DIDACTICAL DESIGN FOR OVERCOMING STUDENTS’ LEARNING OBSTACLES ON THE INVERSE FUNCTION CONCEPT

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Abstract

The inverse function in secondary schools is still a difficult concept to learn. Meanwhile, in national exams or competitions this problem often arises. Didactic design is an attempt to improve the learning process. The purpose of this study is to identify the epistemological obstacle, to develop the learning designs, and to describe the students’ responses regarding the implementation of inverse function’s concept learning designs in the class. This research was held in one of Senior High School in South Tangerang of 38 people from class X. The research method is Didactical Design Research (DDR). This method is conducted from three stages, prospective analysis, metapedia didactic analysis, and retrospective analysis. The result of the study was indicated that the student’s obstacles are according to predictions and didactical design still generates some new epistemological obstacle. The revised didactical design was obtained by updating the initial didactical design to resolve the obstacle. The revised didactical design includes choosing vocabulary, adding new instructions, and expanding predictions and anticipating student responses.

Keywords: didactical design; epistemological obstacle; hypothetical learning trajectory; inverse function

Abstrak


Kata kunci: desain didaktis; hambatan epistemologis; lintasan belajar hipotetis; fungsi invers

How to Cite

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Introduction

The main purpose of learning is to get new information that will be useful for himself and others in the future. Likewise, learning mathematics, learning mathematics is expected to improve the ability of students in various aspects, namely reasoning, critical thinking, logical thinking, creative thinking, active thinking, and systematic thinking that can be used in solving every problem. Mathematics is one of the fields of study which is considered as a sufficient condition to continue education to the next level.

One of the materials contained in the high school mathematics curriculum is a function, whose existence is highly considered because many concepts must be understood as prerequisites for further material. But in reality and based on evidence in the field, there are still students who have difficulty in understanding the concept of functions, especially inverse functions. Learning difficulties or obstacles experienced by students are known as learning obstacles. Brousseau (2002) revealed three types of learning obstacles, namely onto genic obstacle (mental readiness of learning), didactical obstacle (due to teacher’s way of teaching), and epistemological obstacle (learners’ knowledge that has a limited application context). Obstacle learning occurs because of a lack of deepening in understanding mathematical concepts.

The concept of inverse functions is a compulsory mathematical material in High Schools. The inverse function material appears every year in the National Examination therefore students are required to understand the concept of inverse functions well. Based on the average results of the National Mathematics Examination in 2017 and 2018, the results of the National High School Mathematics National Examination in the City of South Tangerang have decreased. From 79 public and private high schools in South Tangerang City, the average of mathematics examination in 2017 is 64.84 and in 2018 is 64.04 (Puspendik, 2018). This shows that students still have difficulty in solving mathematics problems. In addition, the percentage of students’ ability to answer questions correctly in the Mathematics National Examination, especially the inverse function material at the provincial level was 35.39% and the national level was 34.66% (Puspendik, 2018). This shows that the mastery of students related to the concept of the inverse function is not optimal.

Related to students’ difficulties in learning, there are three types of learning obstacles according to Brousseau (2002), namely ontogenetic obstacles (mental readiness of learning), didactical obstacles (due to teacher way of teaching), and epistemological obstacles (learners’ knowledge that has a limited application context). Ontogenetic obstacles arise because of students’ self-limitations. This is caused by the development of mental attitudes and cognitive abilities of students who grow according to certain age stages. Brousseau (2002) explained that a child will acquire knowledge according to abilities and goals that vary depending on the development of his age. Therefore, it is important for an educator to know the age of a student’s mental development in order to adjust the material and learning methods that will be used with students’ self-abilities.

