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The FLRW Universe Metric in 4+1 Spacetime Dimensional with Spherical Coordinate Invariance

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Abstract. The Friedmann-Lemaître-Robertson-Walker (FLRW) universe metric is an abstraction of the distance between two points in a time-evolving universe. The evolution of the FLRW universe can be either expansion or contraction. In this article, the FLRW universe metric in 4+1 spacetime is formulated. When this metric is relatively one dimension higher than the original metric. The addition of these dimensions is based on the assumption that the laws of physics have the same shape in the higher dimensions. A mathematical modeling idea is based on a spatial 4-dimensional isotropic sphere system immersed in this 5-dimensional spatial system. Then, Minkowski's flat spacetime concept was used to couple the spatial dimensions with the temporal dimension. Thus, we find the FLRW universe metric in 4+1 spacetime. The result of formalism shows that there is a radius quantity in the extra metric dimension, and this radius quantity forms the angle with the other two spatial dimensions. Then, we also show that the dimension of the cosmic scale factor will always be relatively higher than the spatial dimension of the metric. This has implications for the expansion or contraction of the FLRW universe model which remains valid in high dimensions.

Keywords: *The FLRW Universe Metric, 4+1 Dimensional Spacetime, Expansion of The Universe, Contraction of The Universe, Higher Dimensional.*

Abstrak. Metrik alam semesta Friedmann-Lemaître-Robertson-Walker (FLRW) merupakan abstraksi jarak antara dua titik di alam semesta yang berevolusi terhadap waktu. Evolusi alam semesta FLRW dapat berupa ekspansi ataupun kontraksi. Pada artikel ini, diformulasikan metrik alam semesta FLRW dalam ruangwaktu 4+1 dimensi. Yang mana, metrik ini relatif lebih tinggi satu dimensi dari metrik aslinya. Penambahan dimensi tersebut berdasarkan pada asumsi bahwa hukum fisika memiliki bentuk yang sama dalam dimensi tinggi. Sebagai ide pemodelan matematis, yakni berdasarkan sistem bola isotropik 4 dimensi spasial yang terbenam pada sistem 5 dimensi spasial. Kemudian, digunakan konsep ruangwaktu datar Minkowski untuk mengkopel dimensi spasial dengan dimensi temporal. Sehingga, didapati metrik alam semesta FLRW dalam ruangwaktu 4+1 dimensi. Hasil formalisme menunjukan bahwa adanya kuantitas radius dalam dimensi ekstra metrik, dan kuantitas radius ini membentuk sudut dengan dua dimensi spasial lainnya. Selain itu, kami juga menunjukan bahwa dimensi faktor skala kosmis akan selalu relatif lebih tinggi dari dimensi spasial metrik. Hal ini berimplikasi pada ekspansi ataupun kontraksi dari model alam semestra FLRW yang tetap berlaku dalam dimensi tinggi.
Kata Kunci: *Metrik Alam Semesta FLRW, Ruangwaktu 4+1 Dimensi, Ekspansi Alam Semesta, Kontraksi Alam Semesta, Dimensi Tinggi.*

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INTRODUCTION

The FLRW universe metric is a model of the distance between two points in the universe that are evolving with time. The evolution of the FLRW universe can be either expansion or contraction [1]. This metric concept was first formulated by Friedmann, Lemaître, Robertson, and Walker. Therefore, this metric is referred to as the FLRW universe metric. However, this metric idea was first coined in 1930 by Robertson and Walker. The FLRW universe metric has the main assumptions on which it is based, namely the principles of homogeneity and isotropic space [2][3]. In other words, there is an even distribution of matter throughout the universe, and the distance from the center of the universe to the distribution of matter in all directions will always be the same on a large scale. Based on the results of astronomical physical observations, the FLRW universe metric is a metric model that is most suitable to the real conditions of the universe [4]. The FLRW universe metric is formulated in 3+1 dimensional spacetime [5]. Where the evolution of the universe can be observed.

Related to the topic discussed in this article, technically formulate the metric of the FLRW universe in high dimensions. From the metric of the FLRW universe which is formulated in the spacetime dimension of 3+1, to the metric in the dimension of spacetime of 4+1. The addition of this dimension is based on the assumption that the laws of physics apply equally to the higher dimensions [6]. Which the essential goal is to ensure the evolution of the FLRW universe that takes place in high dimensions. The idea of the metric formulation of the FLRW universe in 4+1 dimensional spacetime in this article is to use the modeling of a spatial 4-dimensional isotropic sphere system that is immersed in a 5-dimensional spatial system. Then, Minkowski's flat spacetime concept was used to couple the spatial dimensions with the temporal dimensions.

