoric processes. From the figure, the process AB is isothermal where energy

is constant with heat added equal to the work by the system. BC is isochoric or isovolume Constant force from state. The process CD is isothermal where energy is constant with heat added equal to the work by the system. DA is isochoric or isovolume Constant force from state.

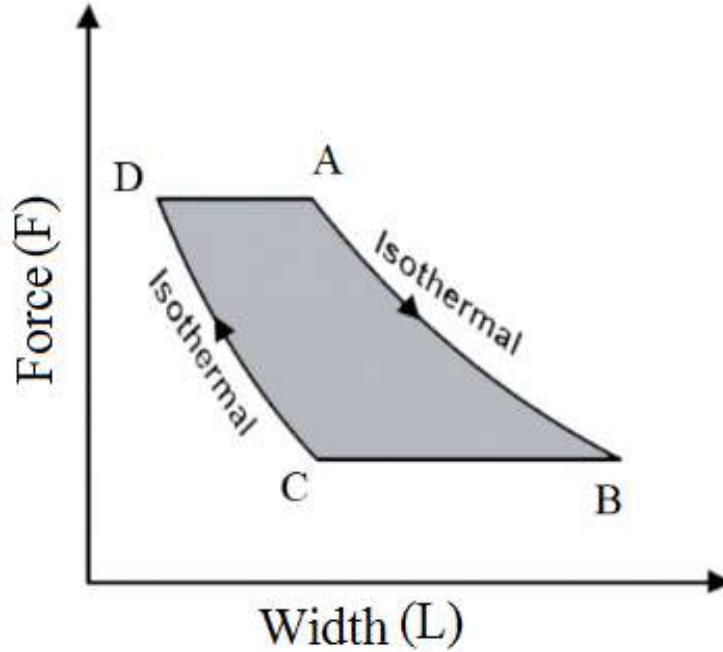


Figure 1. Cycle for Ericson Engine consist two Isothermal processes and two Isobaric processes. In The system Force is acting as pressure and width is acting as volume.

The process AB is isothermal energy is constant which heat added equal to the work by the system. The transition from $n=1$ to the superposition state 1, 2, 3, 4, ..., N. The wave function is superposition of the state

$$\frac{\pi^2 \hbar^2}{2mL_A^2} = E_A$$

$$\Psi_{AB}(x) = \sum_{n=1}^N a_n \varphi_n(x)$$

With normalization condition of the wave function, $\sum_{n=1}^N |a_n|^2 = 1$

The energy is give by

$$E_{AB} = \sum_{n=1}^N |a_n|^2 n^2 \frac{\pi^2 \hbar^2}{2mL^2} = E_A$$

$$S_{Bf} \frac{\pi^2 \hbar^2}{2mL_B^2} = \frac{\pi^2 \hbar^2}{2mL_A^2}$$

$$S_{Bf} L_A^2 = L_B^2$$

Where the coefficient $s_{AB} = \sum_{n=1}^N |a_n(L)|^2 n$, $s_{Bf} = \sum_{n=1}^N |a_n(L_B)|^2 n$
The force

$$F_{AB} = s_{AB} \frac{\pi^2 \hbar^2}{mL^3} = \frac{2E_A}{L}$$

The change in energy is same as heat added the system.

$$Q_{AB} = W_{AB} = \int_{L_A}^{L_B} F_{AB} dL = 2E_A \ln\left(\frac{L_B}{L_A}\right)$$

Where the energy change in this process, $\Delta E_{AB} = 0$

BC is isobaric

$$\psi_{BC}(x) = \sum_{n=1}^N a_n \varphi_n(x)$$

Constant force from state. The energy is given by

$$E_{BC} = s_{BC} \frac{\pi^2 \hbar^2}{2mL^2}$$

$s_{BC} = \sum_{n=1}^N |a_n(L)|^2 n$. The force can be derived from the energy,

$$F_{BC} = s_{BC} \frac{\pi^2 \hbar^2}{mL^3}$$

Constant force give us relation

$$\frac{s_{Cf} \pi^2 \hbar^2}{mL_C^3} = \frac{s_{Bf} \pi^2 \hbar^2}{mL_B^3}$$

$$s_{Cf} L_B^3 = s_{Bf} L_C^3$$

The work done to the system

$$W_{BC} = \int_{L_B}^{L_C} F_{CA} dL = \int_{L_B}^{L_C} \frac{s_{Cf} \pi^2 \hbar^2}{mL_C^3} dL = \frac{s_{Cf} \pi^2 \hbar^2}{mL_C^3} (L_C - L_B)$$