Didactic obstacles relate to the teaching methods of the teacher. This can be assessed from the way of delivering the material, the textbooks used, and the creation of a learning situation by the teacher (Brousseau, 2002). One example of aspects that includes didactical obstacles is the choice of language and the speed of the teacher when delivering subject matter. In addition, the selection of teaching materials in the form of textbooks used can also be material for next analysis.
Epistemological obstacles are related to concept understanding. It is very possible that in the learning process there are errors of understanding by students of the material being studied. This implies a certain understanding (misconception) that will cause obstacles to understanding other concepts. Furthermore, Hercovics identified the epistemological obstacles consisting of several aspects: 1) Deception of intuitive experience, 2) tendency to generalize and 3) use of natural language (Setiawati, 2011). Ruli, Prabawanto, and Mulyana (2019) in their research found that the majority of students had difficulty in determining the x and y coordinate points on the topic of a quadratic form. This shows that epistemological difficulties in students are still an important concern to improve.

Meanwhile, according to Conru (Setiawati, 2011), there are four learning obstacles, namely: cognitive obstacles that occur during the learning process, genetic and psychological obstacles that occur due to students’ personal development, didactical obstacles that occur because of effective teaching methods, and epistemology obstacles that occurs because of the nature of mathematical concepts.

As professional educator, teachers are expected to be able to identify, analyze, and overcome obstacles experienced by students. This is not possible in a short time and requires careful preparation before the learning process is carried out. Efforts can be conducted to reduce epistemological obstacle experienced by students in the inverse function concept by creating learning designs that are based on the Hypothetical Learning Trajectory (HLT) which contains assignments, students’ response predictions, and anticipation of response predictions, and Student Activity Sheets (SAS). The didactical design was developed by considering several learning theories, including Piaget’s Theory (schemata, assimilation, accommodation, and equilibration), Thorndike’s Theory (the law of readiness, the law of practice, and the law of effect), Ausubel’s theory (meaningful learning), and Vygotsky’s theory (scaffolding and social interaction).

The problems studied include: learning obstacle related to the inverse function concept, the didactical design developed to overcome students’ epistemological obstacles to the inverse function concept, students’ responses to the implementation of didactical design of the inverse function concept during the learning process, and didactical design revision of the concept of an inverse function.

**Method**

The method used in this research is qualitative research by applying Didactical Design Research. Suryadi (2013) stated that didactical design research consisted of three stages, namely: 1) prospective analysis, 2) metapedia didactic analysis, and 3) retrospective analysis. The stages in this research are as follows:

- **Prospective Analysis** (Didactical situation analysis stage before learning): a) Determine the scope of mathematical concepts to be used in research, namely the inverse function concept; b) Study, understand, and analyze the material inverse functions; c) Perform repersonalization, namely analyzing textbooks used during the learning process in class related to the inverse function concept and understanding previous research that examines learning difficulties that have been found; d) Develop and consult the learning obstacle test instruments experienced by class XI students at one of state high school and one of private high school in South Tangerang city; e) Conduct the learning obstacle tests and conduct semi-structured interviews to students who
have studied the inverse function material; f) Conduct the observations, documentation, and interviews with subject teachers related to their experiences during the learning process of inverse function material; g) Analyze the results of learning obstacle tests and interviews to identify learning obstacle on inverse functions material; h) Conducting recontextualization, which is making an alternative learning trajectory based on the analysis of the learning obstacle test and analysis of various material sources; i) Develop and consult the initial didactic designs that have been made to people who are experts in their fields. Didactical design is made according to the identified learning obstacle; j) Make predictions of students' responses that are likely to arise during didactical design implementation and anticipate responses that arise.

- Metapedaddidactic analysis: a) Implement a didactical design that has been prepared for class X; b) Analyze situations, student responses, and anticipate student responses when didactical designs are implemented.

- Retrospective analysis: a) Connect the results of the didactical situation analysis of the hypothetical learning trajectory with the analysis of the metapedia didactic; b) Analyze the emergence of learning obstacles that have been identified previously; c) Make improvements to the didactical design flaws that have been prepared previously. d) Compile the final report of the research.

Techniques of data collection in this study are diagnostic tests, observations, interviews, and documentation. The diagnostic test conducted in research is a learning obstacle test to find out the difficulties associated with students' conceptual understanding. Observations were made directly on the learning process in the classroom when implementing didactical designs. An interview is a process of taking data by asking questions to respondents consisting of subject matter teachers and students. Documentation is the collection of data obtained by the researcher himself about his actions and experiences, which can be in the form of learning videos, photos, during the learning process, and other relevant data that aims to obtain objective data.