This high-dimensional concept was inspired by what Kaluza did in 1921. Where Kaluza hypothesized that the universe is not only 3+1 dimensions of spacetime. Based on this hypothesis, Kaluza conducted a unification experiment between a gravitational entity and an electromagnetic entity in a 4+1 dimensional spacetime [7]. In 1926, Klein collaborated with Kaluza and contributed to the idea that the existence of higher dimensions exists on a microscopic scale. The dimensions of the extra space are hypothesized to be circular in a circle with a radius of a very small value [6]. This Kaluza-Klein idea became the hypothesis for the existence of a high-dimensional universe N+1 that world physicists now agree on. The purpose of this study is to prove that the expansion or contraction of the FLRW model of the universe is valid in high dimensions.

FORMALISM

Spatial 4 Dimension System Immersed in 5 Spatial Dimension System

Fundamental mathematical modeling of the metric of the FLRW universe, which is a 4-dimensional isotropic sphere system immersed in a 5-dimensional spatial system. To achieve this, depart from a vector function in Euclid's 5 spatial plane dimensions [8]:

$$x_i = x_1 + x_2 + x_3 + x_4 + x_5 \tag{1}$$

Then to get the metric vector function from Eq. (1), namely by using the differential method to Eq. (1). The differential form describing the metric vector function of Eq. (1) is written:

$$dx_i = dx_1 + dx_2 + dx_3 + dx_4 + dx_5 \tag{2}$$

Since the metric tensor function is the outer product of two metric vectors, the formula for the metric tensor function is expressed in [9]:

$$\begin{aligned} dx \otimes dx &= \eta^{\mu\nu} dx_\mu dx_\nu \\ (dx)^2 &= \eta^{\mu\nu} dx_\mu dx_\nu \end{aligned} \tag{3}$$

Where, the metric contravariance tensor $\eta^{\mu\nu}$ in the matrix representation is in the form of:

$$\eta^{\mu\nu} = \begin{pmatrix} \eta^{11} & \cdots & \eta^{1n} \\ \vdots & \ddots & \vdots \\ \eta^{m1} & \cdots & \eta^{mn} \end{pmatrix} \tag{4}$$

Based on Eq. (3), the metric tensor form of Eq. (2) can be formulated:

$$(dx_i)^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 + (dx_4)^2 + (dx_5)^2 \tag{5}$$

Then, the radius quantity of an isotropic sphere in 5 spatial dimensions can be represented as:

$$R^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 \tag{6}$$

By using the differential method of Eq. (6), a mathematical form is found:

$$0 = xdx_1 + xdx_2 + xdx_3 + xdx_4 + xdx_5 \tag{7}$$

Eq. (7) can be rewritten as:

$$dx_5 = -\frac{x_1 dx_1 + x_2 dx_2 + x_3 dx_3 + x_4 dx_4}{x_5} \tag{8}$$

Then, the results of the integral evaluation of Eq. (8) namely:

$$x_5 = -\frac{(x_1)^2 + (x_2)^2 + (x_3)^2 + (x_4)^2}{2x_5} \tag{9}$$

By substituting Eq. (9) into Eq. (5), we find the metric tensor of the FLRW universe in Cartesian coordinates:

$$(dx_i)^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 + (dx_4)^2 + \frac{1}{(2x_5)^2}\left[\sum_{j=1}^{4} d(x_j)^2\right]^2 \tag{10}$$

Spherical Coordinate Transformation Invariance

To get the transformation result from Eq. (10) in the form of a metric tensor in spherical coordinates, that is, it departs from the formulation of the fundamental transformation:

$$\begin{aligned} x_1 &= r\sin\theta\cos\phi\cos\alpha \\ x_2 &= r\sin\theta\sin\phi\cos\alpha \\ x_3 &= r\cos\theta\cos\alpha \end{aligned} \tag{11}$$