$$= \frac{s_{Cf} \pi^2 \hbar^2}{mL_C^2} - \frac{\pi^2 \hbar^2}{mL_A^2}$$

The energy change

$$\Delta E_{BC} = E_C - E_B = \frac{s_{Cf} \pi^2 \hbar^2}{2mL_C^2} - \frac{s_{Bf} \pi^2 \hbar^2}{2mL_B^2} = \frac{s_{Cf} \pi^2 \hbar^2}{2mL_C^2} - \frac{\pi^2 \hbar^2}{2mL_A^2}$$

Heat energy can be derived from the law of conservation energy.

$$Q_{BC} = \Delta E_{BC} + W_{BC} = \frac{3s_{Cf}\pi^2\hbar^2}{2mL_C^2} - \frac{3\pi^2\hbar^2}{2mL_A^2}$$

The process CD is isothermal energy is constant which heat added equal to the work by the system. The transition from to the superposition state 1, 2, 3, 4, ..., N. The wave function is superposition of the state

$$E_C = s_{Cf} \frac{\pi^2\hbar^2}{2mL_C^2}$$

$$\psi_{CD}(x) = \sum_{n=1}^N a_n \varphi_n(x)$$

With normalization condition of the wave function, $\sum_{n=1}^N |a_n|^2 = 1$

The energy is give by

$$E_{CD} = \sum_{n=1}^N |a_n|^2 n^2 \frac{\pi^2\hbar^2}{2mL^2} = E_C$$

$$s_{Df} \frac{\pi^2\hbar^2}{2mL_D^2} = s_{Cf} \frac{\pi^2\hbar^2}{2mL_C^2}$$

$$s_{Df}L_C^2 = s_{Cf}L_D^2$$

Where the coefficient $s_{Df} = \sum_{n=1}^N |a_n(L_D)|^2 n$

The force

$$F_{CD} = s_{CDf} \frac{\pi^2\hbar^2}{mL^3} = \frac{2E_C}{L}$$

The change in energy is the same as heat added to the system.

$$Q_{CD} = W_{CD} = \int_{L_C}^{L_D} F_{AB} dL = 2E_C \ln\left(\frac{L_D}{L_C}\right)$$

Where the energy change in this process, $\Delta E_{CD} = 0$

DA is isobaric

$$\psi_{DA}(x) = \sum_{n=1}^N a_n \varphi_n(x)$$

Constant force from state The energy is given by

$$E_{DA} = s_{DA} \frac{\pi^2 \hbar^2}{2mL^2}$$

$s_{DA} = \sum_{n=1}^N |a_n|^2 n$. The force can be derived from the energy,

$$F_{DA} = s_{DA} \frac{\pi^2 \hbar^2}{mL^3}$$

Constant pressure give us relation

$$\frac{s_{Df} \pi^2 \hbar^2}{mL_D^3} = \frac{\pi^2 \hbar^2}{mL_A^3}$$

$$L_D^3 = L_A^3 s_{Df}$$

The work done to the system

$$\begin{aligned} W_{DA} &= \int_{L_D}^{L_A} F_{DA} dL = \int_{L_D}^{L_A} \frac{s_{Df} \pi^2 \hbar^2}{mL_D^3} dL = \frac{s_{Df} \pi^2 \hbar^2}{mL_D^3} (L_A - L_D) \\ &= \frac{\pi^2 \hbar^2}{mL_A^2} - \frac{s_{Df} \pi^2 \hbar^2}{mL_D^2} \end{aligned}$$

The energy change

$$\Delta E_{DA} = E_A - E_D = \frac{\pi^2 \hbar^2}{2mL_A^2} - \frac{s_{Df} \pi^2 \hbar^2}{2mL_D^2}$$

Heat energy can be derived from the law of conservation energy.

$$Q_{DA} = \Delta E_{DA} + W_{DA} = \frac{3\pi^2 \hbar^2}{2mL_A^2} - \frac{3s_{Df} \pi^2 \hbar^2}{2mL_D^2}$$