Results and Discussions

The didactic design development process is carried out through three phases that are in accordance with the teacher's thinking process, namely before learning, during learning, and after learning (Suryadi, 2013). In the first phase, a prospective analysis is carried out before carrying out the learning process. The second phase, implementing the metapedia didactic analysis is carried out during the learning process. In the third phase, a retrospective analysis is carried out after carrying out the learning process and results in a didactical design revision.

Prospective Analysis

The curriculum is the part that is involved in conducting a prospective analysis. The school curriculum currently used in Indonesia is the Curriculum 2013. In relation to mathematics, research conducted by Suryadi et al. (2019) found that numerous obstacles were found in the implementation of the Curriculum 2013.

The prospective analysis consists of three analyzes, namely learning obstacle analysis, re-personalization and re-contextualization, and the development of didactical designs related to the concept of inverse functions. Didactical design is prepared by considering the results of the analysis.
of learning obstacles and learning trajectories of students. Based on the results of the learning obstacle test conducted on 50 students in two high schools in the City of South Tangerang, as many as 51% of all students experience learning obstacle that is epistemological in the concept of an inverse function. Analysis of the learning obstacle test results in the classification of learners’ obstacles to the inverse function concept as follows: 1) Learners have difficulty describing inverse relations; 2) Learners have difficulty on concluding the inverse function definition correctly; 3) Learners have difficulty on determining the inverse function of mathematical models on contextual problems; 4) Learners have difficulty on determining the inverse function of linear and fraction functions (rational); 5) Students have difficulty on determining the composition inverse function \((f \circ g)^{-1}(x)\) and \((g \circ f)^{-1}(x)\); 6) Students have difficulty on determining the composition inverse function \(f^{-1}(x) \circ g^{-1}(x)\) and \(g^{-1}(x) \circ f^{-1}(x)\).

Based on the exploration of concepts that have been carried out and the results of the analysis of the learning obstacle test, an alternative learning trajectory is obtained as follows figure 1.

![Learning Trajectory of Inverse Function Concepts](image)

Figure 1. Learning Trajectory of Inverse Function Concepts

After re-personalizing and re-contextualizing, a didactical design was arranged which would produce a hypothetical didactical design in the form of a Hypothetical Learning Trajectory (HLT). The HLT that has been compiled will produce a Student Activity Sheet (SAS).

Based on the learning trajectory inverse function that has been formed, the first concept to be studied is the concept of the inverse function definition and algebraic inverse function. The concept of inverse function definition starts with the problem of determining the domain and code and the range of a linear function and determining its inverse. Problems given to students will help to get an inverse function definition. After that students will learn the concepts of algebraic functions of linear and linear fractions (rational). Students are given problems that can be found in everyday life. On this issue, the researchers chose the context of "the benefits of sending out of town pottery". The problem given will cause students to learn to determine the general form of its function. After that students will determine the inverse. Furthermore, students are given the problem of linear functions directly informal form and determine the inverse function using the concepts they have.

After learning the concept of an algebraic inverse function, students will enter the concept of inverse composition function. In this concept, students will determine the inverse function of the composition of two functions and three functions. Before entering the concept of inverse function composition, students need to understand the concept of algebraic inverse function completely. If students still experience errors, it will cause students to have difficulty understanding the concept of inverse composition function. In this concept, students will indirectly learn the nature of the inverse composition function. Furthermore, students will learn the nature of inverse functions and draw a graph of functions and graphs of inverse functions. Students will find the nature of the inverse function based on the problem they have solved. Learners will also discover the nature of the inverse function graph from the problem of depicting a given graph.

The initial didactic designs were developed on the concept of inverse function as many as
three didactic designs, each of which discussed the concept of an algebraic inverse function, the concept of inverse composition function, and the nature of the inverse function and the inverse function graph. Overall, each learning situation created involves all learning theories that are interrelated with one another.