And:

$$r^2 = (x_1)^2 + (x_2)^2 + (x_3)^2 + (x_4)^2 \tag{12}$$

Then, for the formulation of transformation x_4 to spherical coordinates, starting from a 4-dimensional spatial vector function in Cartesian coordinates [10]:

$$(x_i)^2 = (x_1)^2 + (x_2)^2 + (x_3)^2 + (x_4)^2 \tag{13}$$

By substituting Eq. (11) into Eq. (13), it is obtained:

$$(x_i)^2 = (r\sin\theta\cos\phi\cos\alpha)^2 + (r\sin\theta\sin\phi\cos\alpha)^2 + (r\cos\theta\cos\alpha)^2 + (x_4)^2 \tag{14}$$

Based on Eq. (12) and Eq. (14) which is simplified by using the trigonometric theorem, a mathematical formulation is found:

$$
\begin{aligned}
r^2 &= r^2 \cos^2 \alpha[\sin^2 \theta(\cos^2 \phi + \sin^2 \theta) + \cos^2 \theta] + \left(x_4\right)^2 \\
r^2 &= r^2 \cos^2 \alpha(\sin^2 \theta + \cos^2 \theta) + \left(x_4\right)^2 \\
r^2 &= r^2 \cos^2 \alpha + \left(x_4\right)^2
\end{aligned} \tag{15}
$$

After the two sides are reduced by the term $r^2\cos^2\theta$, we find the transformation form for the extra dimension x_4 to the following spherical coordinates:

$$
\begin{aligned}
\left(x_4\right)^2 &= r^2(1-\cos^2\alpha) \\
\left(x_4\right)^2 &= r^2 \sin^2 \alpha \\
x_4 &= r \sin\alpha
\end{aligned} \tag{16}
$$

Then, by differentiating Eq. (11) and Eq. (16), we find the following line elements:

$$
\begin{aligned}
dx_1 &= \sin\theta\cos\phi\cos\alpha\ dr + r\cos\theta\cos\phi\cos\alpha\ d\theta \\
&+ (-r\sin\theta\sin\phi\cos\alpha)\ d\phi + (-r\sin\theta\cos\phi\sin\alpha)\ d\alpha
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
dx_2 &= \sin\theta\sin\phi\cos\alpha\ dr + r\cos\theta\sin\phi\cos\alpha\ d\theta \\
&+ r\sin\theta\cos\phi\cos\alpha\ d\phi + (-r\sin\theta\sin\phi\sin\alpha)\ d\alpha
\end{aligned} \tag{18}
$$

$$
\begin{aligned}
dx_3 &= \cos\theta\cos\alpha\ dr + (-r\sin\theta\cos\alpha)\ d\theta \\
&+ (-r\cos\theta\sin\alpha)\ d\alpha
\end{aligned} \tag{19}
$$

$$
dx_4 = \sin\alpha\ dr + r\cos\alpha\ d\alpha \tag{20}
$$

Based on the mathematical axiom of Eq. (3), the metric tensor function in spherical coordinates is given by the following set of formulations:

$$
\begin{aligned}
\left(dx_1\right)^2 &= \sin^2\theta\cos^2\phi\cos^2\alpha\ dr^2 + r^2\cos^2\theta\cos^2\phi\cos^2\alpha\ d\theta^2 \\
&+ r^2\sin^2\theta\sin^2\phi\cos^2\alpha\ d\phi^2 + r^2\sin^2\theta\cos^2\phi\sin^2\alpha\ d\alpha^2 \\
&- 2r\sin^2\theta\cos^2\phi\sin\alpha\cos\alpha\ drd\alpha - 2r^2\sin\theta\cos\theta\cos^2\phi\sin\alpha\cos\alpha\ d\theta d\alpha \\
&- 2r\sin^2\theta\sin\phi\cos\phi\cos^2\alpha\ drd\phi - 2r^2\sin\theta\cos\theta\sin\phi\cos\phi\cos^2\alpha\ d\theta d\phi \\
&+ 2r\sin\theta\cos\theta\sin\phi\cos\phi\cos^2\alpha\ drd\theta
\end{aligned} \tag{21}
$$