The net work for one cycle is just the sum of the work for each process.

$$\begin{aligned} W_{\text{nett}} &= W_{AB} + W_{BC} + W_{CD} + W_{DA} \\ &= 2E_A \ln\left(\frac{L_B}{L_A}\right) + \frac{s_{Cf} \pi^2 \hbar^2}{mL_C^2} - \frac{\pi^2 \hbar^2}{mL_A^2} + 2E_C \ln\left(\frac{L_D}{L_C}\right) + \frac{\pi^2 \hbar^2}{mL_A^2} - \frac{s_{Df} \pi^2 \hbar^2}{mL_D^2} \\ &= 2E_A \ln\left(\frac{L_B}{L_A}\right) + 2E_C \ln\left(\frac{L_D}{L_C}\right) \end{aligned}$$

The nett heat added to the system in only at step AB

$$Q_{in} = Q_{AB} =$$

$$\eta = 1 + \frac{Q_{CD}}{Q_{AB}} = 1 + \frac{2E_C \ln\left(\frac{L_D}{L_C}\right)}{2E_A \ln\left(\frac{L_B}{L_A}\right)} = 1 - \frac{E_C \ln\left(\frac{L_C}{L_D}\right)}{E_A \ln\left(\frac{L_B}{L_A}\right)} = 1 - \frac{E_C}{E_A}$$

We have used the fact that $\frac{L_A}{L_B} = \frac{L_D}{L_C}$

CONCLUSION

The one dimensional quantum ericson engine has the same efficiency formulation as the classical ericson engine. We have shown that the multiple-state 1D box quantum ericson Engine efficiency depends on the initial and final energy ratio of thermal compression. It is analog to the efficiency of classical ericson Engine which is determined as a ratio of the temperature of the cold and hot bath contacted, 1 because the energy, as an average, is proportional to the temperature. The width of 1D box parameter does not vary when the system is in contact with either bath. The parameter varies only during the adiabatic transitions from one bath to the other bath. This cycle may deliver work or receive work for appropriate choice of the parameter

REFERENCES

- [1] M.W. Zemansky and R.D. Dittman, Heat and Thermodynamics, seventh ed (McGraw-Hill Companies, Inc, New York, 1997).
- [2] T.D. Kieu, European Physical Journal D 39, 115 (2006).
- [3] C.M. Bender, D.C. Brody, and B.K. Meister, Journal of Physics A: Mathematical and General 33, 4427 (2000).
- [4] B.C.M. Bender, D.C. Brody, and B.K. Meister, Royal Society (2002).
- [5] S. Singh, "Quantum braton engine of non-interacting fermions in one-dimensional box". arXiv preprint arXiv:1908.09281. (2019).
- [6] E. Latifah and A. Purwanto, Journal of Modern Physics 02, 1366 (2011).
- [7] A. Purwanto, H. Sukamto, B.A. Subagyo, and M. Taufiqi, Journal of Applied Mathematics and Physics 04, 1344 (2016).
- [8] A. Purwanto, H. Sukamto, and B.A. Subagyo, Journal of Modern Physics 06, 2297 (2015).
- [9] E. Latifah and A. Purwanto, Journal of Modern Physics 04, 1091 (2013).
- [10] D.P. Setyo, E. Latifah, A. Hidayat, and H. Wisodo, Jurnal Penelitian Fisika Dan Aplikasinya (JPFA) 8, 25 (2018).
- [11] M.S. Akbar, E. Latifah, and H. Wisodo, Journal of Physics: Conference Series 1093, (2018).

- [12] H.T. Quan, Y. Liu, C.P. Sun, and F. Nori, *Physical Review E* 76, 1 (2007).
- [13] H.T. Quan, *Physical Review E - Statistical, Nonlinear, and Soft Matter Physics* 79, (2009).
- [14] V. Ganesan, *Internal Combustion Engines, Second Edition* (The McGraw-Hill Companies, New Delhi, 2003).
- [15] M.J. Moran, H.N. Shapiro, D.D. Boettner, and M.B. Bailey, *Fundamental of Engineering Thermodynamics*, 9th edition (John Wiley and Sons, USA, 2018).