**Metapedia Didactic Analysis**

Metapedia Didactic analysis is an analysis of the results of observations made at the time of implementing a didactical design. Metapedadidactic analysis is carried out based on didactical design and the students’ possible responses accompanied by their anticipation of the learning process. Student responses to the implementation of the didactical design on the concept of the inverse function as a whole in accordance with predictions and anticipation of responses that have been made previously. However, there are some new difficulties that were not predicted beforehand. New anticipation is given spontaneously to some new difficulties that occur and some difficulties whose anticipations are less relevant during the learning process. The learning design used was made in 8 recorded situations.

In situation 1, all difficulties can be anticipated according to predictions that have been made before. In situation 1 also there are no new difficulties that arise beyond predictions. However, all students are able to resolve situation 1 appropriately according to the expected response if given scaffolding. This shows that the didactic design of situation 1 is considered not optimal and some changes must be made to the assignment to be able to overcome the difficulties experienced by students optimally. After that, students together confirm all answers to situation 1, so that all students do not experience misconceptions and better understand the concept of inverse multiplication function.

In situation 2, students are asked to determine the domain, code, and range of functions, then determine the inverse to understand the inverse function of the sum form. In this assignment, some difficulties and anticipations are predicted. However, all predictions do not occur in students. This is because in situation 2, an assignment is almost the same as situation 1, so students do not experience difficulties and are able to complete the assignment appropriately. Learners already understand the basic concepts of the inverse addition function.

In situation 3, all difficulties experienced and anticipations were carried out in accordance with predictions made previously. However, there are difficulties that occur outside of predictions. Difficulties experienced by students in situation 4 have been predicted beforehand so that these difficulties can be anticipated optimally. Students are able to determine $F^{-1}(x)$ correctly. However, students can complete the assignment of situation 4 by being given scaffolding. This shows that situation 4 in the SAS is not optimal and must be improved in order to get an optimal didactical design.

Difficulties in Situation 5 also correspond to predictions made previously. In addition, no new difficulties were found in the assignment of situation 1. After completing the assignment in SAS of situation 1, students were asked to make conclusions related to the nature of the formula of the inverse composition function in general. In the assignment in situation 6, there are some difficulties experienced by students. These difficulties are anticipated according to predictions made previously. In situation 6, there are five prediction difficulties that do not arise. However, there are new difficulties beyond predictions.

In situation 7, all difficulties experienced by students and their anticipation are in accordance with predictions that have been made previously.
However, there is a new difficulty beyond prediction. In situation 8, there are some
difficulties experienced by students in completing
the assignments given.

Retrospective Analysis

Retrospective analysis is the stage where
researchers conduct a didactical design analysis
that has been implemented in the classroom and
examines the results of a hypothetical didactical
situation analysis with the results of the
metapedia didactic analysis. This analysis will
produce an empirical didactical design which is a
revision of the initial didactical design. The
following is a recapitulation table for changes in
the Hypothetical Learning Trajectory concept of
inverse functions.

Table 1. Change of Hypothetical Learning Trajectory

<table>
<thead>
<tr>
<th>Hypothetical Learning Trajectory (HLT)</th>
<th>Revised Learning Trajectory (RLT)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Didactical Situation 1</strong></td>
<td>-Modification and addition of assignments related to review of function material. -Expansion of students’ response predictions and anticipations.</td>
</tr>
<tr>
<td>Students are asked to determine the domain, codomain, and the range based on the function formula given, and map it into diagram. Students are asked to map and determine the direction of the arrow on the inverse function diagram.</td>
<td></td>
</tr>
<tr>
<td><strong>Didactical Situation 2</strong></td>
<td>-Modify and addition of assignment sentences to “Base on assignments in situations 1 and 2 that you have completed, by assuming member A as x and member B as y write the definition of the inverse function formally.” -Expansion of students’ response predictions and anticipations.</td>
</tr>
<tr>
<td>Students are asked to conclude the formal definition of inverse functions.</td>
<td></td>
</tr>
<tr>
<td><strong>Didactical Situation 3</strong></td>
<td>-Addition of illustrations related to model the function of contextual problems. -Modification of assignments to contextual issues that are given.</td>
</tr>
<tr>
<td>Students are asked to write a mathematical model/general formula of the given story problem and understand the purpose of cost per-tons and shipping. Students are asked to determine the inverse function of the general formula in situation 3.</td>
<td></td>
</tr>
<tr>
<td><strong>Didactical Situation 4</strong></td>
<td>-Adding new instructions that make students reasoned in finding the inverse function. -Expansion of students’ response predictions and anticipations.</td>
</tr>
<tr>
<td>Students are asked to change the function of</td>
<td></td>
</tr>
</tbody>
</table>