$$\begin{aligned}(dx_2)^2 &= \sin^2\theta\sin^2\phi\cos^2\alpha\ dr^2 + r^2\cos^2\theta\sin^2\phi\cos^2\alpha\ d\theta^2 \\ &+ r^2\sin^2\theta\cos^2\phi\cos^2\alpha\ d\phi^2 + r^2\sin^2\theta\sin^2\phi\sin^2\alpha\ d\alpha^2 \\ &- 2r\sin^2\theta\sin^2\phi\sin\alpha\cos\alpha\ drd\alpha - 2r^2\sin\theta\cos\theta\sin^2\phi\sin\alpha\cos\alpha\ d\theta d\alpha \\ &+ 2r\sin^2\theta\sin\phi\cos\phi\cos^2\alpha\ drd\phi + 2r^2\sin\theta\cos\theta\sin\phi\cos\phi\cos^2\alpha\ d\theta d\phi \\ &+ 2r\sin\theta\cos\theta\sin^2\phi\cos^2\alpha\ drd\theta - 2r^2\sin^2\theta\cos\theta\sin\phi\cos\phi\sin\alpha\cos\alpha\ d\phi d\alpha\end{aligned} \tag{22}$$

$$\begin{aligned}(dx_3)^2 &= \cos^2\theta\cos^2\alpha\ dr^2 + r^2\sin^2\theta\cos^2\alpha\ d\theta^2 + r^2\cos^2\theta\sin^2\alpha\ d\alpha^2 \\ &- 2r\sin\theta\cos\theta\cos^2\alpha\ drd\theta - 2r\cos^2\theta\sin\alpha\cos\alpha\ drd\alpha \\ &+ 2r^2\sin\theta\cos\theta\sin\alpha\cos\alpha\ d\theta d\alpha\end{aligned} \tag{23}$$

$$(dx_4)^2 = \sin^2\alpha\ dr^2 + r^2\cos^2\alpha\ d\alpha^2 + 2r\sin\theta\cos\alpha\ drd\alpha \tag{24}$$

Then, by substituting Eq. (11), (16), (21-24) into Eq. (10), we get the formalism of the metric tensor in the following simplification results:

$$(dx_i)^2 = R^2(t)\left(\frac{1}{1-Mr^2}\ dr^2 + r^2\cos^2\alpha\sin^2\theta\ d\phi^2 + r^2\cos^2\alpha\ d\theta^2 + r^2\ d\alpha^2\right) \tag{25}$$

Where:

$$R(t) = r^2 + (x_5)^2 \tag{26}$$

And the curvature constant *M* which represents the shape of the space:

$$M = \begin{cases} 1 \leftrightarrow \text{Positive Curvature Space} \\ 0 \leftrightarrow \text{Flat Space} \\ -1 \leftrightarrow \text{Negative Curvature Space}\end{cases} \tag{27}$$

The relation between Eq. (25) and Eq. (27) is obtained from positive R^2 in Eq. (6) which can be converted into negative R^2. If $M = 0$ is determined, then a 4-dimensional isotropic spherical system is found immersed in a 5-dimensional Euclid system. Then to couple the temporal dimension with the spatial dimension, which is based on Minkowski's flat spacetime metric in the 4+1 dimension [9]:

$$(ds)^2 = -(c\ dt)^2 + (dx_i)^2 \tag{28}$$

By substituting Eq. (25) into Eq. (28), we find the metric of the universe FLRW in the spacetime dimension 4+1 at the spherical coordinates:

$$(ds)^2 = -(c\ dt)^2 + R^2(t)\left[\frac{1}{1+Mr^2}\ dr^2 + r^2\left(\cos^2\alpha\sin^2\theta\ d\phi^2 + \cos^2\alpha\ d\theta^2 + d\alpha^2\right)\right] \tag{29}$$

RESULT

This formalism yields the parameters $R(t)$ and M, which are the cosmic scale factor and the spatial curvature constant. For this context, the cosmic scale factor $R(t)$ is the sum of the radius r^2 of the spatial 4-dimensional system with the radius quantity in the spatial dimension $(x_5)^2$, and the cosmic scale factor $R(t)$ is a time-evolving quantity. The first significant difference between the metrics of the FLRW universe in dimensions 3+1 and 4+1 lies in the scale factor $R(t)$. The scale factor $R(t)$ in dimension 3+1 satisfies $r^2 + (x_4)^2$, with the definition of radius r^2 as $(x_1)^2 + (x_2)^2 + (x_3)^2$ [8]. Meanwhile, the scale factor $R(t)$ in the 4+1 spacetime dimension fulfills Eq. (26). Therefore, the scale factor $R(t)$ can be interpreted as a radius quantity with a dimension that is relatively higher than the FLRW metric spatial dimension, to parameterize the radial evolution of the universe. In other words, metric expansion or contraction remains valid in the 4+1 dimension. Based on the pattern of comparisons between the 3+1 and 4+1 dimensional metrics, it might be intuitive to expect that in the other higher dimensions the evolution of the metric will hold over time. The second significant difference lies in the $R^2r^2d\alpha^2$ term in Eq. (29), where the $R^2r^2d\alpha^2$ term only exists in the 4+1 spacetime dimension. It means that there is a radius quantity in the extra dimension $d\alpha^2$ which is orthogonal to the other dimensions. The final significant difference between the metric of the FLRW universe in the dimensions 3+1 and 4+1 is at the angle $\cos^2\alpha$ formed by the extra dimension $d\alpha^2$ with dimension $d\phi^2$ and dimension $d\theta^2$. Where this does not exist in the 3+1 dimension. Based on the results of formalism, the addition of this dimension does not have an impact on the curvature constant of space M. In other words, the curvature constant of space M applies generally to each dimension.