Based on research that has been done before, there are several research results related to
students’ learning difficulties in the subject
matter of the inverse function under study.
Perbowo and Anjarwati (2017) in their research
revealed that there are four types of errors that occur in the process of learning inverse functions, namely students’ mistakes in understanding the concept of material functions and inverse functions, procedural errors in problem-solving, errors in the calculation process, and errors in making conclusions. Waluyo and Surya (2016) in their research stated the difficulties experienced by students in solving the problem of composition and inverse function include the following: 1) Students are not able to understand the purpose of the problem given and do not understand the concept of algebraic form operation on functions, 2) Students do not replace the variable x with the variable y when solving problems, 3) Students are less thorough in performing algebraic form operations on functions. Muzaffar (2013) in his research revealed that misconception experienced by students occur when students understand that $f^{-1}$ must always represent a function, so they believe that there is no inverse mapping of a function if they cannot determine the inverse function.

Based on the research described above, learning obstacles experienced by students who are the objects of this study are on the topic of inverse functions caused by students' lack of understanding of the initial concept of a material function and composition functions, so that students have difficulty in solving inverse function problem, both algebraic inverse function and composition inverse function. Learners also assume the inverse function and the inverse of a function are the same things. In addition, students are less skilled and thorough in solving problems of composition functions, inverse functions, and inverse composition functions, so students cannot write answers correctly. These mistakes cause students to be unable to make conclusions about the definition of inverse functions and the properties of inverse functions appropriately. Learner constructs meaning based on the initial knowledge they have and then transform it into a new concept (Maudy, 2016). However, the classroom situation will affect the students. Therefore, the teacher’s ability to convey learning to students is very important in reducing learning obstacles. This is in line with the results of the research by Maksun et al. (2020), that the knowledge that students acquire in class is not only from didactic situations but also from explanations given by the teacher directly.

In general, the framework in the initial didactical design underwent minor changes with modifications such as vocabulary selection, changes in the form of assignments, expansion of prediction difficulties and anticipation of students’ responses to avoid new difficulties that arise outside of predictions and management of learning time. The follow up of this research is the initial didactical design improvement made by researchers to make it easier for students to understand the concept of inverse functions.

**Conclusions**

The initial didactical design was developed based on the results of the analysis of the learning obstacle by taking into account the hypothetical learning trajectory of students. The didactical design concept of an inverse function consists of three learning activities which generally have the following forms: didactical design of algebraic inverse functions developed to overcome the difficulties of students in understanding the concept of inverse functions of linear and rational functions; didactical design of inverse composition functions specifically developed to overcome the difficulties of students in understanding the concept of inverse function composition of two functions and inverse function composition of three functions; didactical design of the inverse function properties developed to deepen students’ understanding of algebraic inverse functions and
inverse composition functions as well as graphs of inverse functions.

Students’ responses to the implementation of the didactical design concept of the inverse function as a whole in accordance with predictions and anticipation of responses that have been made previously. New anticipation is given spontaneously to some new difficulties that occur and some difficulties whose anticipations are less relevant during the learning process. Didactical design revisions made to the initial didactical design include vocabulary selection, the addition of instructions to assignments, expansion of predictions and anticipation of student responses, and management of learning time.

References