CONCLUSION

Based on the results obtained from this research, it can be concluded that the addition of dimensions in the metric formalism of the FLRW universe has an impact on the radius r^2 in the scale factor $R(t)$ which experiences an increase in the radius $(x_5)^2$ quantity. Thus, it has implications for the dimension of the cosmic scale factor which will always be relatively higher than the spatial dimension of the metric tensor. In other words, the expansion or contraction of the FLRW universe model remains valid in high dimensions. Also, the metric tensor of the FLRW universe has the addition of the extra-dimensional term $R^2r^2d\alpha^2$ and the angle $\cos^2\alpha$ which are formed by the extra dimension $d\alpha^2$ with dimensions $d\phi^2$ and dimensions $d\theta^2$. As an application of this 4+1 dimensional FLRW universe metric, it can be used as an asymptotic spacetime for the 4+1 dimensional Schwarzschild spacetime and the 4+1 dimensional Reissner-Nordström spacetime**.**

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REFERENCES

[1] H. Alatas, A. K. Falah, T. Wibowo, M. A. Qohhar, and B. E. Gunara, "Coordinate-time lapse function of FLRW accelerated expanding universe in dRGT massive gravity theory," *Int. J. Mod. Phys. D*, vol. 29, no. 10, 2020, doi: 10.1142/S021827182050073X.

[2] B. L'Huillier and A. Shafieloo, "Model-independent test of the FLRW metric, the flatness of the Universe, and non-local estimation of H0rd," *J. Cosmol. Astropart. Phys.*, vol. 2017, no. 1, 2017, doi: 10.1088/1475-7516/2017/01/015.

[3] Q. Wang, "Reformulation of the Cosmological Constant Problem," *Phys. Rev. Lett.*, vol. 125, no. 5, 2020, doi: 10.1103/PhysRevLett.125.051301.

[4] S. Cao *et al.*, "Direct test of the FLRW metric from strongly lensed gravitational wave observations," *Sci. Rep.*, vol. 9, no. 1, 2019, doi: 10.1038/s41598-019-47616-4.

[5] H. het Lam and T. Prokopec, "Singularities in FLRW spacetimes," *Phys. Lett. Sect. B Nucl. Elem. Part. High-Energy Phys.*, vol. 775, 2017, doi: 10.1016/j.physletb.2017.10.070.

[6] S. Weinberg, "Massless particles in higher dimensions," *Phys. Rev. D*, vol. 102, no. 9, 2020, doi: 10.1103/PhysRevD.102.095022.

[7] J. Zhu *et al.*, "X-ray reflection spectroscopy with Kaluza–Klein black holes," *Eur. Phys. J. C*, vol. 80, no. 7, 2020, doi: 10.1140/epjc/s10052-020-8198-x.

[8] R. Anugraha, *Pengantar Teori Relativitas dan Kosmologi*. Gadjah Mada University Press, 2005.

[9] I. L. Zulfa, R. P. Alamsyah, and A. Fitriani, "Solusi Metrik Kerr-Newman Untuk Dimensi Ruang-waktu 3+1 Dengan Invariansi Rotasi Anisotropik," *Al-Fiziya J.*

Mater. Sci. Geophys. Instrum. Theor. Phys., vol. 3, no. 2, 2020, doi: 10.15408/fiziya.v3i2.18600.

[10] I. L. Saptiani, "Solusi geometri schwarzschild untuk dimensi 4+1 dengan invarian rotasi isotropik," 2019.